HARMONIC MATERIALS OF MODERN MUSIC

RESOURCES OF THE TEMPERED SCALE

HOWARD HANSON

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To my dear wife, Peggie,
who loves music but does not
entirely approve of the twelve-tone scale,
this book is affectionately dedicated.

Preface

This volume represents the results of over a quarter-century of study of the problems of the relationships of tones. The conviction that there is a need for such a basic text has come from the author's experience as a teacher of composition, an experience which has extended over a period of more than thirty-five years. It has developed in an effort to aid gifted young composers groping in the vast unchartered maze of harmonic and melodic possibilities, hunting for a new "lost chord," and searching for an expressive vocabulary which would reach out into new fields and at the same time satisfy their own esthetic desires.

How can the young composer be guided in his search for the far horizons? Historically, the training of the composer has been largely a matter of apprenticeship and imitation; technic passed on from master to pupil undergoing, for the most part, gradual change, expansion, liberation, but, at certain points in history, radical change and revolution. During the more placid days the apprenticeship philosophy—which is in effect a study of styles—was practical and efficient. Today, although still enormously important to the development of musical understanding, it does not, *by itself*, give the young composer the help he needs. He might, indeed, learn to write in the styles of Palestrina, Purcell, Bach, Beethoven, Wagner, Debussy, Schoenberg, and Stravinsky and still have difficulty in coming to grips with the problem of his own creative development. He needs a guidance which is more basic, more concerned with a study of the *material* of the art and

less with the *manner* of its use, although the two can never be separated.

This universality of concept demands, therefore, an approach which is radical and even revolutionary in its implications. The author has attempted to present here such a technic in the field of tonal relationship. Because of the complexity of the task, the scope of the work is limited to the study of the relationship of tones in melody or harmony without reference to the highly important element of rhythm. This is not meant to assign a lesser importance to the rhythmic element. It rather recognizes the practical necessity of isolating the problems of tonal relationship and investigating them with the greatest thoroughness if the composer is to develop a firm grasp of his tonal vocabulary.

I hope that this volume may serve the composer in much the same way that a dictionary or thesaurus serves the author. It is not possible to bring to the definition of musical sound the same exactness which one may expect in the definition of a word. It *is* possible to explain the derivation of a sonority, to analyze its component parts, and describe its position in the tonal cosmos. In this way the young composer may be made more aware of the whole tonal vocabulary; he may be made more sensitive to the subtleties of tone fusion; more conscious of the tonal alchemy by which a master may, with the addition of one note, transform and illuminate an entire passage. At the same time, it should give to the young composer a greater confidence, a surer grasp of his material and a valid means of self-criticism of the logic and consistency of his expression.

It would not seem necessary to explain that this is not a "method" of composition, and yet in these days of systems it may be wise to emphasize it. The most complete knowledge of tonal material cannot create a composer any more than the memorizing of Webster's dictionary can produce a dramatist or poet. Music is, or should be, a means of communication, a vehicle for the expression of the inspiration of the composer. Without that inspiration, without the need to communicate, without—in other

words—the creative spirit itself, the greatest knowledge will avail nothing. The creative spirit must, however, have a medium in which to express itself, a vocabulary capable of projecting with the utmost accuracy and sensitivity those feelings which seek expression. It is my hope that this volume may assist the young composer in developing his own vocabulary so that his creative gift may express itself with that simplicity, clarity, and consistency which is the mark of all great music.

Since this text differs radically from conventional texts on "harmony," it may be helpful to point out the basic differences together with the reason for those differences.

Traditional theory, based on the harmonic technics of the seventeenth, eighteenth, and nineteenth centuries, has distinct limitations when applied to the music of the twentieth—or even the late nineteenth—century. Although traditional harmonic theory recognizes the twelve-tone equally tempered scale as an underlying basis, its fundamental scales are actually the seven-tone major and minor scales; and the only chords which it admits are those consisting of superimposed thirds within these scales together with their "chromatic" alterations. The many other combinations of tones that occur in traditional music are accounted for as modifications of these chords by means of "non-harmonic" tones, and no further attempt is made to analyze or classify these combinations.

This means that traditional harmony systematizes only a very small proportion of all the possibilities of the twelve-tones and leaves all the rest in a state of chaos. In contemporary music, on the other hand, many other scales are used, in addition to the major and minor scales, and intervals other than thirds are used in constructing chords.

I have, therefore, attempted to analyze *all* of the possibilities of the twelve-tone scale as comprehensively and as thoroughly as traditional harmony has analyzed the much smaller number of chords it covers. This vast and bewildering mass of material is classified and thus reduced to comprehensible and logical order

chiefly by four devices: interval analysis, projection, involution, and complementary scales.

Interval analysis is explained in Chapter 2 and applied throughout. All interval relationship is reduced to six basic categories: the perfect fifth, the minor second, the major second, the minor third, the major third, and the tritone, each—except the tritone—considered in both its relationship *above* and *below* the initial tone. This implies a radical departure from the classic theories of intervals, their terminology, and their use in chord and scale construction. Most of Western music has for centuries been based on the perfect-fifth category. Important as this relationship has been, it should not be assumed that music based on other relationships cannot be equally valid, as I believe the examples will show.

Projection means the construction of scales or chords by any logical and consistent process of addition and repetition. Several types of projection are employed in different sections of the book.

If a series of specified intervals, arranged in a definite *ascending* order, is compared with a similar series arranged in *descending* order, it is found that there is a clear structural relationship between them. The second series is referred to here as the *involution* of the first. (The term *inversion* would seem to be more accurate, since the process is literally the "turning upsidedown" of the original chord or scale. It was felt, however, that confusion might result because of the traditional use of the term *inversion*.)

The relation of any sonority and its involution is discussed in Chapter 3, and extensively employed later on.

Complementary scales refer to the relationship between any series of tones selected from the twelve-tones and the other tones which are omitted from the series. They are discussed in Parts V and VI. This theory, which is perhaps the most important—and also the most radical—contribution of the text, is based on the fact that every combination of tones, from two-tone to six-tone, has its complementary scale composed of similar proportions of the same intervals. If consistency of harmonic-melodic expression is important in musical creation, this theory should bear the most

intensive study, for it sets up a basis for the logical expansion of tonal ideas once the germinating concept has been decided upon in the mind of the composer.

The chart at the end of the text presents graphically the relationship of all of the combinations possible in the twelve-tone system, from two-tone intervals to their complementary ten-tone scales.

I must reiterate my passionate plea that this text not be considered a "method" nor a "system." It is, rather, a compendium of harmonic-melodic material. Since it is inclusive of all of the basic relationships within the twelve-tones, it is hardly likely that any composer would in his lifetime use all, or even a large part, of the material studied. Each composer will, rather, use only those portions which appeal to his own esthetic taste and which contribute to his own creative needs. Complexity is no guarantee of excellence, and a smaller and simpler vocabulary used with sensitivity and conviction may produce the greatest music.

Although this text was written primarily for the composer, my colleagues have felt that it would be useful as a guide to the analysis of contemporary music. If it is used by the student of theory rather than by the composer, I would suggest a different mode of procedure, namely, that the student study carefully Parts I and II, Chapters I to 16, without undertaking the creative exercises—although if there is sufficient time the creative exercises will enlighten and inform the theorist as well as the composer.

During the first part of this study he should try to find in the works of contemporary composers examples of the various hexad formations discussed. He will not find them in great abundance, since contemporary composers have not written compositions primarily to illustrate the hexad formations of this text! However, when he masters the theory of complementary scales, he will have at his disposal an analytical technic which will enable him to analyze factually any passage or phrase written in the twelve-tone equally tempered scale.

H. H.

Rochester, New York

Acknowledgments

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His warm thanks go to the various music publishers for their generous permission to quote from copyrighted works and finally and especially to Appleton-Century-Crofts for their co-operation and for their great patience.

Finally, my devoted thanks go to my hundreds of composition students who have borne with me so loyally all these many years.

H. H.

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HARMONIC MATERIALS OF MODERN MUSIC

1

Equal Temperament

Since the subject of our study is the analysis and relationship of all of the possible sonorities contained in the twelve tones of the *equally tempered* chromatic scale, in both their melodic and harmonic implications, our first task is to explain the reasons for basing our study upon that scale. There are two primary reasons. The first is that a study confined to equal temperament is, although complex, a *finite* study, whereas a study of the theoretical possibilities within just intonation would be infinite. A simple example will illustrate this point. If we construct a major third, E, above C, and superimpose a second major third, G♯, above E, we produce the sonority C-E-G♯. Now if we superimpose yet another major third above the G♯, we reach the tone B♯. In equal temperament, however, B♯ is the *enharmonic* equivalent of C, and the four-tone sonority C-E-G♯-B♯ is actually the three tones C-E-G♯ with the lower tone, C, duplicated at the octave. In just intonation, on the contrary, B♯ would not be the equivalent of C. A projection of major thirds above C in *just* intonation would therefore approach infinity.

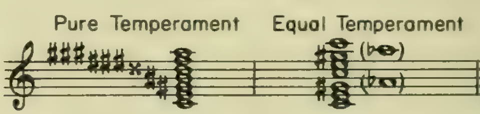
The second reason is a corollary of the first. Because the pitches possible in just intonation approach infinity, just intonation is not a practical possibility for keyboard instruments or for keyed and valve instruments of the woodwind and brass families. Just intonation would be possible for stringed instruments, voices, and one brass instrument, the slide trombone. However, since much of our music is concerted, using all

of these resources simultaneously, and since it is unlikely that keyboard, keyed, and valve instruments will be done away with, at least within the generation of living composers, the system of equal temperament is the logical basis for our study.

Another advantage of equal temperament is the greater simplicity possible in the symbolism of the pitches involved. Because enharmonic equivalents indicate the same pitch, it is possible to concentrate upon the *sound* of the sonority rather than upon the complexity of its spelling.

Referring again to the example already cited, if we were to continue to superimpose major thirds in just intonation we would soon find ourselves involved in endless complexity. The major third above B♯ would become D double-sharp; the major third above D double-sharp would become F triple-sharp; the next major third, A triple-sharp; and so on. In equal temperament, after the first three tones have been notated—C-E-G♯—the G♯ is considered the equivalent of A♭ and the succeeding major thirds become C-E-G♯-C, merely octave duplicates of the first three.

Example 1-1

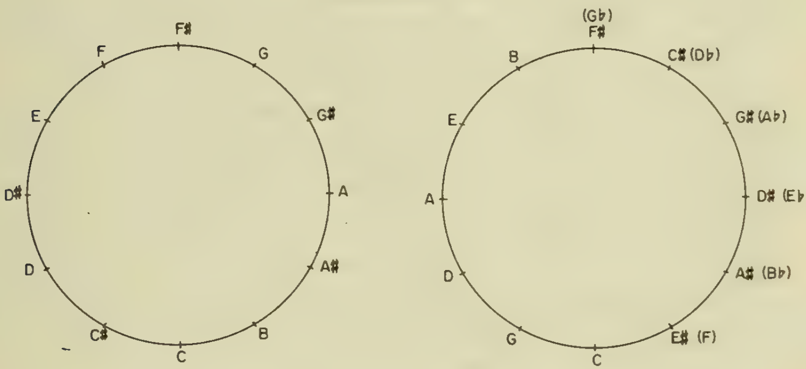


This point of view has the advantage of freeing the composer from certain inhibiting preoccupations with academic symbolization as such. For the composer, the important matter is the *sound* of the notes, not their "spelling." For example, the sonority G-B-D-F *sounds* like a dominant seventh chord whether it is spelled G-B-D-F, G-B-D-E♯, G-B-C𝄪-E♯, G-C♭-C𝄪-F, or in some other manner.

The equally tempered twelve-tone scale may be conveniently thought of as a circle, and any point on the circumference may be considered as representing any tone and/or its octave. This

circumference may then be divided into twelve equal parts, each representing a minor second, or half-step. Or, with equal validity, each of the twelve parts may represent the interval of a perfect fifth, since the superposition of twelve perfect fifths also embraces all of the twelve tones of the chromatic scale—as in the familiar "key-circle." We shall find the latter diagram particularly useful. Beginning on C and superimposing twelve minor seconds or twelve perfect fifths clockwise around the circle, we complete the circle at B♯, which in equal temperament has the same pitch as C. Similarly, the pitch names of C♯ and D♭, D♯ and E♭, and so forth, are interchangeable.

EXAMPLE 1-2



The term *sonority* is used in this book to cover the entire field of tone relationship, whether in terms of melody or of harmony. When we speak of G-B-D-F, for example, we mean the relationship of those tones used either as tones of a melody or of a harmony. This may seem to indicate a too easy fusion of melody and harmony, and yet the problems of tone relationship are essentially the same. Most listeners would agree that the sonority in Example 1-3*a* is a dissonant, or "harsh," combination of tones when sounded together. The same effect of dissonance, however, persists in our aural memory if the tones are sounded consecutively, as in Example 1-3*b*:

EXAMPLE 1-3

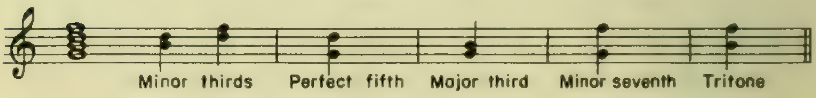
(*a*)

(*b*)

The first problem in the analysis of a sonority is the analysis of its component parts. A sonority sounds as it does primarily because of the relative degree of consonance and dissonance of its elements, the position and order of those elements in relation to the tones of the harmonic series, the degree of acoustical clarity in terms of the doubling of tones, timbre of the orchestration, and the like. It is further affected by the environment in which the sonority is placed and by the manner in which experience has conditioned the ears of the listener.

Of these factors, the first would seem to be basic. For example, the most important aural fact about the familiar sonority of the dominant seventh is that it contains a greater number of minor thirds than of any other interval. It contains also the consonances of the perfect fifth and the major third and the mild dissonances of the minor seventh and the tritone. This is, so to speak, the chemical analysis of the sonority.

EXAMPLE 1-4



It is of paramount importance to the composer, since the composer should both love and understand the beauty of sound. He should "savor" sound as the poet savors words and the painter form and color. Lacking this sensitivity to sound, the composer is not a composer at all, even though he may be both a scholar and a craftsman.

This does not imply a lack of importance of the secondary analyses already referred to. The historic position of a sonority in various styles and periods, its function in tonality—where tonality is implied—and the like are important. Such multiple analyses strengthen the young composer's grasp of his material, providing always that they do not obscure the fundamental analysis of the *sound as sound*.

Referring again to the sonority G-B-D-F, we should note its historic position in the counterpoint of the sixteenth century and its harmonic position in the tonality of the seventeenth, eighteenth, and nineteenth centuries, but we should first of all observe its construction, the elements of which it is formed. All of these analyses are important and contribute to an understanding of harmonic and melodic vocabulary.

As another example of multiple analysis, let us take the familiar chord C-E-G-B. It contains two perfect fifths, two major thirds, one minor third, and one major seventh.

EXAMPLE 1-5

It may be considered as the combination of two perfect fifths at the interval of the major third; two major thirds at the perfect fifth; or perhaps as the combination of the major triad C-E-G and the minor triad E-G-B or the triads* C-G-B and C-E-B:

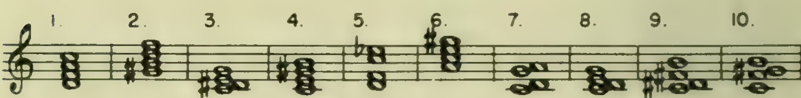
EXAMPLE 1-6

*The word *triad* is used to mean *any* three-tone chord.

Historically, it represents one of the important dissonant sonorities of the baroque and classic periods. Its function in tonality may be as the subdominant or tonic seventh of the major scale, the mediant or submediant seventh of the "natural" minor scale, and so forth.

Using the pattern of analysis employed in Examples 1-4, 1-5, and 1-6, analyze as completely as possible the following sonorities:

EXAMPLE 1-7



2

The Analysis of Intervals

In order again to reduce a problem of theoretically infinite proportions to a finite problem, an additional device is suggested. Let us take as an example the intervallic analysis of the major triad C-E-G:

Example 2-1

This triad is commonly described in conventional analysis as a combination of a perfect fifth and a major third above the lowest or "generating" tone of the triad. It is obvious, however, that this analysis is incomplete, since it omits the concomitant interval of the minor third between E and G. This completes the analysis as long as the triad is in the simple form represented above. If, however, the chord is present in a form in which there are many doublings in several octaves, such a complete analysis becomes more complex.

If we examine the scoring of the final chord in *Death and Transfiguration* by Richard Strauss we find a sixteen-tone chord:

Example 2-2

These sixteen tones combine to form one hundred and twenty different intervals. The relationship between C and G is represented not only by the intervals

EXAMPLE 2-3

but also by the intervals

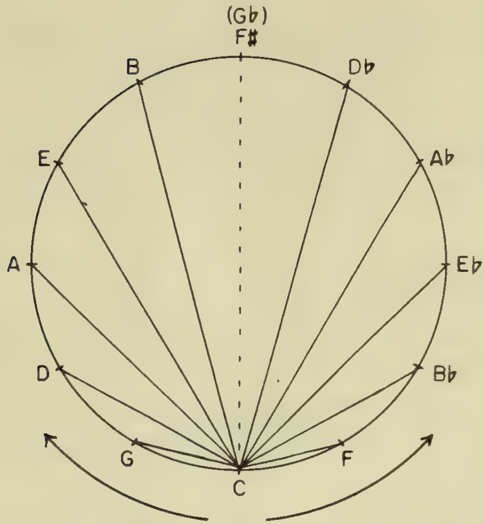
EXAMPLE 2-4

in which case we commonly call the second relationship the "inversion" of the first. The same is true of the relation of C to E and E to G.

However, the composite of all of the tones still gives the impression of the C major triad in spite of the complexity of doubling. In other words, the interval C to G performs the same *function in the sonority* regardless of the manner of the doubling of voices.

The similarity of an interval and its inversion may be further illustrated if one refers again to the arrangement of the twelve-tone scale in the circle of fifths:

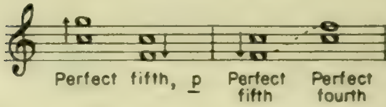
EXAMPLE 2-5



Here it will be seen that C has two perfect-fifth relationships, C to G and C to F; the one, C to G, proceeding clockwise (ascending) and the other, C to F, proceeding counterclockwise (descending). In the same manner, C has two major-second relationships, C to D and C to B♭; two major-sixth relationships, C to A and C to E♭; two major-third relationships, C to E and C to A♭; and two major-seventh relationships, C to B and C to D♭. It has only one tritone relationship, C up to F♯, or C down to G♭. It will be helpful in our analysis if we use only one symbol to represent both the interval under consideration and its inversion. This is not meant to imply that the interval and its inversion are the same, but rather that they perform the same function in a sonority.

Proceeding on this theory, we shall choose the symbol *p* to represent the relationship of the perfect fifth *above or below* the first tone, even though when the lower tone of each of the two intervals is raised an octave the relationship becomes actually a perfect fourth:

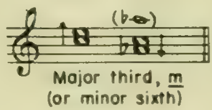
EXAMPLE 2-6



The symbolization is arbitrary, the letter *p* being chosen because it connotes the designation "perfect," which applies to both intervals.

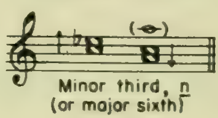
The major third above or below the given tone will be designated by the letter *m*:

EXAMPLE 2-7



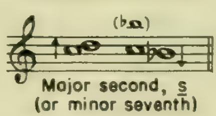
The minor third above or below the given tone will be represented by the letter *n*:

EXAMPLE 2-8



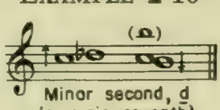
the major second above or below, by *s*:

EXAMPLE 2-9



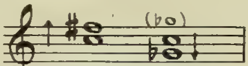
the dissonant minor second by *d*:

EXAMPLE 2-10



and the tritone by *t*:

EXAMPLE 2-11



The letters *pmn,* therefore, represent intervals commonly considered consonant, whereas the letters *sdt* represent the intervals commonly considered dissonant. The symbol *pmn, sdt** would therefore represent a sonority which contained one perfect fifth or its inversion, the perfect fourth; one major third or its inversion, the minor sixth; one minor third or its inversion, the major sixth; one major second or its inversion, the minor seventh; one minor second or its inversion, the major seventh; and one augmented fourth or its inversion, the diminished fifth; the three symbols at the left of the comma representing consonances, those at the right representing dissonances. A sonority represented, for example, by the symbol sd^2, indicating a triad composed of one major second and two minor seconds, would be recognized as a highly dissonant sound, while the symbol *pmn* would indicate a consonant sound.

The complexity of the analysis will depend, obviously, upon the number of different tones present in the sonority. A three-tone sonority such as C-E-G would contain the three intervals C to E, C to G, and E to G. A four-tone sonority would contain 3+2+1 or 6 intervals; a five-tone sonority, 4+3+2+1 or 10 intervals, and so on.

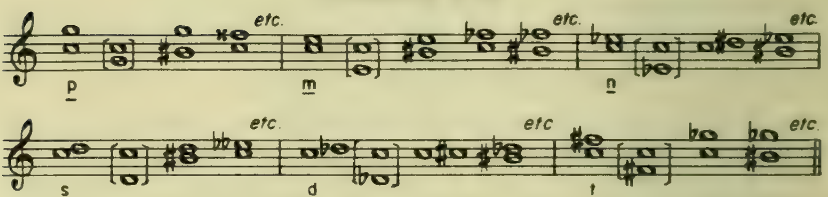
Since we are considering all tones in equal temperament, our task is somewhat simplified. C to D♯, for example, represents the same *sound* as the interval C to E♭; and since the sound is

* For the sake of uniformity, analyses of sonorities will list the constituent intervals in this order.

the same, they would both be represented by the single symbol *n*. A table of intervals with their classification would, therefore, be as follows:

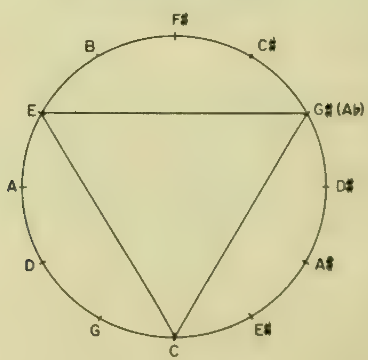
C-G (or G-C), B♯-G, C-F𝄪, etc. = *p*
C-E (or E-C), B♯-E, C-F♭, B♯-F♭, etc. = *m*
C-E♭ (or E♭-C), C-D♯, B♯-E♭, etc. = *n*
C-D (or D-C), B♯-D, C-E♭♭, etc. = *s*
C-D♭ (or D♭-C), C-C♯, B♯-D♭, etc. = *d*
C-F♯ (or F♯-C), C-G♭, B♯-G♭, etc. = *t*

Example 2-12



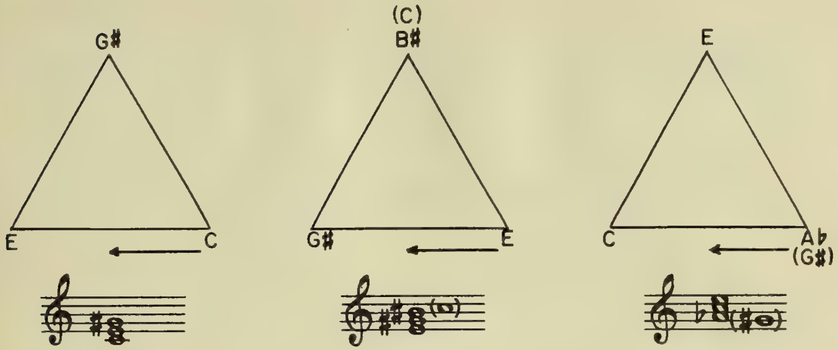
For example, the augmented triad C-E-G♯ contains the major third C to E; the major third E to G♯, and the interval C to G♯. Since, however, C to G♯ sounds like C to A♭, the inversion of which is A♭ to C—also a major third—the designation of the augmented triad would be three major thirds, or m^3. A diagram of these three notes in equal temperament quickly illustrates the validity of this analysis. The joining of the three notes C-E-G♯ (A♭) forms an equilateral triangle—a triangle having three equal sides and angles:

Example 2-13



It is, of course, a figure which has the same form regardless of which side is used as its base:

Example 2-14

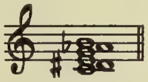


Similarly the augmented triad sounds the same regardless of which of the three tones is the lowest:

G♯	B♯ (C)	E
E	G♯	C
C	E	G♯ (A♭)

One final illustration will indicate the value of this technique of analysis. Let us consider the following complex-looking sonority in the light of conventional academic analysis:

Example 2-15



The chord contains six notes and therefore has 5+4+3+2+1, or 15 intervals, as follows:

C-D♯ and A♭-B	=	augmented seconds
C-E and G-B	=	major thirds
C-G and E-B	=	perfect fifths
C-A♭ and D♯-B	=	minor sixths

C-B	=	major seventh
D♯-E and G-A♭	=	minor seconds
D♯-G and E-A♭	=	diminished fourths
D♯-A♭	=	double-diminished fifth
E-G	=	minor third

However, in the new analysis it converts itself into only four types of intervals, or their inversions, as follows:

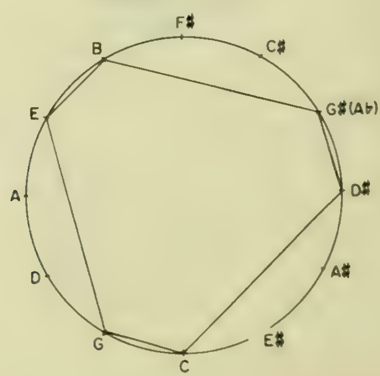
3 perfect fifths: C-G, E-B, and A♭-E♭ (D♯).
6 major thirds: C-E, E♭ (D♯)-G, E-G♯ (A♭), G-B, A♭-C, and B-D♯.
3 minor thirds: C-E♭ (D♯), E-G, and G♯ (A♭)-B.
3 minor seconds: D♯-E, G-A♭, and B-C.

The description is, therefore, $p^3m^6n^3d^3$.

EXAMPLE 2-16

A diagram will indicate the essential simplicity of the structure:

EXAMPLE 2-17

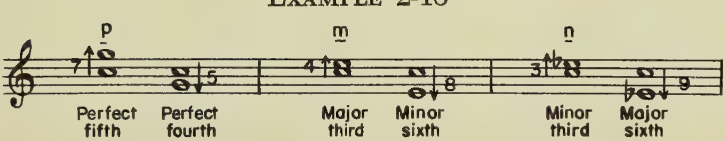


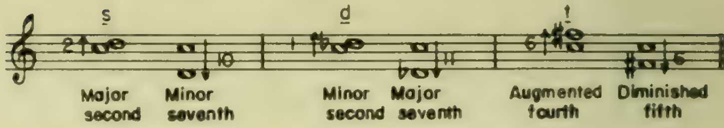
It has been my experience that although the young composer who has been thoroughly grounded in academic terminology may at first be confused by this simplification, he quickly embraces the new analysis because it conforms directly to his own aural impression.

In analyzing intervals, the student will find it practical to form the habit of "measuring" all intervals in terms of the "distance" in half-steps between the two tones. Seven half-steps (up or down), for example, will be designated by the symbol *p*; four half-steps by the symbol *m*; three half-steps by the symbol *n*, and so forth, regardless of the spelling of the tones which form the interval:

p	perfect fifth	7 half-steps
	perfect fourth	5 " "
m	major third	4 " "
	minor sixth	8 " "
n	minor third	3 " "
	major sixth	9 " "
s	major second	2 " "
	minor seventh	10 " "
d	minor second	1 " "
	major seventh	11 " "
t	augmented fourth	6 " "
	diminished fifth	6 " "

Example 2-18

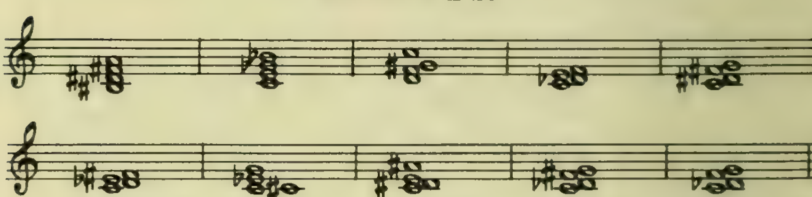




In speaking of sonorities we shall apparently make little distinction between tones used successively in a melody and tones used simultaneously in a harmony. It is true that the addition of the element of rhythm, the indispensable adjunct of melody, with its varying degrees of emphasis upon individual notes by the devices of time length, stress of accent, and the like, creates both great and subtle variance from the sonority played as a "block" of sound. Nevertheless, the basic relationship is the same. A melody may grow out of a sonority or a melody may itself *be* a sonority.

Analyze the following sonorities in the same manner employed In Examples 2-15 and 2-16, pages 13 and 14, giving first the conventional interval analysis, and second the simplified analysis:

Example 2-19



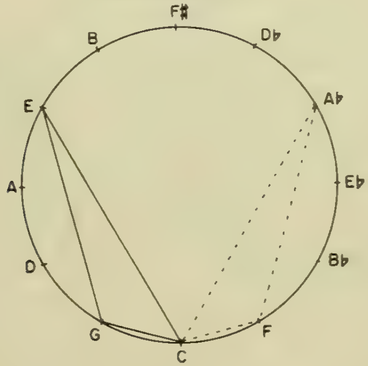
Repeat the same process with the chords in Example 1-7, page 6.

3

The Theory of Involution

Reference has already been made to the two-directional aspect of musical relationship, that is, the relationship "up" and "down" in terms of pitch, or the relationship in clockwise or counterclockwise rotation on the circle already referred to. It will be readily apparent that every sonority in music has a counterpart obtained by taking the *inverse ratio* of the original sonority. The projection *down* from the lowest tone of a given chord, using the same intervals in the order of their occurrence in the given chord, we may call the *involution* of the given chord. This counterpart is, so to speak, a "mirror" of the original. For example, the major triad C-E-G is formed by the projection of a major third and a perfect fifth above C. However, if this same relationship is projected downward, the interval C to E has as its counterpart the interval ↓C to A♭; and the interval C to G has as its counterpart ↓C to F.

Example 3-1

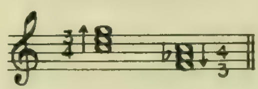


It will be noted that the involution of a sonority always contains the same intervals found in the original sonority.

There are three types of involutions: simple, isometric, and enharmonic.

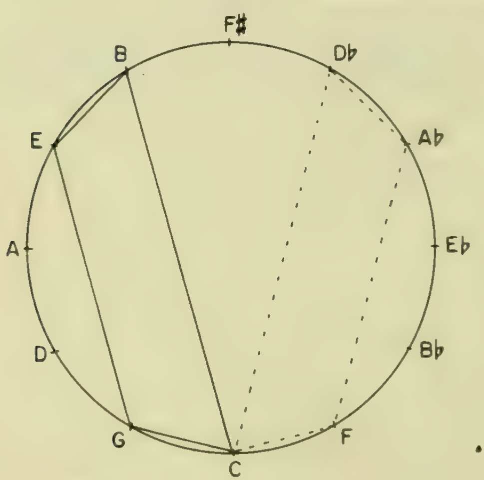
In *simple involution*, the involuted chord differs in sound from the given chord. Let us take, for example, the major triad C-E-G, which is formed by the projection of a major third and a perfect fifth above C. Its involution, formed by the projection downward from C of a major third and a perfect fifth, is the minor triad ↑F-A♭-C. The major triad C-E-G and its involution, the minor triad ↑F-A♭-C, each contain a perfect fifth, a major third, and a minor third, and can be represented by the symbols *pmn*.

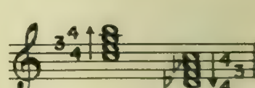
EXAMPLE 3-2



In the second type of involution, which we may call *isometric involution*, the involuted sonority has the same *kind* of sound as the original sonority. For example, the tetrad C-E-G-B has as its involution ↑D♭-F-A♭-C.

EXAMPLE 3-3

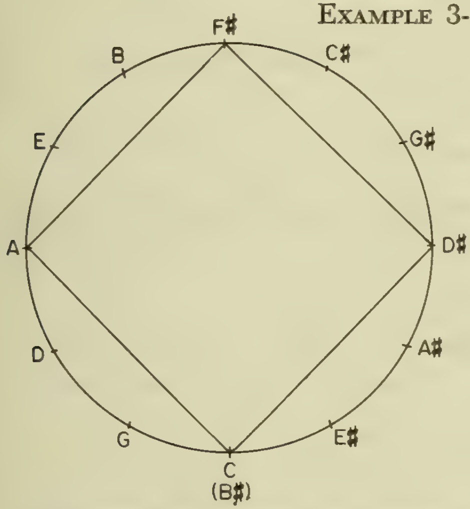




Each of these is a major seventh chord, containing two perfect fifths, two major thirds, a minor third, and a major seventh, and can be characterized by the symbols p^2m^2nd, the exponents in this instance representing two perfect fifths and two major thirds.

In the third type, *enharmonic involution*, the involuted sonority and the original sonority contain the same tones in different octaves (except for one common tone). For example, the augmented triad C-E-G♯ involutes to produce the augmented triad $\uparrow$F♭-A♭-C, F♭ and A♭ being the equal-temperament equivalents of E and G♯. Another common example of enharmonic involution is the diminished seventh chord:

EXAMPLE 3-4

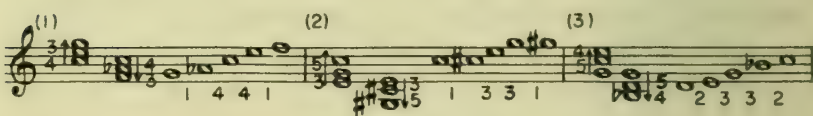


All sonorities which are formed by the combination of a sonority with its involution are *isometric* sonorities, since they will have the same order of intervals whether considered "up" or "down," clockwise or counterclockwise. We have already seen that the involution of the triad C-E-G is $\downarrow$C-A♭-F. The two together produce the sonority $F_3A\flat_4C_4E_3G$, which has the same order of intervals upward or downward.*

*The numbers indicate the number of half-steps between the tones of the sonority.

If the tone E of the triad C_4E_3G is used as the *axis of involution,* a different five-tone sonority will result, since the involution of E_3G_5C will be $\downarrow E_3C\sharp_5G\sharp$, forming together the sonority $G\sharp_5C\sharp_3E_3G_5C$. If the tone G is used as the axis of involution, the involution of G_5C_4E will be $\downarrow G_5D_4B\flat$, forming together the sonority $B\flat_4D_5G_5C_4E$. These resultant sonorities will all be seen to be *isometric* in structure. (See Note, page 24.)

Example 3-5



If two tones are used as the axes of involution, the result will be a four-tone isometric sonority:

Example 3-6

In the first of the above examples, C and G constitute the "double axis"; in the second C and E; and in the third E and G.

The discussion of involution up to this point does not differ greatly from the "mirror" principle of earlier theorists, whereby "new" chords were formed by "mirroring" a familiar chord and combining the "mirrored" or involuted chord with the original.

At this point, however, we shall expand the principle to the point where it becomes a basic part of our theory. When a major triad is involuted—as in Example 3-2—deriving the minor triad as the "mirrored" image of the major triad seems to place the minor triad in a position of secondary importance, as the *reflected image* of the major triad.

In the principle of involution presented here, no such secondary importance is intended; for if the minor triad is the reflected image of the major triad, it is equally true that the major triad is

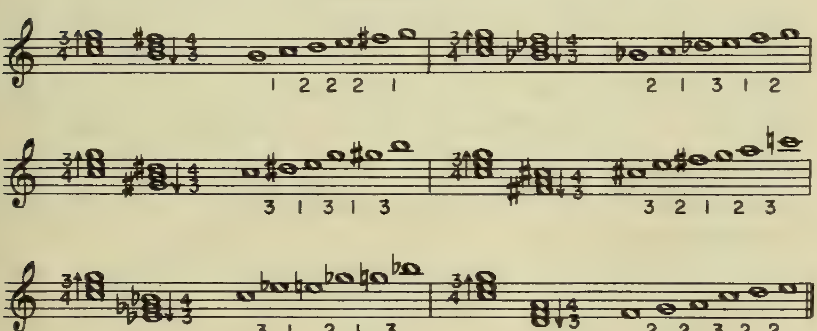
also the reflected image of the minor triad. For example, the involution of the major triad C_4E_3G is the minor triad $\downarrow C_4A\flat_3F$, and the involution of the minor triad $C_3E\flat_4G$ is the major triad $\downarrow C_3A_4F$.

In order to avoid any implication that the involution is, so to speak, a less important sonority, we shall in analyzing the sonorities construct both the first sonority and its involution upward by the simple process of reversing the intervallic order. For example, if the first triad is C_4E_3G the involution of this triad will be any triad which has the same order of half-steps *in reverse,* for example $F_3A\flat_4C$, the comparison being obviously 4-3 versus 3-4.

In this sense, therefore, the involution of a major triad can be considered to be *any* minor triad whether or not there is an axis of involution present.

In Example 3-7, therefore, the B minor, B♭ minor, G♯ minor, F♯ minor, E♭ minor, and D minor triads are all considered as possible involutions of the C major triad, although there is no axis of involution. When the C major triad is combined with any one of them, the resultant formation is a six-tone isometric sonority.

Example 3-7



Note that the combination of any sonority with its involuted form *always* produces an isometric sonority, that is, a sonority which can be arranged in such a manner that its formation of intervals is the same whether thought up or down. For example, the first combination in Example 3-7, if begun on B, has the configuration $B_1C_2D_2E_2F\sharp_1G$, which is the same whether considered from B to G or from G to B.

The second combination, C major and B♭ minor, must be begun on B♭ or E to make its isometric character clear: $B\flat_2C_1D\flat_3E_1F_2G$ or $E_1F_2G_3B\flat_2C_1D\flat$.

The isometric character of the third combination, C major and G♯ minor, is clear regardless of the tone with which we begin: $C_3D\sharp_1E_3G_1G\sharp_3B$; $D\sharp_1E_3G_1G\sharp_3B_1C$, etc.

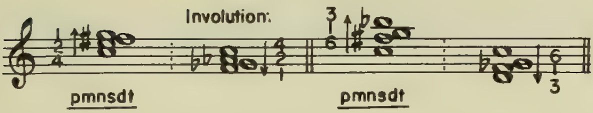
If, however, for the sake of comparison, we combine a major triad with *another major* triad, for example, the combination of C major with D major, the resultant formation is not isometric, since it is impossible to arrange these tones so that the configuration is the same up or down:

$C_2D_2E_2F\sharp_1G_2A$; $D_2E_2F\sharp_1G_2A_3C$; $E_2F\sharp_1G_2A_3C_2D$;
$F\sharp_1G_2A_3C_2D_2E$; $G_2A_3C_2D_2E_2F\sharp$; $A_3C_2D_2E_2F\sharp_1G$.

There is one more phenomenon which should be noted. There are a few sonorities which *have the same components but which are not involutions one of the other*, although each has its own involution. Examples are the tetrads C-E-F♯-G and C-F♯-G-B♭. Each contains one perfect fifth, one major third, one minor third, one major second, one minor second, and one tritone (*pmnsdt*), but one is *not* the involution of the other—although each has *its own* involution.

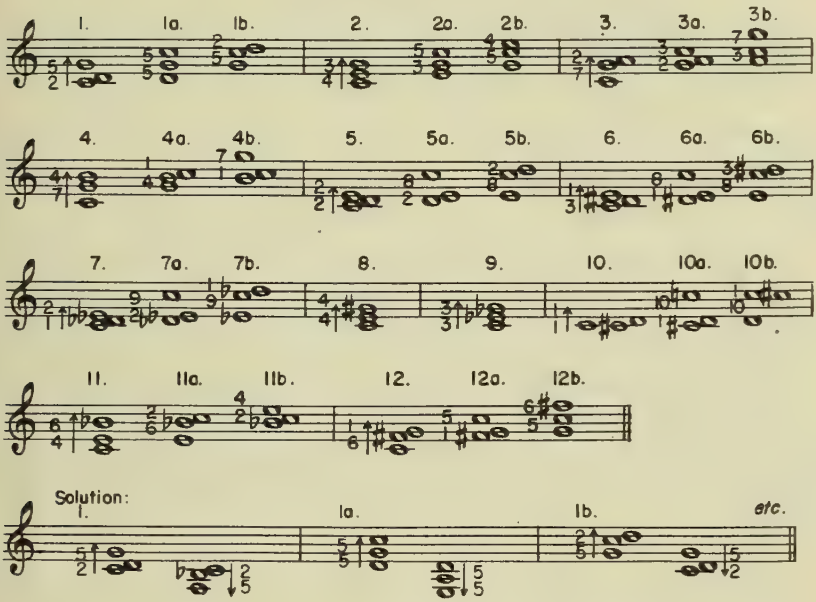
We shall describe such sonorities, illustrated in Example 3-8, as *isomeric* sonorities.

EXAMPLE 3-8



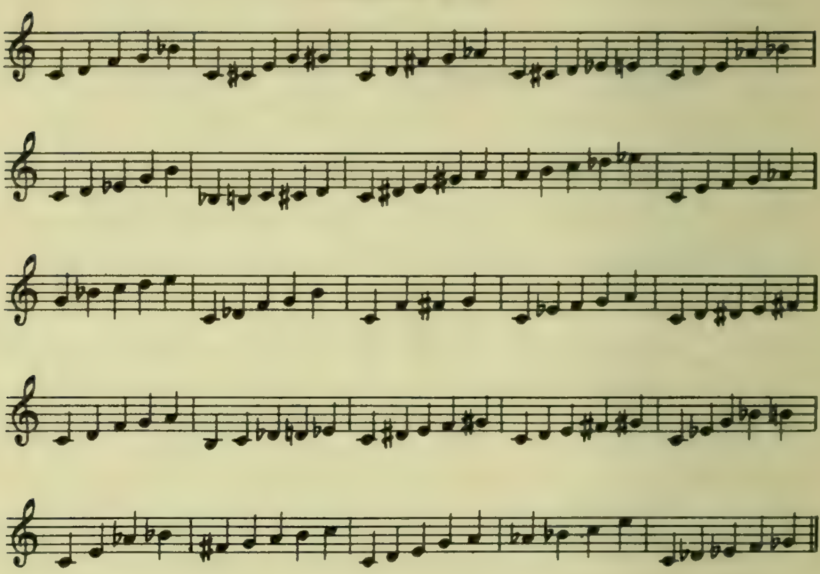
Using the *lowest* tone of each of the following three-tone sonorities as the axis of involution, write the involution of each by projecting the sonority *downward,* as in Example 3-5.

EXAMPLE 3-9



The following scales are all isometric, formed by the combination of one of the three-tone sonorities in Example 3-9 with its involution. Match the scale in Example 3-10 with the appropriate sonority in Example 3-9.

Example 3-10



Note: We have defined an isometric sonority as one which has the same order of intervals regardless of the direction of projection. The student should note that this bidirectional character of a sonority is not always immediately evident. For example, the perfect-fifth pentad in the position $C_2D_2E_3G_2A_3$(C) does not at first glance seem to be isometric. However in the position $D_2E_3G_2A_3C_2$(D), its isometric character is readily apparent.

Part I

THE SIX BASIC TONAL SERIES

4

Projection of the Perfect Fifth

WE HAVE SEEN that there are six types of interval relationship, if we consider such relationship both "up" and "down": the perfect fifth and its inversion, the perfect fourth; the major third and its inversion, the minor sixth; the minor third and its inversion, the major sixth; the major second and its inversion, the minor seventh; the minor second and its inversion, the major seventh; and the tritone,–the augmented fourth or diminished fifth–which we are symbolizing by the letters, *p, m, n, s, d,* and *t,* respectively.

In a broader sense, the combinations of tones in our system of equal temperament–whether such sounds consist of two tones or many–tend to group themselves into sounds which have a preponderance of one of these basic intervals. In other words, most sonorities fall into one of the six great categories: perfect-fifth *types,* major-third *types,* minor-third *types,* and so forth. There is a smaller number in which two of the basic intervals predominate, some in which three intervals predominate, and a few in which four intervals have equal strength. Among the six-tone sonorities or scales, for example, there are twenty-six in which one interval predominates, twelve which are dominated equally by two intervals, six in which three intervals have equal strength, and six sonorities which are practically neutral in "color," since four of the six basic intervals are of equal importance.

The simplest and most direct study of the relationship of tones

is, therefore, in terms of the projection of each of the six basic intervals discussed in Chapter 2. By "projection" we mean the building of sonorities or scales by superimposing a series of *similar* intervals one above the other. Of these six basic intervals, there are only two which can be projected with complete consistency by superimposing one above the other until all of the tones of the equally tempered scale have been used. These two are, of course, the perfect fifth and the minor second. We shall consider first the perfect-fifth projection.

Beginning with the tone C, we add first the perfect fifth, G, and then the perfect fifth, D, to produce the triad C-G-D or, reduced to the compass of an octave, C-D-G. This triad contains, in addition to the two fifths, the concomitant interval of the major second. It may be analyzed as p^2s.

EXAMPLE 4-1

The tetrad adds the fifth above D, or A, to produce C-G-D-A, or reduced to the compass of the octave, C-D-G-A. This sonority contains three perfect fifths, two major seconds, and—for the first time in this series—a minor third, A to C.

EXAMPLE 4-2

The analysis is, therefore, p^3ns^2.

The pentad adds the next fifth, E, forming the sonority C-G-D-A-E, or the melodic scale C-D-E-G-A, which will be recognized as the most familiar of the pentatonic scales. Its components are four perfect fifths, three major seconds, two

minor thirds, and—for the first time—a major third. The analysis is, therefore, $p^4mn^2s^3$.

EXAMPLE 4-3

The hexad adds B, C-G-D-A-E-B, or melodically, producing C-D-E-G-A-B,

EXAMPLE 4-4

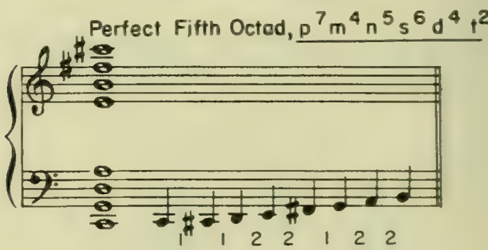
its components being five perfect fifths, four major seconds, three minor thirds, two major thirds, and—for the first time—the dissonant minor second (or major seventh), $p^5m^2n^3s^4d$.
The heptad adds F♯:

EXAMPLE 4-5

producing the first scale which in its melodic projection contains no interval larger than a major second—in other words, a scale without melodic "gaps." It also employs for the first time the interval of the tritone (augmented fourth or diminished fifth), C to F♯. This sonority contains six perfect fifths, five major seconds, four minor thirds, three major thirds, two minor seconds, and one tritone: $p^6m^3n^4s^5d^2t$. (It will be noted that the heptad is the first sonority to contain all of the six basic intervals.)

The octad adds C♯:

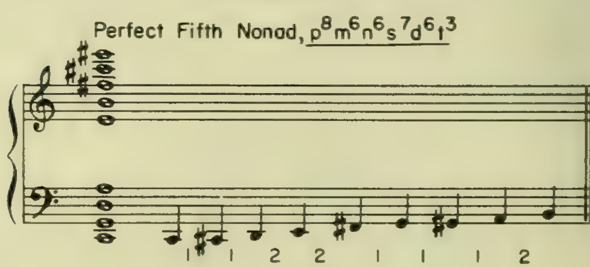
Example 4-6



Its components are seven perfect fifths, six major seconds, five minor thirds, four major thirds, four minor seconds, and two tritones: $p^7m^4n^5s^6d^4t^2$.

The nonad adds G♯:

Example 4-7



Its components are eight perfect fifths, seven major seconds, six minor thirds, six major thirds, six minor seconds, and three tritones: $p^8m^6n^6s^7d^6t^3$.

The decad adds D♯:

EXAMPLE 4-8

Its components are nine perfect fifths, eight major seconds, eight minor thirds, eight major thirds, eight minor seconds, and four tritones: $p^9m^8n^8s^8d^8t^4$.

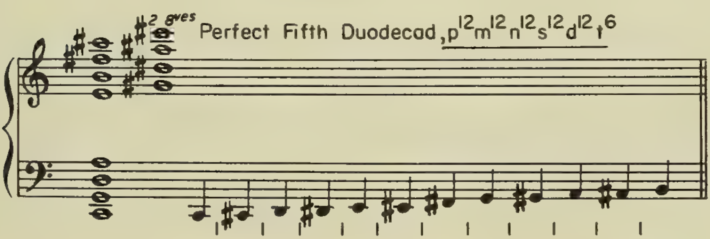
The undecad adds A♯:

EXAMPLE 4-9

Its components are ten perfect fifths, ten major seconds, ten minor thirds, ten major thirds, ten minor seconds, and five tritones: $p^{10}m^{10}n^{10}s^{10}d^{10}t^5$.

The duodecad adds the last tone, E♯:

EXAMPLE 4-10



Its components are twelve perfect fifths, twelve major seconds, twelve minor thirds, twelve major thirds, twelve minor seconds, and six tritones: $p^{12}m^{12}n^{12}s^{12}d^{12}t^{6}$.

The student should observe carefully the progression of the intervallic components of the perfect-fifth projection, since it has important esthetic as well as theoretical implications:

doad:	p
triad:	p^2s
tetrad:	p^3ns^2
pentad:	$p^4mn^2s^3$
hexad:	$p^5m^2n^3s^4d$
heptad:	$p^6m^3n^4s^5d^2t$
octad:	$p^7m^4n^5s^6d^4t^2$
nonad:	$p^8m^6n^6s^7d^6t^3$
decad:	$p^9m^8n^8s^8d^8t^4$
undecad:	$p^{10}m^{10}n^{10}s^{10}d^{10}t^5$
duodecad:	$p^{12}m^{12}n^{12}s^{12}d^{12}t^6$

In studying the above projection from the two-tone sonority to the twelve-tone sonority built on perfect fifths, several points should be noted. The first is the obvious affinity between the perfect fifth and the major second, since the projection of one perfect fifth upon another always produces the concomitant interval of the major second. (It is interesting to speculate as to whether or not this is a partial explanation of the fact that the "whole-tone" scale was one of the first of the "exotic" scales to make a strong impact on occidental music.)

The second thing which should be noted is the relatively greater importance of the minor third over the major third in the perfect-fifth projection, the late arrival of the dissonant minor second and, last of all, the tritone.

The third observation is of the greatest importance because of its esthetic implications. From the first sonority of two tones, related by the interval of the perfect fifth, up to the seven-tone sonority, there is a steady and regular progression. Each new

tone adds one *new* interval, in addition to adding one more to each of the intervals already present. However, when the projection is carried beyond seven tones, no *new* intervals can be added. In addition to this loss of any new material, there is also a gradual decrease in the difference of the *quantitative* formation of the sonority. In the octad there are the same number of major thirds and minor seconds. In the nonad the number of major thirds, minor thirds, and minor seconds is the same. The decad contains an equal number of major thirds, minor thirds, major seconds, and minor seconds. When the eleven- and twelve-tone sonorities are reached, there is no differentiation whatsoever, except in the number of tritones.*

The sound of a sonority—either as harmony or melody—depends not only upon what is present, but equally upon what is *absent*. The pentatonic scale in the perfect-fifth series sounds as it does not only because it contains a preponderance of perfect fifths and because of the presence of major seconds, minor thirds, and the major third in a regularly decreasing progression, but also because it *does not contain* either the dissonant minor second or the tritone.

On the other hand, as sonorities are projected beyond the six-tone series they tend to lose their individuality. *All* seven-tone series, for example, contain *all* of the six basic intervals, and the difference in their proportion decreases as additional tones are added.

This is probably the greatest argument against the rigorous use of the atonal theory in which all twelve tones of the chromatic scale are used in a single melodic or harmonic pattern, since such patterns tend to lose their identity, producing a monochromatic effect with its accompanying lack of the essential element of contrast.

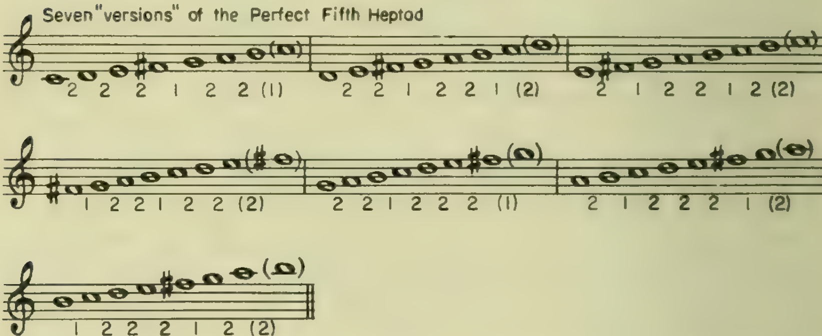
All of the perfect-fifth scales are isometric in character, since if any of the projections which we have considered are begun on

* See page 139 and 140.

the final tone of that projection and constructed *downward*, the resultant scale will be the same as if the projection were upward. The seven-tone scale $C_2D_2E_2F\sharp_1G_2A_2B$, for example, begun on the final tone of the projected fifths—that is, F♯—and projected downward produces the same tones: $\downarrow F\sharp_2E_2D_2C_1B_2A_2G$.

Every scale may have as many *versions* of its basic order as there are tones in the scale. The seven-tone scale, for example, has seven versions, beginning on C, on D, on E, and so forth.

Example 4-11



The student should distinguish carefully between an *involution* and the different *versions* of the *same* scale. An involution is the same order of progression but in the opposite direction and is significant only if a *new* chord or scale results.

Referring to page 29, you will see that the perfect-fifth pentatonic scale on C, C-D-E-G-A, contains a major triad on C and a minor triad on A. The six-tone perfect-fifth scale adds the major triad on G and the minor triad on E. Analyze the seven-, eight-, nine-, ten-, eleven- and twelve-tone scales of the perfect-fifth projection and determine where the major, minor, diminished, and augmented triads occur in each.

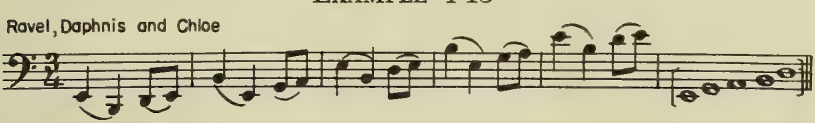
Construct the complete perfect-fifth projection beginning on the tone A. Indicate where the major, minor, diminished, and augmented triads occur in each.

Since the perfect-fifth projection includes the most familiar scales in occidental music, innumerable examples are available. The most provocative of these would seem to be those which produce the greatest impact with the smallest amount of tonal material. To illustrate the economical use of material, one can find no better example than the principal theme of Beethoven's overture, *Leonore,* No. 3. The first eight measures use only the first five tones of the perfect-fifth projection: C-D-E-G-A. The next measure adds F and B, which completes the tonal material of the theme.

EXAMPLE 4-12

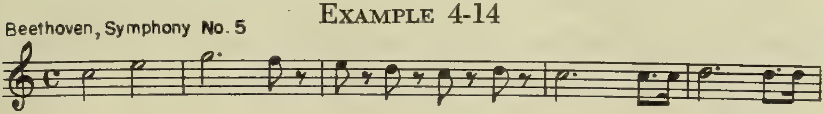
In the same way, Ravel uses the first five tones of the perfect-fifth projection G-D-A-E-B—or, in melodic form, E-G-A-B-D—in building to the first climax in the opening of *Daphnis and Chloe,* Suite No. 2.

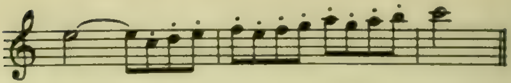
EXAMPLE 4-13



The principal theme of the last movement of the Beethoven Fifth Symphony is only slightly less economical in its use of material. The first six measures use only the pentatonic scale C-D-E-F-G, and the seventh measure adds A and B.

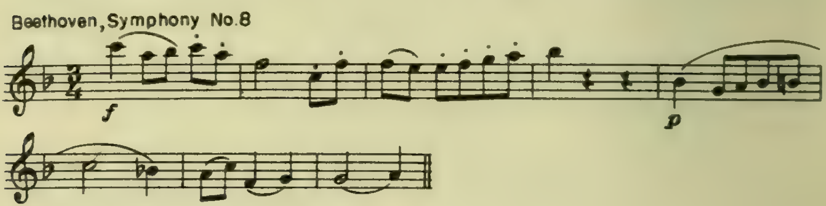
EXAMPLE 4-14





However, even Beethoven with his sense of tonal economy extended his tonal material beyond the seven-tone scale without implying modulation. The opening theme of the Eighth Symphony, for example, uses only the six tones F-G-A-B♭-C-E of the F major scale in the first four measures but reaches beyond the seven-tone perfect-fifth scale for an additional tone, B♮ (the perfect fifth above E) in the fifth measure.

EXAMPLE 4-15



Such chromatic tones are commonly analyzed as chromatic passing tones, non-harmonic tones, transient modulations, and the like, but the student will find it useful also to observe their position in an "expanded" scale structure.

Study the thematic material of the Beethoven symphonies and determine how many of them are constructed in the perfect-fifth projection.

A useful device of many contemporary composers is to begin a passage with only a few tones of a particular projection and then gradually to expand the medium by adding more tones of the *same* projection. For example, the composer might begin a phrase in the perfect-fifth projection by using only the first four tones of the projection and then gradually expand the scale by adding the fifth tone, the sixth tone, and so forth.

Examine the opening of Stravinsky's *Petrouchka.* The first five measures are formed of the pure four-tone perfect-fifth tetrad G-D-A-E. The sixth measure adds B♮, which forms the perfect-fifth pentad G-D-A-E-B. The following measure adds a C♯, forming the hexad G-A-B-C♯-D-E. This hexad departs momentarily from the pure perfect-fifth projection, since it is a combination of a perfect-fifth and major-second projection—G-D-A-E-B + G-A-B-C♯.

Measure 11 substitutes a C for the C♯ and measure 12 substitutes a B♭ for the previous B, forming the hexad $G_2A_1B♭_2C_2D_2E$ which is the involution of the previous hexad $G_2A_2B_2C♯_1D_2E$. Measure 13 adds an F, establishing the seven-tone perfect-fifth scale B♭-F-C-G-D-A-E.

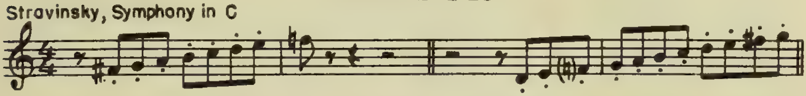
Continue this type of analysis to rehearsal number 7, determining how much of the section is a part of the perfect-fifth projection.

Analyze the thematic material of the second movement of the Shostakovitch Fifth Symphony. How much of this material conforms to the perfect-fifth projection?

Excellent examples of the eight-tone perfect-fifth projection are found in the beginning of all three movements of the Stravinsky Symphony in C. In the first movement, for example, the first seven measures are built on the tonal material of the seven-tone perfect-fifth scale on C: C-G-D-A-E-B-F♯. In the eighth measure, however, the scale is expanded one perfect fifth downward by the addition of the F♮ in the violas, after which *both* F and F♯ are integral parts of the scale. Note the scale passage in the trumpet:

EXAMPLE 4-16

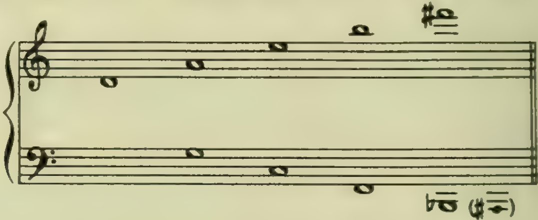
Stravinsky, Symphony in C



Similarly, the following theme from the first movement of the Prokofieff Sixth Symphony may be analyzed as the expansion of the perfect-fifth projection to nine tones:

EXAMPLE 4-17

Prokofieff, Symphony No. 6



Even when *all* of the tones of the chromatic scale are used, the formation of individual sonorities frequently indicates a simpler basic structure which the composer had in mind. For example, the first measure of the *Lyrische Suite* by Alban Berg employs all of the tones of the chromatic scale. Each sonority in the measure, however, is unmistakably of perfect-fifth construction:

EXAMPLE 4-18

Alban Berg, Lyrische Suite

Analyze the first movement of the Stravinsky Symphony in C and determine how much of it is written in the perfect-fifth projection.

In any analysis, always try to discover how the work is constructed, that is, how much should be analyzed as one fragment of the composition. It will be observed, for example, that some composers will use one scale pattern for long periods of time without change, whereas others will write in a kind of mosaic pattern, one passage consisting of many small and different patterns.

5

Harmonic-Melodic Material of the Perfect-Fifth Hexad

SINCE, as has been previously stated, all seven-tone scales contain *all* of the six basic intervals, and since, as additional tones are added, the resulting scales become increasingly similar in their component parts, the student's best opportunity for the study of different types of tone relationship lies in the six-tone combinations, which offer the greatest number of different scale types. We shall therefore concentrate our attention primarily upon the various types of hexads, leaving for later discussion those scales which contain more than six tones.

In order to reduce the large amount of material to a manageable quantity, we shall disregard the question of inversions. That is, we shall consider C-E-G a major triad whether it is in its fundamental position—C-E-G; in its first inversion—E-G-C; or in its second inversion—G-C-E. In the same way, we shall consider the pentad C-D-E-G-A as one type of sonority, that is, as a sonority built of four perfect fifths, regardless of whether its form is C-D-E-G-A, D-E-G-A-C, E-G-A-C-D, and so forth. It is also clear that we shall consider all enharmonic equivalents in equal temperament to be equally valid. We shall consider C-E-G a major triad whether it is spelled C-E-G, C-F♭-G, B♯-E-G, or in some other manner.

Examining the harmonic-melodic components of the perfect-fifth hexad, we find that it contains six types of triad formation. These are in order of their appearance:

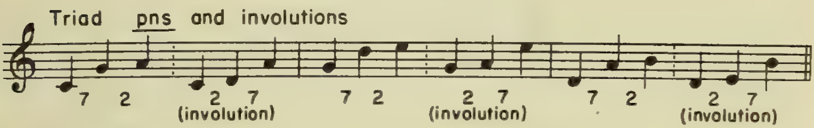
1. The basic triad C_2D_5G, p^2s, consisting of two superimposed

perfect fifths with the concomitant major second, which is duplicated on G, D, and A:

EXAMPLE 5-1

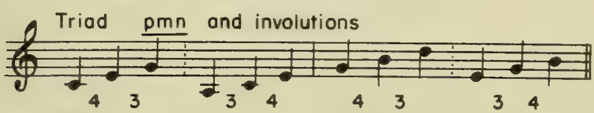
2. The triad C_7G_2A, *pns,* with the involution C_2D_7A, which consists of a perfect fifth, a major second, and a major sixth (or minor third). These triads are duplicated on G and on D:

EXAMPLE 5-2



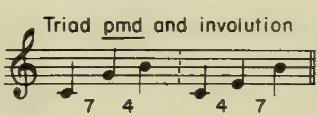
3. The triad C_4E_3G, *pmn,* with the involution A_3C_4E, which consists of a perfect fifth, a major third, and a minor third, forming the familiar major and minor triads. The major triad is duplicated on G, and the minor triad on E:

EXAMPLE 5-3



4. The triad C_7G_4B, *pmd,* with the involution C_4E_7B, consisting of the perfect fifth, major seventh (minor second), and major third:

EXAMPLE 5-4

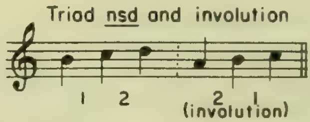


5. The triad C_2D_2E, ms^2, which consists of two superimposed major seconds with the concomitant major third, an isometric triad, which is reproduced on G:

EXAMPLE 5-5

6. The triad B_1C_2D, *nsd,* with the involution A_2B_1C, which consists of a minor third, a major second, and a minor second:

EXAMPLE 5-6

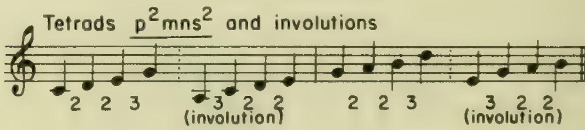


The tetrads of the perfect-fifth hexad consist of seven types. The *first* is the basic tetrad $C_2D_5G_2A$, p^3ns^2, already discussed in the previous chapter, duplicated on G and D:

EXAMPLE 5-7

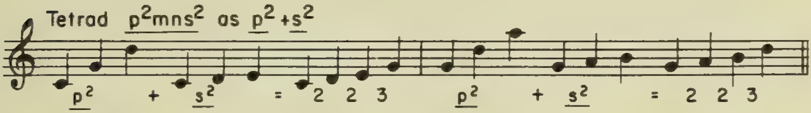
The *second* is the tetrad $C_2D_2E_3G$, also duplicated on G ($G_2A_2B_3D$), and the involutions $A_3C_2D_2E$ and $E_3G_2A_2B$. This tetrad contains two perfect fifths, two major seconds, one major third, and one minor third: p^2mns^2.

EXAMPLE 5-8



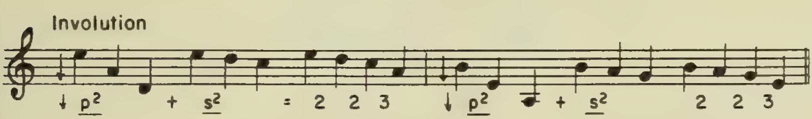
It is one of the most consonant of the tetrads, containing no strong dissonance and no tritone. Not only does it contain an equal number of perfect fifths and major seconds, but it is also the first example of the simultaneous projection of two different intervals above the same tone, since it consists of the two perfect fifths above C plus the two major seconds above C, that is, C-G-D plus C-D-E, or—above G—G-D-A plus G-A-B. (These formations will be discussed in Part III.)

Example 5-9



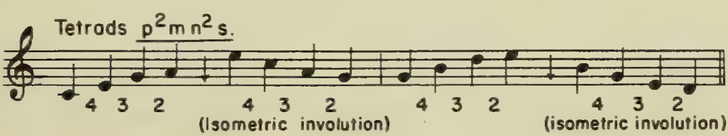
The involutions may also be considered to be formed by the simultaneous projection of two perfect fifths and two major seconds *downward*, that is ↓E-A-D + ↓E-D-C: and ↓B-E-A + ↓B-A-G:

Example 5-10



The *third* is the tetrad $C_4E_3G_2A$, duplicated on G ($G_4B_3D_2E$), also a predominantly consonant tetrad, which consists of two perfect fifths, C to G and A to E; two minor thirds, A to C and E to G; the major third, C to E; and the major second, G to A: p^2mn^2s. This is an isometric tetrad since, if we begin on the tone E and form the same tetrad *downward*, ↓$E_4C_3A_2G$, we produce the identical tones:

Example 5-11

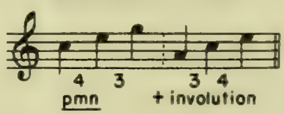


It may be considered to be formed of the relationship of two perfect fifths at the interval of the minor third, indicated by the symbol *p* @ *n*; or of two minor thirds at the interval of the perfect fifth, indicated by the symbol *n* @ *p*:

EXAMPLE 5-12

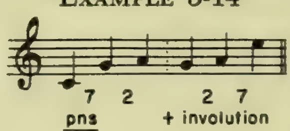
It contains the major triad C_4E_3G and the involution A_3C_4E;

EXAMPLE 5-13



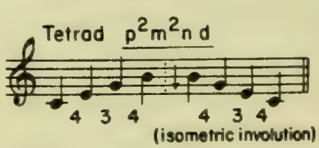
and the triad C_7G_2A, *pns*, with the involution G_2A_7E:

EXAMPLE 5-14



The *fourth* tetrad, $C_4E_3G_4B$, is also isometric, since if we begin on the tone B and form the same tetrad downward, we produce the identical tones, $\downarrow B_4G_3E_4C$:

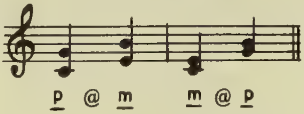
EXAMPLE 5-15



It is a more dissonant chord than those already discussed, for it contains two perfect fifths, C to G and E to B; two major thirds,

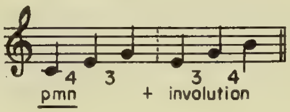
C to E and G to B; one minor third, E to G; and the dissonant major seventh (or minor second), C to B: p^2m^2nd. It may be considered to be formed of two perfect fifths at the interval relationship of the major third, C to G, plus E to B; or of two major thirds at the relationship of the perfect fifth, C to E plus G to B:

EXAMPLE 5-16



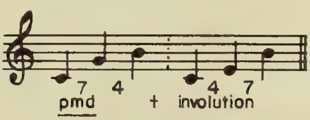
It contains the major triad C_4E_3G and the involution, the minor triad E_3G_4B;

EXAMPLE 5-17



and the triad C_7G_4B, *pmd,* and the involution C_4E_7B:

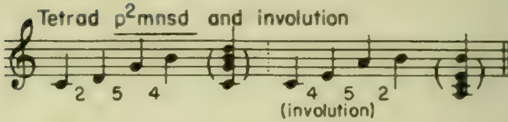
EXAMPLE 5-18



The *fifth* tetrad $C_2D_5G_4B$, *p^2mnsd,* consists of two perfect fifths, C to G and G to D, with the dissonance, B. This tetrad may also be considered as the major triad G-B-D with the added fourth above, or fifth below, G, that is, C. It is the first of the tetrads of this projection which contains all of the intervals of the parent hexad.

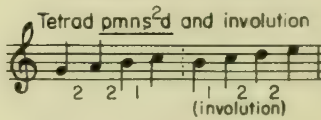
Together with this tetrad is found the involution $C_4E_5A_2B$, which consists of the minor triad A-C-E with the perfect fifth above, or the perfect fourth below, E, namely, B:

EXAMPLE 5-19



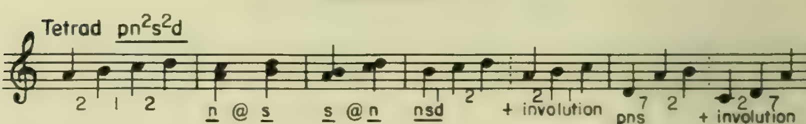
The *sixth* tetrad, $G_2A_2B_1C$, $pmns^2d$, contains one perfect fifth, one major third, one minor third, two major seconds, and a minor second. We also find the involution $B_1C_2D_2E$:

EXAMPLE 5-20



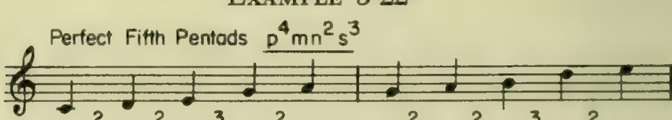
And *finally*, we have the isometric tetrad $A_2B_1C_2D$, pn^2s^2d, which consists of a perfect fifth, two minor thirds, two major seconds, and a minor second. It may be analyzed as the combination of two minor thirds at the interval of the major second, or two major seconds at the interval of the minor third. It contains the triad B_1C_2D, *nsd*, and the involution A_2B_1C; also the triad D_7A_2B, *pns*, and the involution C_2D_7A:

EXAMPLE 5-21



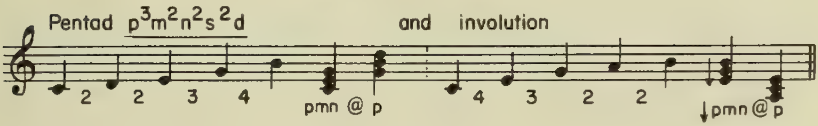
The parent hexad contains three pentad types. The *first* is the basic perfect-fifth pentad $C_2D_2E_3G_2A$, $p^4mn^2s^3$, also duplicated on G, $G_2A_2B_3D_2E$:

EXAMPLE 5-22



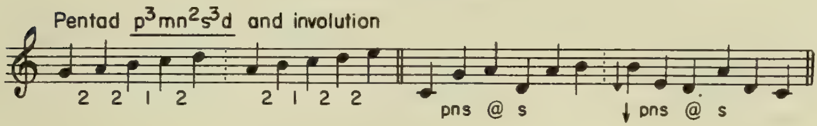
The *second* pentad, $C_2D_2E_3G_4B$, $p^3m^2n^2s^2d$, predominates in perfect fifths, like its parent scale, but has an equal number of major thirds, minor thirds, and major seconds. It may be identified more easily as the superposition of one major triad upon the fifth of another, C-E-G + G-B-D; its involution is $C_4E_3G_2A_2B$ with, of course, the same analysis, and consists of two minor triads projected *downward*, ↓B-G-E plus ↓E-C-A:

Example 5-23



The *final* pentad consists of the tones $G_2A_2B_1C_2D$, $p^3mn^2s^3d$. This pentad will be seen to have an equal number of perfect fifths and major seconds, two minor thirds, one major third, and one minor second. The involution is $A_2B_1C_2D_2E$:

Example 5-24



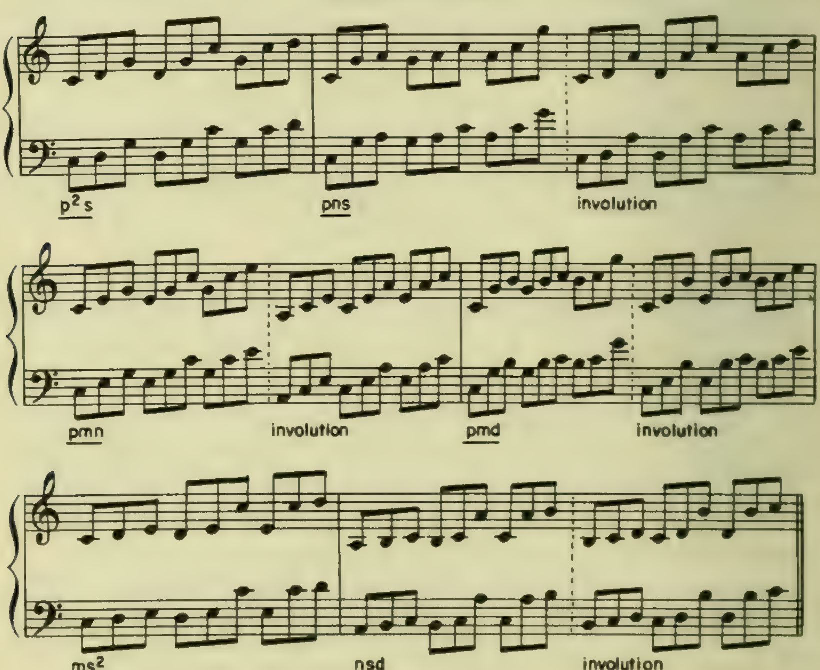
These pentads may be analyzed further as consisting of two triads *pns* at the interval of the major second, projected up or down.

The scales formed of perfect fifths, which have been discussed in this and the previous chapter, account for a very large segment of all occidental music. The five-tone scale in this series is the most important of all the pentatonic scales and has served as the basis of countless folk melodies. The seven-tone scale upon examination proves to be the most familiar of all occidental scales, the series which embraces the Gregorian modal scales, including the familiar major scale and the "natural" minor scale.

We have found in the previous chapter that the perfect-fifth hexad contains two isometric triads, p^2s and ms^2, and four triads with involutions, *pns*, *pmn*, *pmd*, and *nsd*. These triads are among the basic words, or perhaps one should say, syllables, of our musical vocabulary. They should be studied with the greatest thoroughness since, unlike words, it is necessary not only to "understand" them but to *hear* them.

For this reason the young composer might well begin by playing Example 5-25, which contains all of the triad types of the perfect-fifth hexad, over and over again, listening carefully until all of these sounds are a part of his basic tonal vocabulary. I suggest that the student play the first measure at least three times, *with the sustaining pedal down,* so that he is fully conscious of the triad's harmonic as well as melodic significance; and then proceed with measure two, and so forth.

EXAMPLE 5-25

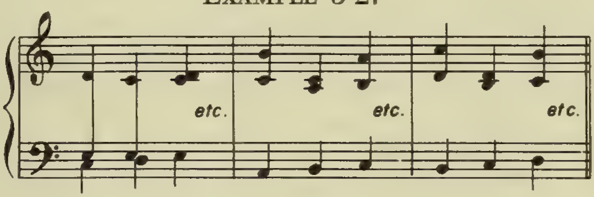


In Example 5-26 play the same triads but as "block" chords, listening carefully to the sound of each.

EXAMPLE 5-26

When the student comes to measures 8 and 9, and 10, the triads may sound too "muddy" and unclear in close position. Experiment with these sounds by "spreading" the triads to give them harmonic character, as in Example 5-27.

EXAMPLE 5-27



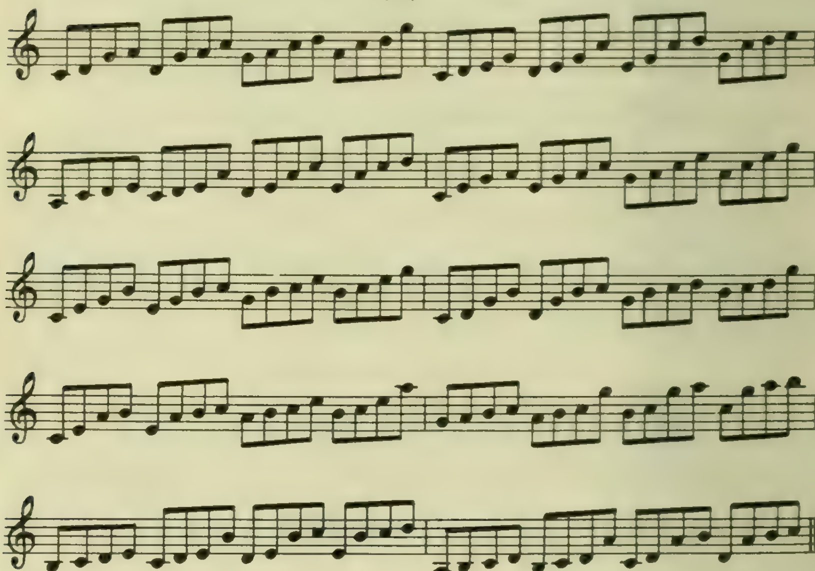
The sound of each of these triads will be affected both by its position and by the doubling of its tones. In the Stravinsky *Symphony of Psalms,* familiar sonorities take on new and sometimes startling character merely by imaginative differences in the doubling of tones. In Example 5-28, go back over the ten triad forms and experiment with the different character the triad can assume both in different positions and with different doublings.

EXAMPLE 5-28

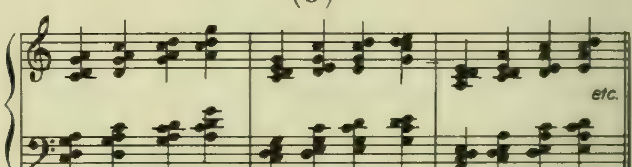
In Example 5-29*a* play the tetrads in arpeggiated form, and in Example 5-29*b* play them as "block" harmonies.

EXAMPLE 5-29

(*a*)

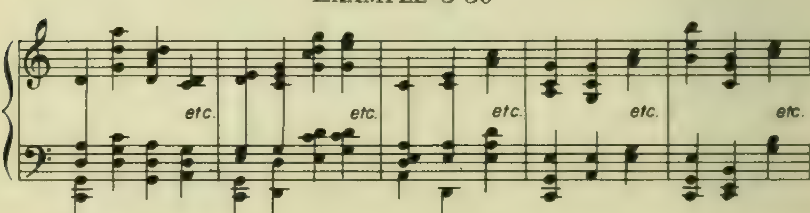


(*b*)



In Example 5-30 experiment with different positions and different doublings of the tones of the tetrads.

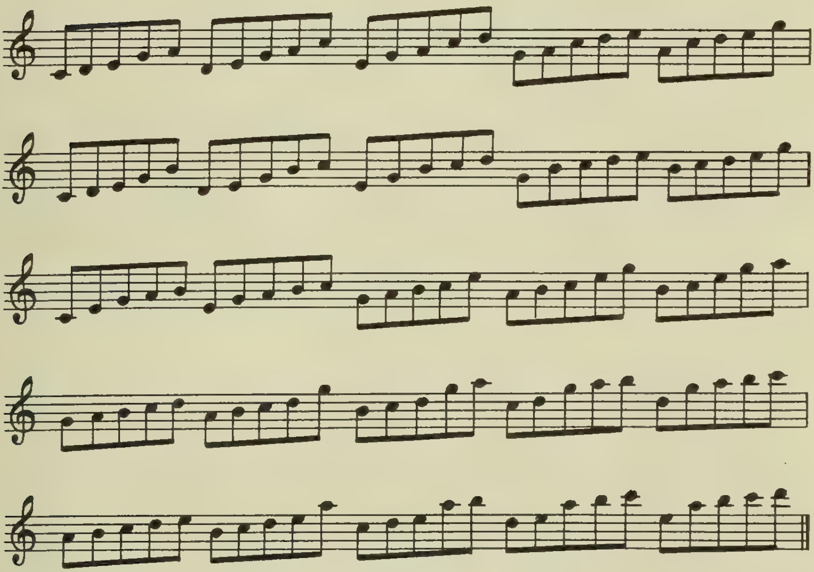
EXAMPLE 5-30



In Example 5-31, repeat the same process with the five pentad types.

EXAMPLE 5-31

(*a*)



(*b*)

In Example 5-32 repeat the same procedure with the hexad.

EXAMPLE 5-32

(*a*)

(*b*)

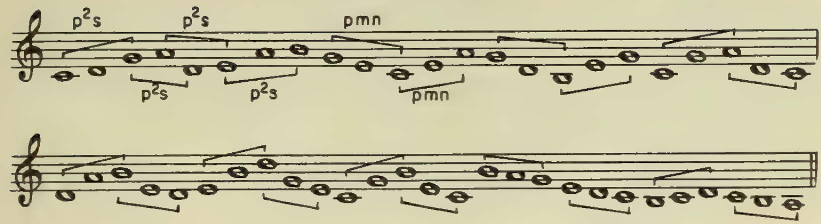
The student will find upon experimentation that although the basic tetrad seems to keep much of the same character regardless of its position, the remaining tetrads vary considerably in sound according to the position of the tetrad—particularly with regard to the bass tone. Play Example 5-30 again, noting the changes which occur in the sound when different tones of the tetrad are placed in the lowest part.

Repeat the experiment in relation to the five pentads in Example 5-31*b* and the one hexad in Example 5-32*b* and notice that as the sonority becomes more complex, the arrangement of the tones of the sonority becomes increasingly important. (Note especially the complete change in the character of the sonority in the second measure of Example 31*b* when the G major triad is shifted from its position above the C major triad to a position below it.)*

The melody in Example 5-33 includes all of the triads in the perfect-fifth hexad in melodic form. Play the example through several times and then finish the analysis.

*See *Note*, page 55.

Example 5-33



Example 5-34 harmonizes each triad by the same tones in the left hand in block harmony. Play this through several times and notice how the change of harmony in the left hand gives to the melodic line a certain pulse which we may call harmonic rhythm. Experiment with the changing of this harmonic rhythm by shifting the grouping of the tones in the melody, thereby changing the harmonic accompaniment. (For example, group the eighth, ninth, and tenth notes in the melody together and harmonize them with an E minor triad under the melodic tone B, and shift the following A minor triad one eighth note earlier.) Continue this type of change throughout the melody.

Example 5-34

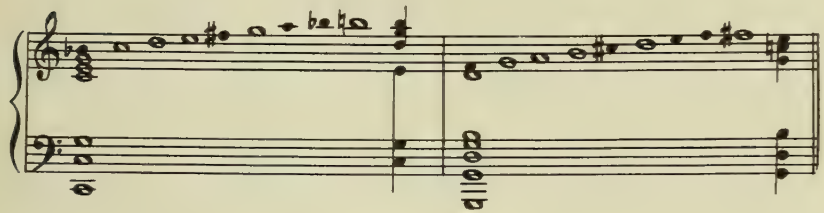
Example 5-35 contains all of the tetrads, the pentads, and the hexad of the six-tone perfect-fifth scale. Play this exercise several times in chorale style and listen to each change of harmony. Now analyze each sonority on the principle that we have discussed in the previous chapter.

Example 5-35

Finally, using as much or as little as you wish of the material which we have been studying, compose a short work in your own manner. Do not, however, use even *one* tone which is not in the material which we have studied. If you have studied orchestration, it would be desirable to score the composition for string orchestra and if possible have it performed, since only through actual performance can the composer test the results of his tonal thinking. Use all of your ingenuity, all of your knowledge of form and of counterpoint in this exercise.

Note: It is interesting to speculate upon the reason why two sonorities containing identical tones should sound so differently. The most logical explanation is perhaps that Nature has a great fondness for the major triad and for those sonorities that most closely approximate the overtone series which she has arranged for most sounding bodies—with the exception of bells and the like. The human ear seems to agree with Nature and prefers the arrangement of any sonority in the form which most closely approximates the overtone series. In the case of the combination of the C major and the G major triads, for example, if C is placed in the bass, the tones D-E-G-B are all found approximated in the first fifteen partials of the tone C. If G is placed in the bass, however, the tone C bears no close resemblance to any of the lower partials generated by the bass tone.

Example 5-36



6

Modal Modulation

Most melodies have some tonal center, one tone about which the other tones of the melody seem to "revolve." This is true not only of the classic period with its highly organized key centers, but also of most melodies from early chants and folk songs to the music of the present day—with, of course, the exceptions of those melodies of the "atonal" school, which deliberately avoid the repetition of any one tone until all twelve have been used. (Even in some of these melodies it is possible to discern evidence of a momentary tonal center.)

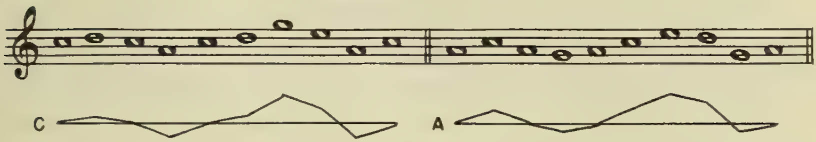
The advantage of a tonal center would seem to be the greater clarity which a melody derives from being organized around some central tone. Such organization avoids the sense of confusion and frustration which frequently arises when a melody wanders about without any apparent aim or direction. The tonal center, however, is not something which is immutably fixed. It may, in fact, be any one tone of a group of tones which the composer, by melodic and rhythmic emphasis or by the configuration of the melodic line, nominates as the tonal center.

For example, we may use the pentatonic scale C-D-E-G-A with C as the tonal center, by having the melody begin on C, depart from it, revolve about it, and return to it. Or we might in the same manner nominate the tone A as the tonal center, using the same tones but in the order A-C-D-E-G. Or, again, we might make either D, E, or G the tonal center of the melody.

One illustration should make this principle clear. If we begin

a melody on C, proceed upward to D, return to C, proceed downward to A, return to C, proceed upward to D, then upward to G, down to E, down to A and then back to C, we produce a melodic line the configuration of which obviously centers about C. If, using the same tones, we now take the same general configuration of the melodic line beginning with A, we produce a melody of which A is the tonal center:

Example 6-1



Finally, we may move from one tonal center to another, *within the same tonal group,* by changing our emphasis from one tone to another. In other words, we might begin a melody which was centered about C, as above, and then transfer that emphasis to the tone A. Such a transition from one tonal center to another is usually called a modulation. Since, however, the term *modulation* generally implies the adding—or more properly, the substitution—of new tones, we may borrow an old term and call this type of modulation *modal modulation,* since it is the same principle by which it is possible to modulate from one Gregorian mode to another without the addition or substitution of new tones. (For example, the scale C-D-E-F-G-A-B-C begun on the tone D will be recognized as the Dorian mode; begun on the tone E, as the Phrygian mode. It is therefore possible to "modulate" from the Dorian to the Phrygian mode simply by changing the melodic line to center about the tone E rather than D.

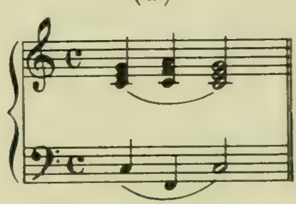
The six-tone perfect-fifth scale has four consonant triads which may serve as natural key centers: two major triads and two minor triads. The perfect-fifth hexad C-D-E-G-A-B, for example, contains the C major triad, the G major triad, the A minor triad, and the E minor triad. We may, as we have seen, nominate any

one of them to be the key center merely by seeing to it that the melodic and harmonic progressions revolve about that particular triad. We may modulate from one of these four key centers to any of the others simply by transferring the tonal seat of government from one to another.

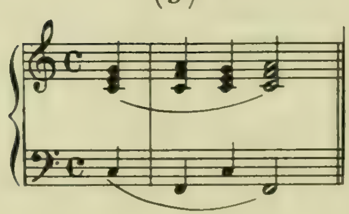
This transferral of attention from one tone as key center to another in a melody has already been discussed on page 57. We can assist this transition from one modal tonic to another (harmonically) by stressing the chord which we wish to make the key center both by rhythmic and agogic accent, that is, by having the key center fall on a strong rhythmic pulse and by having it occupy a longer time value. The simplest of illustrations will make this clear. In the following example, 6-2*a*, the first three triads seem to emphasize C major as the tonic, while in Example 6-2*b* we make F the key center merely by shifting the accent and changing the relative time values. In the slightly more complicated Example 6-2*c*, the key center will be seen to be shifted from A minor to E minor merely by shifting the melodic, harmonic, and rhythmic emphasis.

Example 6-2

(*a*)



(*b*)



(c)

Compose a short sketch in three-part form using the hexad C-D-E-G-A-B. Begin with the A minor triad as the key center, modulating after twelve or sixteen measures to the G major triad as the key center and ending the first part in that key. Begin the second part with G major as the key center and after a few measures modulate to the key center of E minor. At the end of part two, modulate to the key center of C major for a few measures and back to the key of A minor for the beginning of the third part. In the third part, pass as rapidly as convenient from the key center of A minor to the key center of E minor, then to the key center of G major and back to A minor for the final cadence.

In writing this sketch, try to use as much of the material available in the hexad formation as possible. In other words, do not rely too heavily upon the major and minor triads. Since these modulations are all *modal* modulations, it is clear that the only tones to appear in the sketch will be the tones with which we started, C-D-E-G-A-B.

At first glance it may seem difficult or impossible to write an interesting sketch and to make convincing modal modulations with only six tones. It *is* difficult, but by no means impossible, and the discipline of producing *multum in parvo* will prove invaluable.

7

Key Modulation

In projecting the perfect-fifth relationship, we began with the tone C for convenience. It is obvious, however, that in equal temperament the starting point could have been any of the other tones of the chromatic scale. In other words, the pentatonic scale $C_2D_2E_3G_2A$ may be duplicated on D♭, as $D\flat_2E\flat_2F_3A\flat_2B\flat$; on D, as $D_2E_2F\sharp_3A_2B$; and so forth. It is therefore possible to use the familiar device of key modulation to modulate from any scale to an identical scale formation begun upon a different tone.

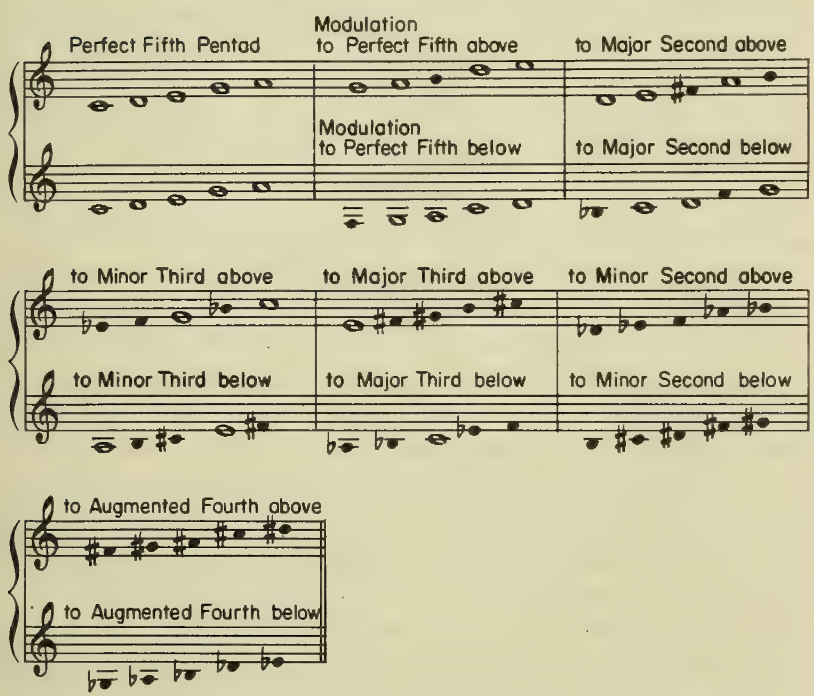
The closeness of relationship of such a modulation depends upon the number of common tones between the scale in the original key and the scale in the key to which the modulation is made. The pentatonic scale C-D-E-G-A, as we have already observed, contains the intervals $p^4mn^2s^3$. Therefore the key modulation to the fifth above or to the fifth below is the closest in relationship. It will have the greatest number of common tones, for the scale contains four perfect fifths. Since the scale contains three major seconds, the modulation to the key a major second above or below is the next closest relationship; the modulation to the key a minor third above or below is the next order of key relationship; the modulation to the key a major third above or below is next in order; and the last relationship is to the key a minor second above or below, or to the key related to the original tonic by the interval of the tritone.

A practical working-out of these modulations will illustrate this principle:

C-D-E-G-A modulating to the:

perfect	fifth	above	gives	G-A-*B*-D-E	(one *new* tone)
"	"	below	"	*F*-G-A-C-D	(" " ")
major	second	above	"	D-E-*F*♯-A-*B*	(two " tones)
"	"	below	"	*B*♭-C-D-*F*-G	(" " ")
minor	third	above	"	*E*♭-*F*-G-*B*♭-C	(three " ")
"	"	below	"	A-*B*-*C*♯-E-*F*♯	(" " ")
major	third	above	"	E-*F*♯-*G*♯-*B*-*C*♯	(four " ")
"	"	below	"	*A*♭-*B*♭-C-*E*♭-*F*	(" " ")
minor	second	above	"	*D*♭-*E*♭-*F*-*A*♭-*B*♭	(all new tones)
"	"	below	"	*B*-*C*♯-*D*♯-*F*♯-*G*♯	(" " ")
tritone		above or below	gives	*F*♯-*G*♯-*A*♯-*C*♯-*D*♯	(all new tones)

Example 7-1



The student should learn to distinguish as clearly as possible—though there will be debatable instances—between, for example, (1) a modulation from the pentatonic scale C-D-E-G-A to the pentatonic scale A-B-C♯-E-F♯, and (2) the eight-tone perfect-fifth scale, C-C♯-D-E-F♯-G-A-B, which contains all of the tones of both pentatonic scales. In the former instance, the two pentatonic scales preserve their identity and there is a clear point at which the modulation from one to the other occurs. In the latter case, all of the eight tones have equal validity in the scale and all are used within the same melodic-harmonic pattern.

In the first of the two following examples, 7-2, there is a definite point where the pentatonic scale on C stops and the pentatonic scale on A begins.

Example 7-2

In the second example, 7-3, all of the eight tones are members of one melodic scale.

Example 7-3

Although modal modulation is the most subtle and delicate form of modulation, of particular importance to the young composer in an age in which it seems to be the fashion to throw the entire tonal palette at the listener, it does not add new material to the tonal fabric. This task is accomplished either by the "expansion" technic referred to on page 36 or by the familiar device of key modulation.

Key modulation offers the advantages of allowing the composer to remain in the same tonal milieu and at the same time to

add new tones to the pattern. A composer of the classic period might—at least in theory—modulate freely to any of the twelve major keys and still confine himself to *one* type of tonal material, that of the major scale. Such modulations might be performed deliberately and leisurely—for example, at cadential points in the formal design—or might be made rapidly and restlessly within the fabric of the structure. In either case, the general impression of a "major key" tonal structure could be preserved.

This same device is equally applicable to any form of the perfect-fifth projection, or to any of the more exotic scale forms. The principle is the same. The composer may choose the tonal pattern which he wishes to follow and cling to it, even though he may in the process modulate to every one of the twelve possible key relationships.

It is obvious that the richest and fullest use of modulation would involve both modal modulation *and* key modulation used successively or even concurrently.

Write an experimental sketch, using as your basic material the perfect-fifth-pentatonic scale C-D-E-G-A. Begin in the key of C, being careful to use *only* the five tones of the scale and modulate to the same scale on E (E-F♯-G♯-B-C♯). Now modulate to the scale on F♯ (F♯-G♯-A♯-C♯-D♯) and from F♯ to E♭ (E♭-F-G-B♭-C). Now perform a combined modal and key modulation by going from the pentatonic scale on E♭ to the pentatonic scale on B (B-C♯-D♯-F♯-G♯), but with G♯ as the key center. Conclude by modulating to the pentatonic scale on F, with D as the key center (F-G-A-C-D), and back to the original key center of C.

You will observe that the first modulation—C to E—retains only one common tone. The second modulation, from E to F♯, retains three common tones. The third, from F♯ to E♭, has two common tones. The fourth, from E♭ to B, like the first modulation, has only one common tone. The fifth, from B to F, has no common tones, and the sixth, from F to C, has four common tones.

If you play the key centers successively, you will find that

only one transition offers any real problem: the modulation from B, with G♯ as the key center, to F, with D as the key center. It will require some ingenuity on your part to make this sound convincing.

Work out the modulations of the perfect-fifth *hexad* at the intervals of the perfect fifth, major second, minor third, major third, minor second and tritone, as in Example 7-1.

8

Projection of the Minor Second

THERE IS ONLY ONE interval, in addition to the perfect fifth, which, projected above itself, gives all of the tones of the twelve-tone scale. This is, of course, the minor second, or its inversion, the major seventh.

Proceeding, therefore, as in the case of the perfect-fifth projection, we may superimpose one minor second upon another, proceeding from the two-tone to the twelve-tone series.

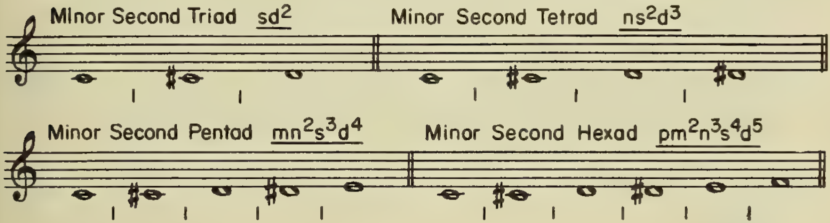
Examining the minor-second series, we observe that the basic triad C-C♯-D contains two minor seconds and the major second C-D: sd^2.

The basic tetrad, C-C♯-D-D♯, adds another minor second, another major second, and the minor third: ns^2d^3.

The basic pentad, C-C♯-D-D♯-E, adds another minor second, another major second, another minor third, and a major third: $mn^2s^3d^4$.

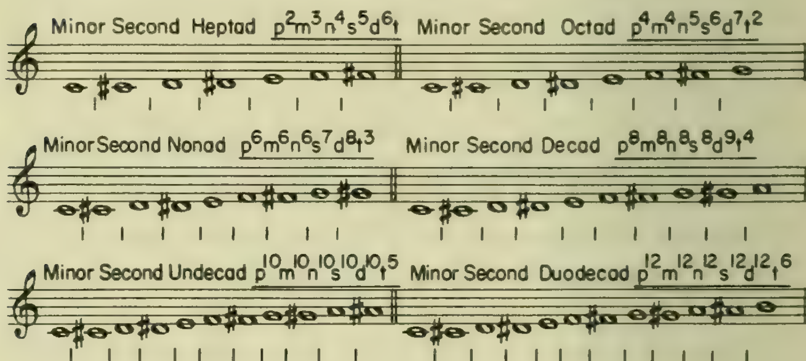
The basic hexad, C-C♯-D-D♯-E-F, adds another minor second, another major second, another minor third, another major third, and a perfect fourth: $pm^2n^3s^4d^5$:

EXAMPLE 8-1



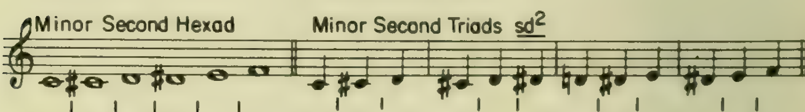
The seven-, eight-, nine-, ten-, eleven- and twelve-tone minor-second scales follow, with the interval analysis of each. The student will notice the same phenomenon which was observed in the perfect-fifth projection: whereas each successive projection from the two-tone to the seven-tone scale adds one new interval, after the seven-tone projection has been reached no *new* intervals can be added. Furthermore, from the seven-tone to the eleven-tone projection, the *quantitative* difference in the proportion of intervals also decreases progressively as each new tone is added.

EXAMPLE 8-2



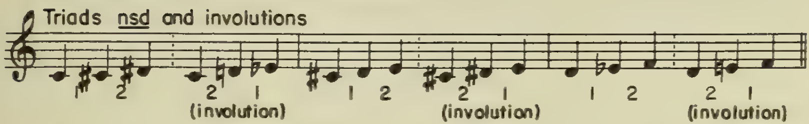
Proceeding again, as in Chapter 5, we may now examine the harmonic-melodic material of the minor-second hexad. First, we have the basic triad C-C♯-D, sd^2, duplicated on the tones C♯, D, and D♯:

EXAMPLE 8-3



The triad $C_1C\sharp_2D\sharp$, *nsd*, a form observed in the perfect-fifth hexad, duplicated on C♯ and D, with their involutions:

EXAMPLE 8-4

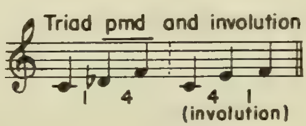


The triad $C_1C\sharp_3E$, *mnd,* duplicated on C♯, with their involutions:

EXAMPLE 8-5

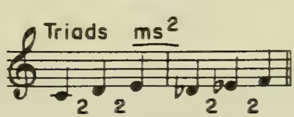
The triad $C_1D\flat_4F$, *pmd,* with its involution C_4E_1F; which has already been found in the perfect-fifth hexad:

EXAMPLE 8-6



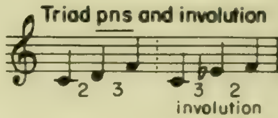
The isometric triad C-D-E, *ms*^{2}, which has already occurred as a part of the perfect-fifth hexad; duplicated on D♭;

EXAMPLE 8-7



and the triad C_2D_3F, *pns*, with its involution, $C_3E\flat_2F$, which form also has been encountered in the perfect-fifth series:

EXAMPLE 8-8

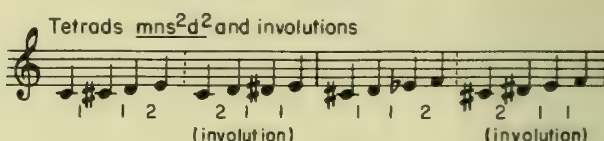


The minor-second hexad contains the basic tetrad $C_1C\sharp_1D_1D\sharp$, ns^2d^3, duplicated on C♯ and on D:

EXAMPLE 8-9

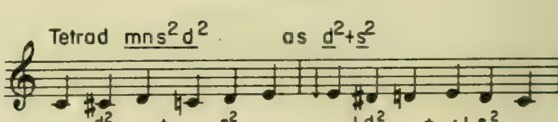
The tetrad $C_1C\sharp_1D_2E$, mns^2d^2, duplicated on C♯, with their respective involutions;

EXAMPLE 8-10



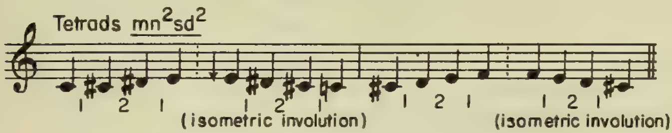
which may be analyzed as the simultaneous projection of two minor seconds and two major seconds above C, or, in its involution, below E:

EXAMPLE 8-11



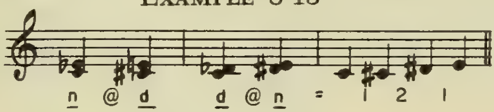
The isometric tetrad $C_1C\sharp_2D\sharp_1E$, mn^2sd^2, duplicated on C♯;

EXAMPLE 8-12



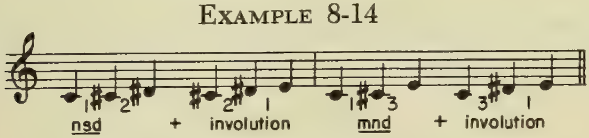
which may be analyzed as two minor thirds at the relationship of the minor second, or two minor seconds at the relationship of the minor third:

EXAMPLE 8-13



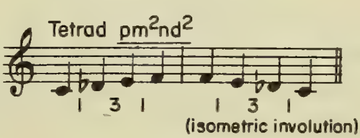
or as a combination of the triad *nsd* and the involution on C♯, or the triad *mnd* and its involution:

EXAMPLE 8-14



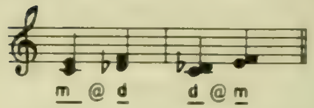
The isometric tetrad $C_1D\flat_3E_1F$, pm^2nd^2;

EXAMPLE 8-15



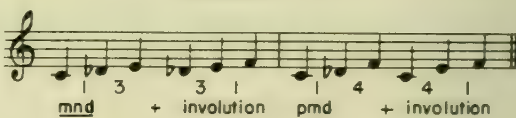
which may be analyzed as consisting of two major thirds at the interval of the minor second, or of two minor seconds at the interval of the major third;

EXAMPLE 8-16



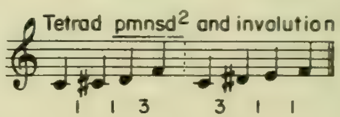
or as a combination of the triad *mnd*, and the involution on D♭, or the triad *pmd*, and its involution:

EXAMPLE 8-17



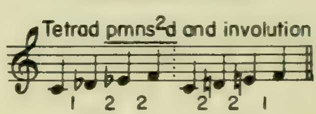
The tetrad $C_1C\sharp_1D_3F$, $pmnsd^2$, and its involution:

EXAMPLE 8-18



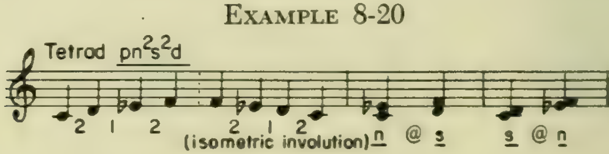
The tetrad $C_1D\flat_2E\flat_2F$, $pmns^2d$, and its involution, which has already been found in the perfect-fifth projection;

EXAMPLE 8-19



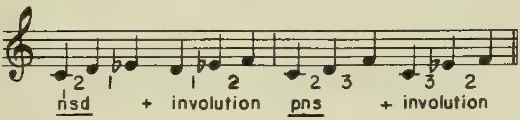
and the isometric tetrad $C_2D_1E\flat_2F$, pn^2s^2d, which is also a part of the perfect-fifth hexad, and which may be analyzed as a combination of two minor thirds at the interval of the major second, or of two major seconds at the interval of the minor third:

EXAMPLE 8-20



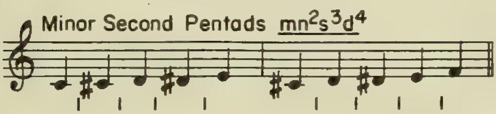
The student will observe that the tetrad C-D-E♭-F may also be analyzed as a combination of the triad *nsd* and the involution on D, or the triad *pns* and its involution:

EXAMPLE 8-21



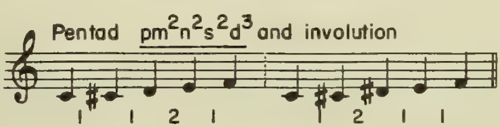
Finally, the pentads in the minor-second hexad consist of the basic pentad $C_1C\sharp_1D_1D\sharp_1E$, $mn^2s^3d^4$, duplicated on C♯;

EXAMPLE 8-22



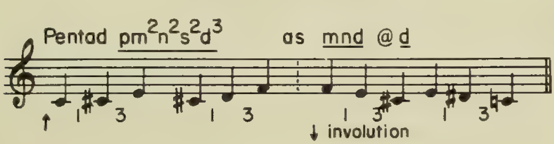
the pentad $C_1C\sharp_1D_2E_1F$, $pm^2n^2s^2d^3$, with its involution, $C_1C\sharp_2D\sharp_1E_1F$;

EXAMPLE 8-23



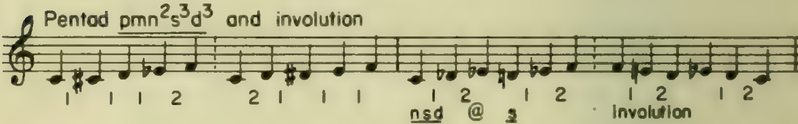
which may be analyzed as the relationship of two triads *mnd*, at the interval of the minor second:

EXAMPLE 8-24



and the pentad $C_1C\sharp_1D_1E\flat_2F$, *$pmn^2s^3d^3$*, with its involution, $C_2D_1D\sharp_1E_1F$, which may be analyzed as the combination of two triads *nsd*, at the interval of the major second:

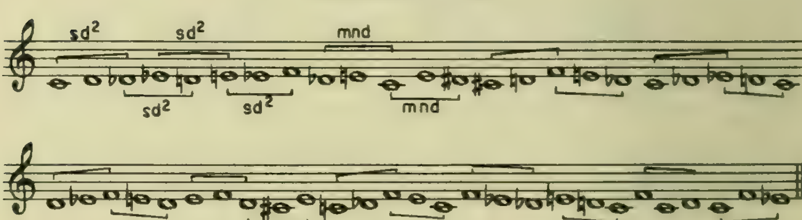
Example 8-25



The minor-second hexad is, quite obviously, a highly dissonant scale. For this reason it has perhaps less harmonic than melodic value. It may be effectively used in two-line or three-line contrapuntal passages where the impact of the thick and heavy dissonance is somewhat lessened by the rhythmic movement of the melodic lines.

Example 8-26 constitutes a mild puzzle. It is constructed to have the same arithmetic, or perhaps I should say geometric relationships, as the melodic line in Example 5-33. It should take only a short examination to discover what this relationship is.

Example 8-26



The six-tone minor-second scale will be found to be too limited in compass to give the composer much opportunity in this restricted form. Nevertheless, it is valuable to become intimately acquainted with the small words and syllables which

go to make up the vocabulary of this series, since these small words constitute an important part of the material of some contemporary music. Therefore, I suggest that you play through Example 8-26 slowly and thoughtfully, since it contains all of the triads of the minor-second hexad. Since I have kept all of these triads in close position, the melody is even "wormier" than such melodies need be.

Complete the analysis of all of the melodic triads under the connecting lines and then play through the melody at a more rapid tempo with the phrasing as indicated in Example 8-27. See if you can sing the melody through without the aid of a piano and come out on pitch on the final E♭.

Example 8-27

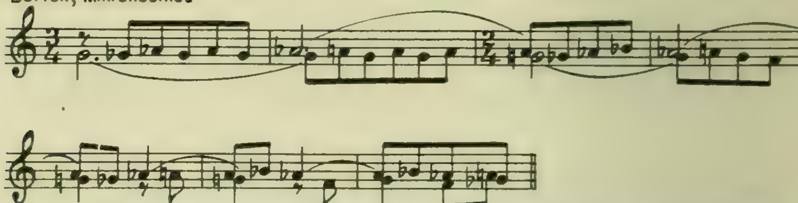
Example 8-28 is a four-measure theme constructed in the minor-second hexad. Continue its development in two-part simple counterpoint, allowing one modulation to the "key" of G—G-G♯-A-A♯-B-C—and modulating back again to the original "key" of C.

Example 8-28

It is difficult to find many examples of the effective use of the minor-second hexad in any extended form in musical literature because of its obvious limitations. A charming example is found in "From the Diary of a Fly" from the *Mikrokosmos* of Béla Bartók. The first nine measures are built on the six-tone scale F-G♭-G♮-A♭-A♮-B♭. The tenth measure adds the seventh tone, C♭.

EXAMPLE 8-29

Bartok, Mikrokosmos



On the other hand, examples of the utilization of the *entire* chromatic scale within a short passage abound in contemporary music, one of the most imaginative of which can be found in the first movement of the *Sixth Quartet* of the same composer:

EXAMPLE 8-30

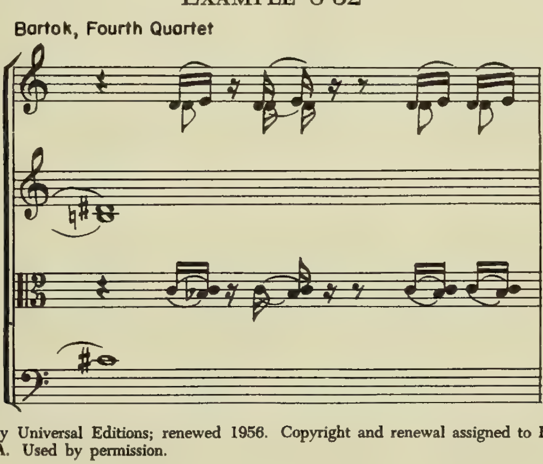
Bartok, Sixth Quartet

A more obvious example of the use of the minor-second scale is found at the beginning of the second movement of the Bartók *Fourth String Quartet*:

EXAMPLE 8-31

A more subtle example—and one very characteristic of the Hungarian master—is found in the twenty-fifth measure of the first movement of the same quartet. Here the tonal material consists of the seven-tone minor-second scale B♭-B♮-C-C♯-D-D♯-E, but divided into two major-second segments, the cello and second violin holding the major-second triad, B-C♯-D♯, and the first violin and viola utilizing the major-second tetrad, B♭-C-D-E:

EXAMPLE 8-32



Analyze the first movement of the Bartók *Sixth Quartet* to determine how much of it is constructed in the minor-second projection.

Modulation of the minor-second pentad follows the same principle as the perfect-fifth pentad. Modulation at the minor second produces one new tone, at the major second two new tones, at the minor third three new tones, at the major third four new tones, and at the perfect fifth and tritone five new tones.

Work out all of the modulations of the minor-second pentad and hexad.

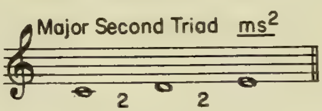
9

Projection of the Major Second

Since the major second is the concomitant interval resulting from the projection of either two perfect fifths or of two minor seconds, it would seem to be the most logical interval to choose for our next series of projections.*

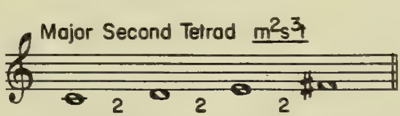
The basic triad of the major-second series is C_2D_2E,

Example 9-1



two major seconds with their concomitant interval of the major third: ms^2. We have already observed this triad as a part of both the perfect-fifth and the minor-second hexads. The third major second produces the tetrad $C_2D_2E_2F\sharp$, adding the new interval of the tritone, C to F♯. The analysis of this sonority becomes three major seconds, two major thirds, and one tritone: m^2s^3t.

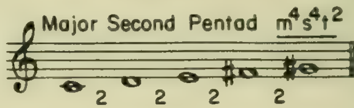
Example 9-2



* The major second would also seem to follow the perfect fifth and minor second, since it can be projected to a pure six-tone scale, whereas the minor third and the major third can be projected only to four and three tones, respectively.

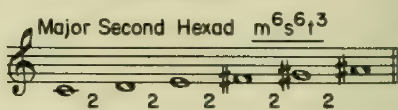
Superimposing another major second produces the pentad $C_2D_2E_2F\sharp_2G\sharp$, which consists of four major seconds; four major thirds, C to E, D to F♯, E to G♯, G♯ (A♭) to C; and two tritones, C to F♯ and D to G♯: $m^4s^4t^2$.

EXAMPLE 9-3



The superposition of one more major second produces the "whole-tone" scale $C_2D_2E_2F\sharp_2G\sharp_2A\sharp$:

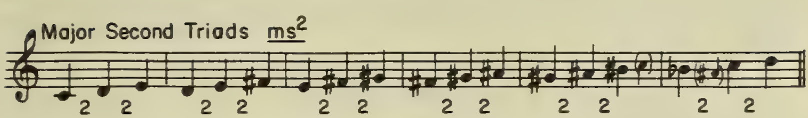
EXAMPLE 9-4



This scale will be seen to consist of six major thirds—C to E, D to F♯, E to G♯, F♯ to A♯, G♯ to B♯ (C) and A♯ (B♭) to D; six major seconds—C to D, D to E, E to F♯, F♯ to G♯, G♯ to A♯, and A♯ (B♭) to C; and three tritones—C to F♯, D to G♯, and E to A♯. Its analysis is $m^6s^6t^3$. It will be obvious that the scale cannot be projected beyond the hexad as a pure major-second scale, since the next major second would be B♯, the enharmonic equivalent of C.

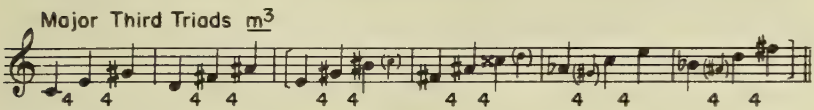
The major-second hexad is an enharmonic isometric scale; not only is its form the same whether thought of clockwise or counterclockwise, up or down, but its involution produces the *identical* tones. Analyzing its components, we find that it has three different types of triads: the basic triad C_2D_2E, ms^2, duplicated on D, E, F♯, G♯, and A♯;

EXAMPLE 9-5



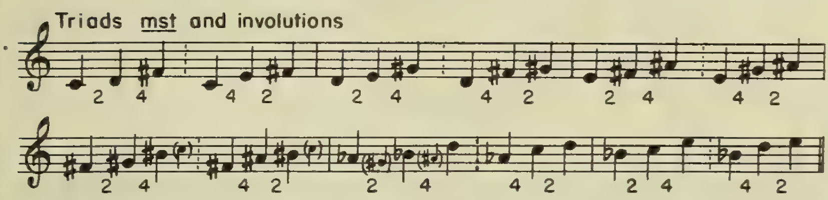
the augmented triad $C_4E_4G\sharp$, m^3, duplicated on D (since the remaining four augmented triads are merely inversions of those on C and D);

EXAMPLE 9-6



and the triad $C_2D_4F\sharp$, *mst*, and its involution, $C_4E_2F\sharp$, also duplicated on the other five notes of the scale:

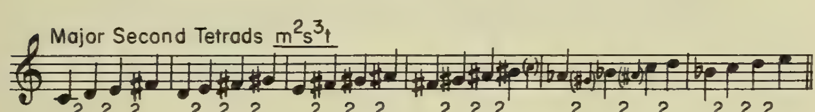
EXAMPLE 9-7



The basic triad we have already analyzed as containing two major seconds and a major third, ms^2. The augmented triad contains three major thirds, C to E, E to G♯, and G♯ (A♭) to C, m^3. The triad $C_2D_4F\sharp$ and its involution $C_4E_2F\sharp$, contain one major second, one major third, and one tritone, *mst*.

The major-second hexad contains three different types of tetrads: the basic tetrad $C_2D_2E_2F\sharp$, m^2s^3t, duplicated on D, E, F♯, A♭, and B♭;

EXAMPLE 9-8

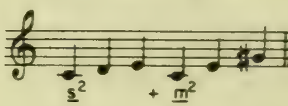


the isometric tetrad $C_2D_2E_4G\sharp$, duplicated on D, E, G♭, A♭, and B♭, containing three major thirds, two major seconds, and one tritone, m^3s^2t;

Example 9-9

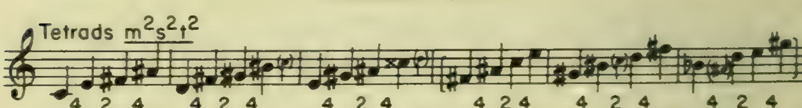
which may also be considered to be formed by the simultaneous projection of two major seconds and two major thirds;

Example 9-10



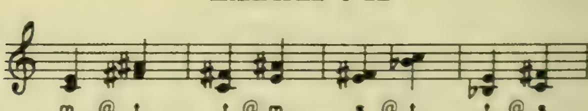
and the isometric tetrad $C_4E_2F\sharp_4A\sharp$, duplicated on D and E, which contains two major thirds, two major seconds, and two tritones, $m^2s^2t^2$:

Example 9-11



This may also be analyzed as two major thirds at the interval of the tritone; as two tritones, at the interval of the major third; as two major seconds at the interval of the tritone, or as two tritones at the interval of the major second.

Example 9-12



This highly isometric sonority was a favorite of Scriabine, particularly in the *Poeme de l'Extase.*

There is only one type of pentad in the six-tone major-second scale, since the remaining five pentads are merely *transpositions* of the first:

EXAMPLE 9-13

An examination of this series will show both its strength and weakness. Its strength lies in the complete consistency of its material. It is one of the most homogeneous of all scales, since it is made up exclusively of major thirds, major seconds, and tritones. It is only mildly dissonant in character, since it contains no primary dissonances (the minor second or major seventh).

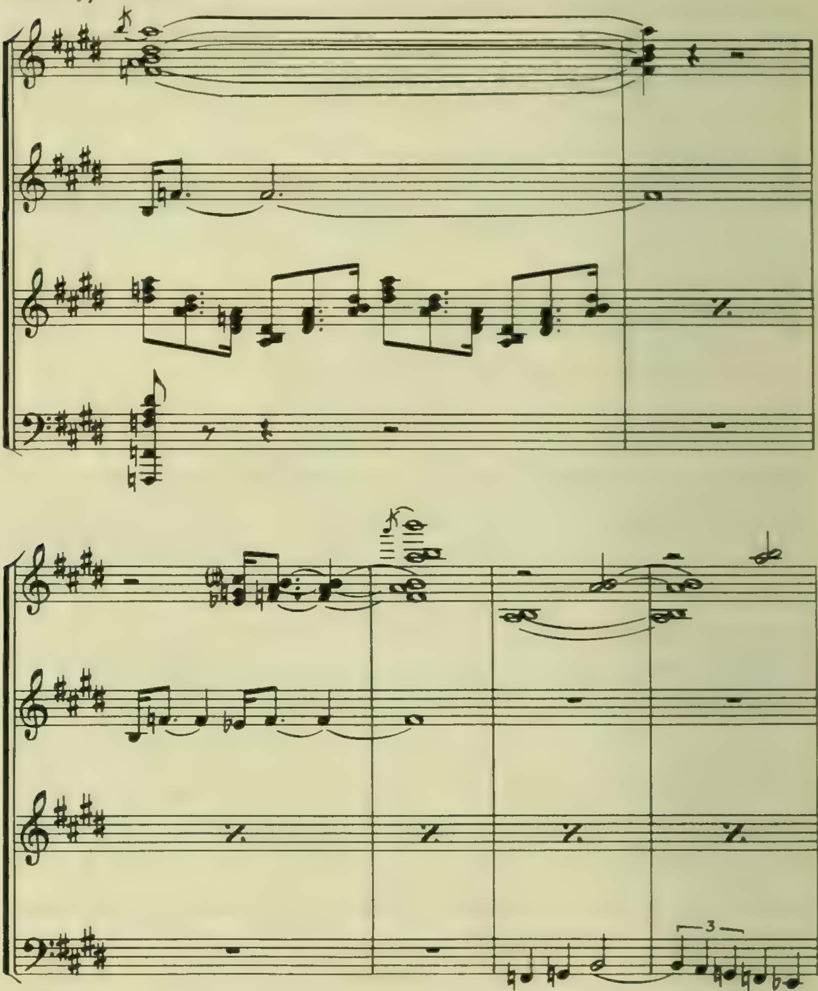
Its very homogeneity is also its weakness, for the absence of contrasting tonal combinations gives, in prolonged use, a feeling of monotony. Also, the absence of the perfect fifth deprives the scale of any consonant "resting-place," or tonic, so that its progressions sound vague, lacking in contrast, and without direction. Nevertheless, it is an important part of the tonal vocabulary and, in the hands of a genius, adds a valuable color to the tonal palette which should not be lightly discarded by the young composer. Its effective use is illustrated in Debussy's "Voiles," the first thirty measures of the first section of which are written entirely in the whole-tone scale.

The same composer's "La Mer" contains extended use of the

same scale in the excerpt below:

Example 9-14

Debussy, "La Mer"



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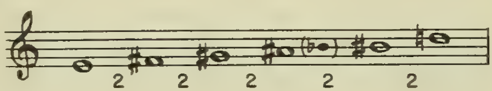
An example of the whole-tone scale where it might not be expected is found in the opening of an early song, "Nacht," of

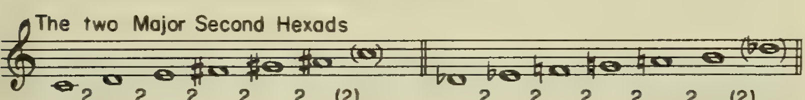
Alban Berg, the first five measures of which are in one of the two forms of the whole tone scale:

EXAMPLE 9-15



It will be observed that whereas the perfect-fifth and minor-second series may be transposed to eleven different pitches, giving ample opportunity for modulation, there is only one effective modulation for the whole-tone scale—the modulation to the whole-tone scale a half-tone above or below it, that is, from the scale C-D-E-F♯-G♯-A♯ to the scale D♭-E♭-F-G-A-B. Modal modulation is impractical, since the whole-tone scales on C, D, E, etc., all have the same configuration:

EXAMPLE 9-16



In the introduction to *Pelléas et Mélisande* Debussy begins with the material of the perfect-fifth pentad for the first four measures—C-D-E-G-A, changes to the pure whole-tone scale for

the fifth, sixth, and seventh measures, and returns to the perfect fifth-series in measures 8 to 11:

EXAMPLE 9-17

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From the same opera we find interesting examples of the use of whole-tone patterns *within the twelve-tone scale* by alternating rapidly between the two whole-tone systems:

EXAMPLE 9-18

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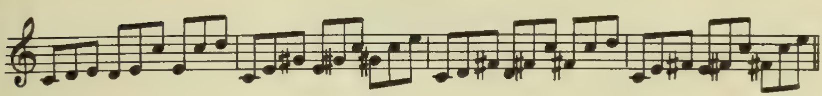
Whereas the minor-second hexad may not be as bad as it

sounds, the careless use of the whole-tone scale frequently makes it sound worse than it is, particularly when used by casual improvisors. Because of the homogeneity of its material, it is often used in the most obvious manner, which destroys the subtle nuances of which it is capable and substitutes a "glob" of "tone color."

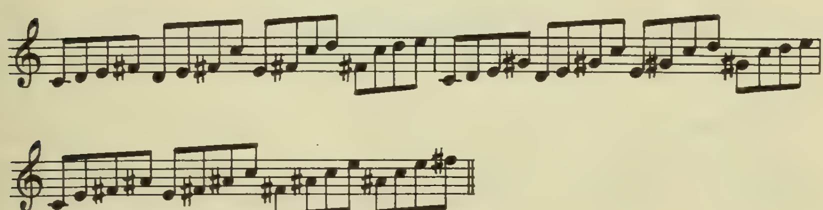
The author is not making a plea for the return of the whole-tone scale in its unadulterated form, but it must be said that this scale has qualities that should not be too lightly cast aside. Example 9-19*a* gives the triads; 19*b* the tetrads, 19*c* the pentad, and 19*d* the hexad, which are found in the six-tone scale. Play them carefully, analyze each, and note their tonal characteristics in the different positions or inversions.

Example 9-19

(*a*)

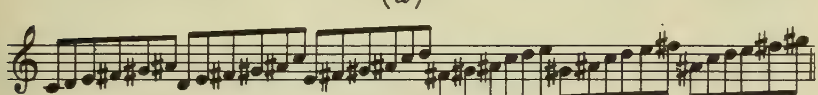


(*b*)



(*c*)

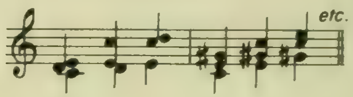
(*d*)



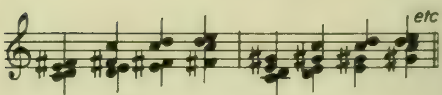
Play the triad types in block form as in Example 9-20*a*. Repeat the same process for the tetrad types in 20*b*; for the pentad type in 20*c*; and for the hexad in 20*d*.

Example 9-20

(*a*)



(*b*)



(*c*)

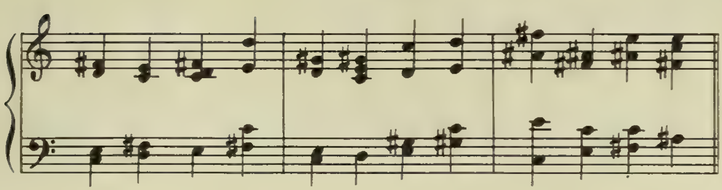
(*d*)

In Example 9-21*a*, experiment with the triad types in various positions. Repeat the same process for the tetrads, as in 21*b*; for the pentad, as in 21*c*; for the hexad, as in 21*d*.

Example 9-21

(*a*)

(*b*)



(*c*)

(*d*)

Experiment with different doublings and positions of all of the above sonorities, as in Example 9-22.

EXAMPLE 9-22

Have the material of Example 9-21 played for you in different order and take it down from dictation, trying to reproduce not only the notes but their exact position.

Analyze in detail the first section of Debussy's "Voiles" and note

not only his use of the widest resources of the scale but also his employment of the devices of change of position and doubling.

In detailed analysis it seems generally wise to analyze every note in a passage regardless of its relative importance, rather than dismissing certain notes as "nonharmonic" or "unessential" tones, for all tones in a passage are important, even though they may be only appoggiaturas or some other form of ornamentation. Occasionally, however, the exclusion of such "unessential" tones seems obvious. The thirty-first measure of Debussy's "Voiles" offers an excellent example of such an occasion. Every note in every measure preceding and following this measure in the first section of the composition is in the six-tone major-second scale, A♭-B♭-C-D-E-F♯, with the exception of the two notes G and D♭ in measure 31. Since both of these notes were quite obviously conceived as passing tones, it would seem unrealistic to analyze them as integral parts of the tonal complex.

Example 9-23

In using any of the tonal material presented in these chapters, one all-important principle should be followed: that the composer should train himself to *hear* the sounds which he uses *before* he writes them. There is reason to fear that some young composers—and some not so young—have been tempted at times to use tonal relationships which are too complex for their own aural comprehension. This is comparable to the use by a writer of words which he does not himself understand—an extremely hazardous practice!

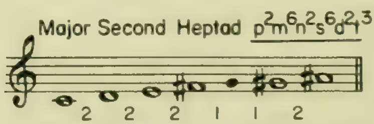
When you feel confident of your understanding of the material, write a short sketch which begins with the use of the major-second hexad on C, modulates to the major-second hexad on G, and returns at the end to the original hexad on C. See to it that you do not *mix* the two scales, so that the sketch consists entirely of major-second material.

10

Projection of the Major Second Beyond the Six-Tone Series

We have already observed that the major-second scale in its pure form cannot be extended beyond six tones, since the sixth major second duplicates the starting tone. We can, however, produce a seven-tone scale which consists of the six-tone major-second scale with a foreign tone added, and then proceed to superimpose major seconds above this foreign tone. We may select this foreign tone arbitrarily from any of the tones which are not in the original whole-tone scale. If we take, for example, the perfect fifth above C as the foreign tone to be added, we produce the seven-tone scale $C_2D_2E_2F\sharp_1G_1{}^*G\sharp_2A\sharp(C_2)$. (The foreign tone is indicated by an asterisk to the right of the letter name.) This again proves to be an isometric scale having the same configuration of half-steps downward, 2221122; since if we begin on the tone D and form the scale downward with the same order of whole- and half-steps, we shall produce the same scale, $\downarrow D_2C_2B\flat_2A\flat_1G_1F\sharp_2E_{(2)}(D)$:

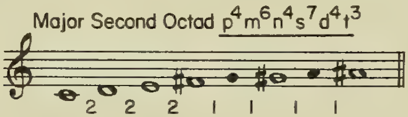
Example 10-1



* It should be noted that the choice of G as the added foreign tone is arbitrary. The addition of any other foreign tone would produce only a different *version* of the same scale; for example, $C_1C\sharp_1D_2E_2F\sharp_2G\sharp_2A\sharp_{(2)}(C)$.

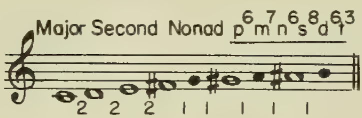
We may now form the eight-tone scale by adding a major second above G, that is, A: $C_2D_2E_2F\sharp_1G_1{}^*G\sharp_1A_1{}^*A\sharp_{(2)}(C)$:

EXAMPLE 10-2



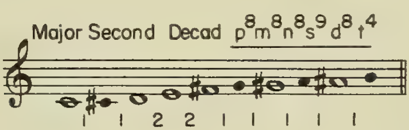
The nine-tone scale becomes, then, the above scale with the major second above A added, that is, B:
$C_2D_2E_2F\sharp_1G_1{}^*G\sharp_1A_1{}^*A\sharp_1B_{(1)}{}^*(C)$:

EXAMPLE 10-3



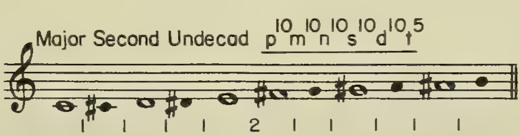
The ten-tone scale adds the major second above B, namely, C♯,
$C_1C\sharp_1{}^*D_2E_2F\sharp_1G_1{}^*G\sharp_1A_1{}^*A\sharp_1B_{(1)}{}^*(C)$:

EXAMPLE 10-4



The eleven-tone scale adds the major second above C♯, namely, D♯, $C_1C\sharp_1{}^*D_1D\sharp_1{}^*E_2F\sharp_1G_1{}^*G\sharp_1A_1{}^*A\sharp_1B_{(1)}{}^*(C)$:

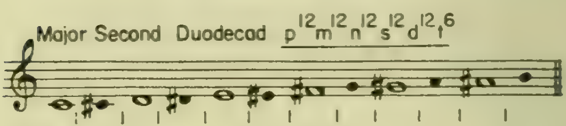
EXAMPLE 10-5



The twelve-tone scale adds the major second above D♯, that is, E♯, and merges with the chromatic scale,

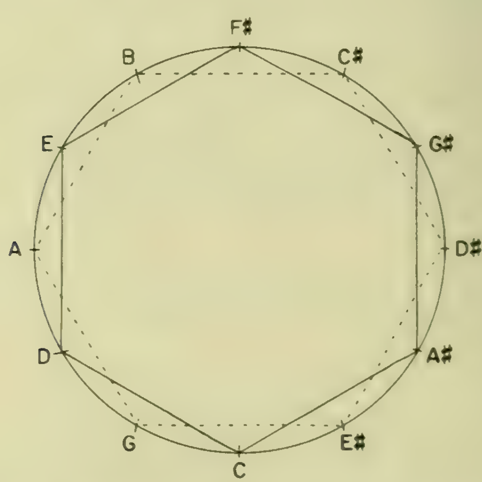
$$C_1 C\sharp_1 {}^\circ D_1 D\sharp_1 {}^\circ E_1 E\sharp_1 {}^\circ F\sharp_1 G_1 {}^\circ G\sharp_1 A_1 {}^\circ A\sharp_1 B_{(1)} {}^\circ (C):$$

EXAMPLE 10-6



If we diagram this projection in terms of the twelve-tone perfect-fifth series, we find that we have produced two hexagons, the first consisting of the tones C-D-E-F♯-G♯-A♯, and the second consisting of the tones G-A-B-C♯-D♯-E♯. We employ first all of the tones of the first hexagon, and then move to the second hexagon a perfect fifth above the first and again proceed to add the six tones found in that hexagon.

EXAMPLE 10-7



The following table gives the complete projection of the major-second scale with the intervallic analysis of each:

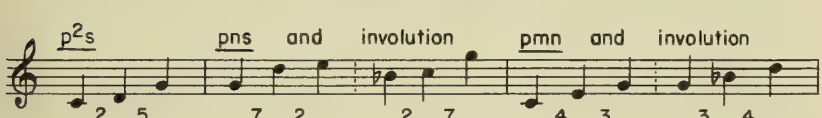
C D	s
C D E	ms^2
C D E F♯	m^2s^3t
C D E F♯ G♯	$m^4s^4t^2$
C D E F♯ G♯ A♯	$m^6s^6t^3$
C D E F♯ G G♯ A♯	$p^2m^6n^2s^6d^2t^3$
C D E F♯ G G♯ A A♯	$p^4m^6n^4s^7d^4t^3$
C D E F♯ G G♯ A A♯ B	$p^6m^7n^6s^8d^6t^3$
C C♯ D E F♯ G G♯ A A♯ B	$p^8m^8n^8s^9d^8t^4$
C C♯ D D♯ E F♯ G G♯ A A♯ B	$p^{10}m^{10}n^{10}s^{10}d^{10}t^5$
C C♯ D D♯ E E♯ F♯ G G♯ A A♯ B	$p^{12}m^{12}n^{12}s^{12}d^{12}t^6$

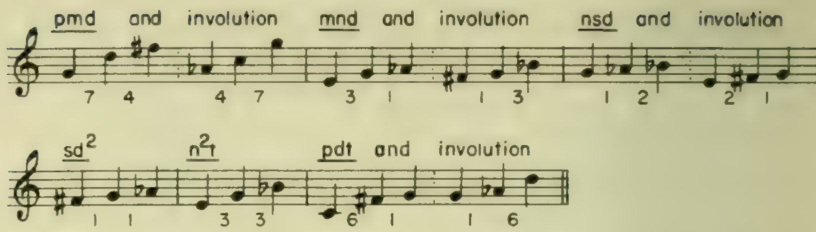
We have already observed that the six-tone major-second scale contains only the intervals of the major third, the major second, and the tritone. The addition of the tone G to the six-tone scale preserves the preponderance of these intervals but adds the new intervals of the perfect fifth, C to G and G to D; the minor thirds, E to G and G to B♭; and the minor seconds, F♯ to G and G to A♭.

It adds the isometric triad p^2s, C_2D_5G; the triad *pns*, G_7D_2E, and the involution $B♭_2C_7G$; the triad *pmn*, C_4E_3G, and the involution $G_3B♭_4D$; the triad *pmd*, $G_7D_4F♯$, and the involution $A♭_4C_7G$; the triad *mnd*, $E_3G_1A♭$, and the involution $F♯_1G_3B♭$; the triad *nsd*, $G_1A♭_2B♭$, and the involution $E_2F♯_1G$; the two isometric triads, sd^2, $F♯_1G_1A♭$, and n^2t, $E_3G_3B♭$; and the triad *pdt*, $C_6F♯_1G$, with the involution $G_1A♭_6D$.

The addition of these triad forms to the three which are a part of the major-second hexad, ms^2, m^3, and *mst*, gives this seven-tone scale *all of the triad types* which are possible in the twelve-tone scale.

EXAMPLE 10-8



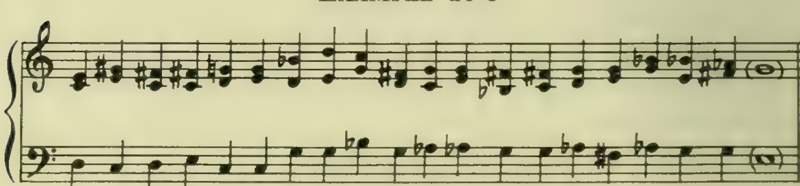


The seven-tone impure major-second scale therefore has certain advantages over the pure six-tone form, since it preserves the general characteristic of the preponderance of major seconds, major thirds, and tritones but adds a wide variety of new tonal material.

For the reasons given earlier, we shall spend most of our time experimenting with various types of six-tone projections, since we find in the six-tone scales the maximum of individuality and variety. We shall make an exception in the case of the major-second projection, however, and write one sketch in the seven-tone major-second scale, since the addition of the foreign tone to the major-second hexad adds variety to this too homogeneous scale without at the same time entirely destroying its character. It is a fascinating scale, having some of the characteristics of a "major" scale, some of the characteristics of a "minor" scale, and all of the characteristics of a whole-tone scale.

Begin by playing Example 10-9, which contains all of the triads of the scale. Listen carefully to each triad and then complete the analysis.

Example 10-9



Example 10-10 contains all of the tetrad types, but in no regular order. Play the example through several times as sensi-

tively as possible, perhaps with a crescendo in the third and fourth measures to the first beat of the fifth measure, and then a diminuendo to the end. Note the strong harmonic accent between the last chord of the fifth measure and the first chord of the sixth measure, even though the tones of the two chords are identical.

Have another student play the example for you and write it accurately from dictation. Now analyze all of the chords as to formation *including* the sonorities formed by passing tones.

Example 10-10

The following measure from Debussy's *Pelléas et Mélisande* offers a simple illustration of the seven-tone major-second scale, the foreign tone, E♭, merely serving as a passing tone:

Example 10-11

Debussy, "Pelleos and Melisonde"

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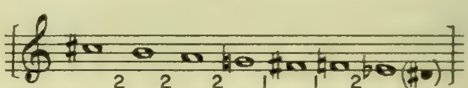
A somewhat more complicated illustration is found in the Alban Berg song, "Nacht," already referred to as beginning in the pure whole-tone scale:

EXAMPLE 10-12



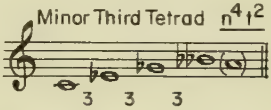
The student should now be ready to write a free improvisatory sketch employing the materials of this scale (Example 10-1). He will notice that the scale has two natural resting points, one on C major and one on G minor. Begin the sketch in G minor, modulate modally to C, establish C as the key center, and then modulate back to the original key center of G. See that only the tones C-D-E-F♯-G-A♭-B♭ are employed in this sketch, but get as much variety as possible from the harmonic-melodic material of the scale.

11

Projection of the Minor Third

THE NEXT SERIES of projections which we shall consider is the projection of the minor third. Beginning with the tone C we superimpose the minor third E♭, then the minor third G♭, forming the diminished triad $C_3E\flat_3G\flat$, which consists of two minor thirds and the concomitant tritone, from C to G♭. Upon this we superimpose the minor third above G♭, B♭♭, which we shall call by its enharmonic equivalent, A, forming the familiar tetrad of the "diminished seventh," consisting of four minor thirds: C to E♭, E♭ to G♭, G♭ to B♭♭ (A), and A to C; and two tritones: C to G♭ and E♭ to A; symbol, n^4t^2:

EXAMPLE 11-1

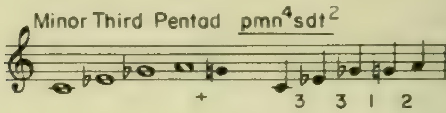


As in the case of the major-second scale, which could not be projected in pure form beyond six tones, so the minor third cannot be projected in pure form beyond four tones, since the next minor third above A duplicates the starting tone, C. If we wish to extend this projection beyond four tones we must, again, introduce an arbitrary foreign tone, such as the perfect fifth, G, and begin a new series of minor-third projections upon the foreign tone.*

* The choice of the foreign tone is not important, since the addition of *any* foreign tone would produce either a different version, or the involution, of the *same* scale.

The minor-third pentad, therefore, becomes $C_3E\flat_3G\flat_1G\natural_2A$:

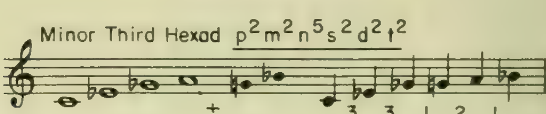
EXAMPLE 11-2



It contains, in addition to the four minor thirds and two tritones already noted, the perfect fifth, C to G; the major third, E♭ to G; the major second, G to A; and the minor second, G♭ to G. The analysis of the scale is, therefore, pmn^4sdt^2. The scale still has a preponderance of minor thirds and tritones, but also contains the remaining intervals as well.

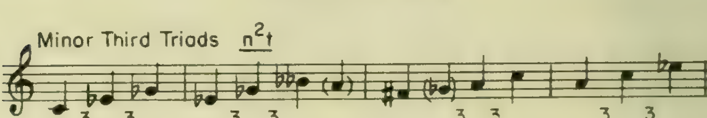
The six-tone scale adds a minor third above the foreign tone G, that is, B♭, the melodic scale now becoming $C_3E\flat_3G\flat_1G_2A_1B\flat$. The new tone, B♭, adds another minor third, from G to B♭; a perfect fifth, from E♭ to B♭; a major third, from G♭ to B♭; a major second, from B♭ to C, and the minor second, A to B♭, the analysis being $p^2m^2n^5s^2d^2t^2$:

EXAMPLE 11-3



The component triads of the six-tone minor-third scale are the basic diminished triad $C_3E\flat_3G\flat$, n^2t, which is also duplicated on E♭, G♭, and A;

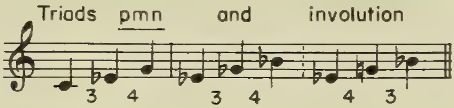
EXAMPLE 11-4



the minor triads $C_3E\flat_4G$ and $E\flat_3G\flat_4B\flat$, *pmn,* with the one involution, the major triad $E\flat_4G_3B\flat$, which are characteristic of

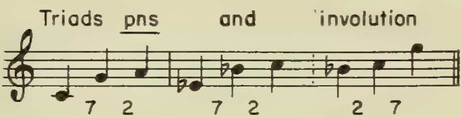
the perfect-fifth series;

EXAMPLE 11-5



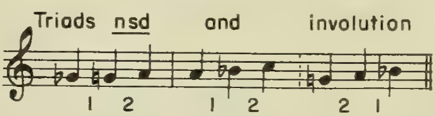
the triads C_7G_2A and $E\flat_7B\flat_2C$, *pns,* with the one involution $B\flat_2C_7G$; found in the perfect-fifth and minor-second series;

EXAMPLE 11-6



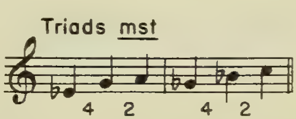
the triads $G\flat_1G\natural_2A$ and $A_1B\flat_2C$, *nsd,* with the one involution $G_2A_1B\flat$, which we have also met as parts of the perfect-fifth and minor-second projection;

EXAMPLE 11-7



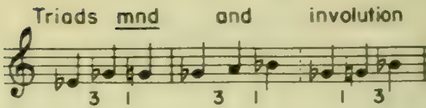
the triads $E\flat_4G_2A$ and $G\flat_4B\flat_2C$, *mst,* with no involution, which we have encountered as part of the major-second hexad;

EXAMPLE 11-8



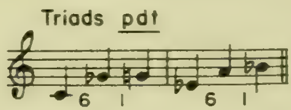
the triads $E\flat_3G\flat_1G\natural$ and $G\flat_3A_1B\flat$, *mnd,* with the one involution $G\flat_1G\natural_3B\flat$; which is a part of the minor-second hexad;

EXAMPLE 11-9



and the triads $C_6G\flat_1G$ and $E\flat_6A_1B\flat$, *pdt*, without involution, which are new in hexad formations:

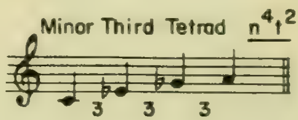
EXAMPLE 11-10



The student should study carefully the sound of the new triads which the minor-third series introduces. He will, undoubtedly, be thoroughly familiar with the first of these, the diminished triad, but he will probably be less familiar with the triad *pdt*. Since, as I have tried to emphasize before, *sound* is the all-important aspect of music, the student should play *and listen to* these "new" sounds, experimenting with different inversions and different doublings of tones until these sounds have become a part of his tonal vocabulary.

The tetrads of the six-tone minor-third scale consist of the basic tetrad $C_3E\flat_3G\flat_3A$, the familiar diminished seventh chord, consisting of four minor thirds and two tritones, n^4t^2, already discussed;

EXAMPLE 11-11

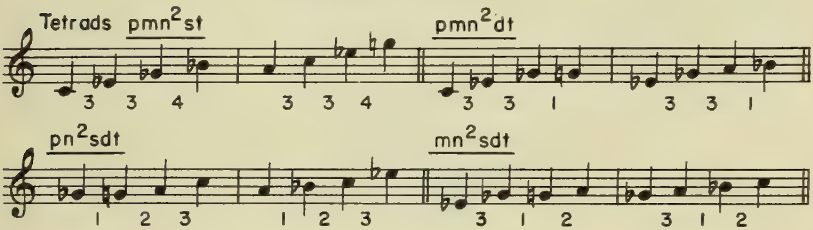


the isometric tetrads $C_3E\flat_4G_3B\flat$, p^2mn^2s, and $G_2A_1B\flat_2C$, pn^2s^2d, both of which we have already met as a part of the perfect-fifth hexad, the latter also in the minor-second hexad;

EXAMPLE 11-12

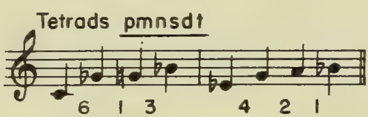
four new tetrad types, all consisting of a diminished triad plus one "foreign" tone: $C_3E\flat_3G\flat_4B\flat$ and $A_3C_3E\flat_4G$, *pmn²st;* $C_3E\flat_3G\flat_1G\natural$ and $E\flat_3G\flat_3A_1B\flat$, *pmn²dt;* $G\flat_1G\natural_2A_3C$ and $A_1B\flat_2C_3$ $E\flat$, *pn²sdt;* $E\flat_3G\flat_1G\natural_2A$ and $G\flat_3A_1B\flat_2C$, *mn²sdt;*

EXAMPLE 11-13



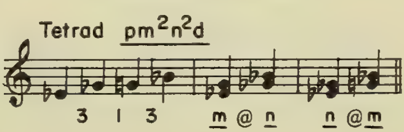
the tetrads $C_6G\flat_1G\natural_3B\flat$, and $E\flat_4G_2A_1B\flat$, both having the analysis *pmnsdt,* the first appearance in any hexad of the twin tetrads referred to in Chapter 3, Example 3-8;

EXAMPLE 11-14



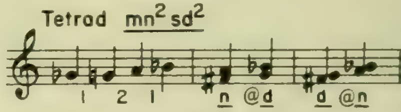
and the two isometric tetrads $E\flat_3G\flat_1G\natural_3B\flat$, *pm²n²d,* which will be seen to consist of two major thirds at the interval of the minor third, or two minor thirds at the relationship of the major third;

EXAMPLE 11-15



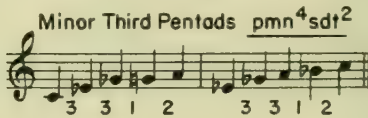
and $G\flat_1 G\natural_2 A_1 B\flat$, *mn*^{2}*sd*^{2}, which consists of two minor thirds at the interval relationship of the minor second, or two minor seconds at the interval of the minor third:

EXAMPLE 11-16



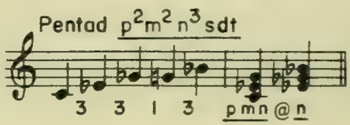
The pentads consist of the basic pentads $C_3 E\flat_3 G\flat_1 G\natural_2 A$, and $E\flat_3 G\flat_3 A_1 B\flat_2 C$, *pmn*^{4}*sdt*^{2};

EXAMPLE 11-17



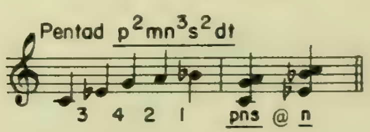
the pentad $C_3 E\flat_3 G\flat_1 G\natural_3 B\flat$, *p*^{2}*m*^{2}*n*^{3}*sdt*, which may also be analyzed as a combination of two minor triads at the interval of the minor third;

EXAMPLE 11-18



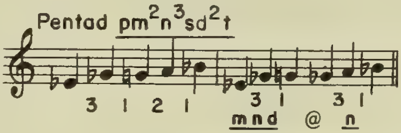
the pentad $C_3 E\flat_4 G_2 A_1 B\flat$, *p*^{2}*mn*^{3}*s*^{2}*dt*, which may also be analyzed as two triads *pns* at the interval of the minor third;

EXAMPLE 11-19



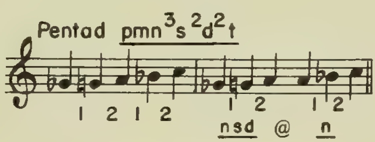
the pentad $E\flat_3G\flat_1G\natural_2A_1B\flat$, $pm^2n^3sd^2t$, which may also be analyzed as the combination of two triads *mnd* at the interval of the minor third;

EXAMPLE 11-20



and the pentad $G\flat_1G\natural_2A_1B\flat_2C$, $pmn^3s^2d^2t$, which may be analyzed as the combination of two triads *nsd* at the interval of the minor third;

EXAMPLE 11-21



The contrast between the six-tone major-second scale and the six-tone minor-third scale will be immediately apparent. Whereas the former is limited to various combinations of major thirds, major seconds, and tritones, the latter contains a wide variety of harmonic and melodic possibilities. The scale predominates. of course, in the interval of the minor third and the tritone, but contains also a rich assortment of related sonorities.

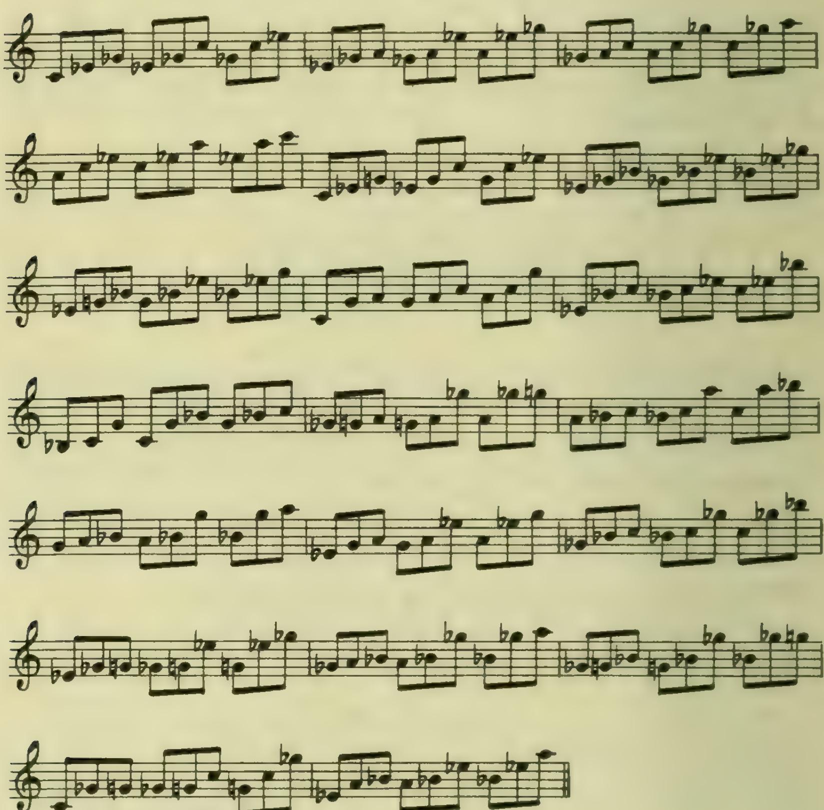
Subtle examples of the minor-third hexad are found in Debussy's *Pelléas et Mélisande,* such as:

EXAMPLE 11-22

Debussy, "Pelleas and Melisande"

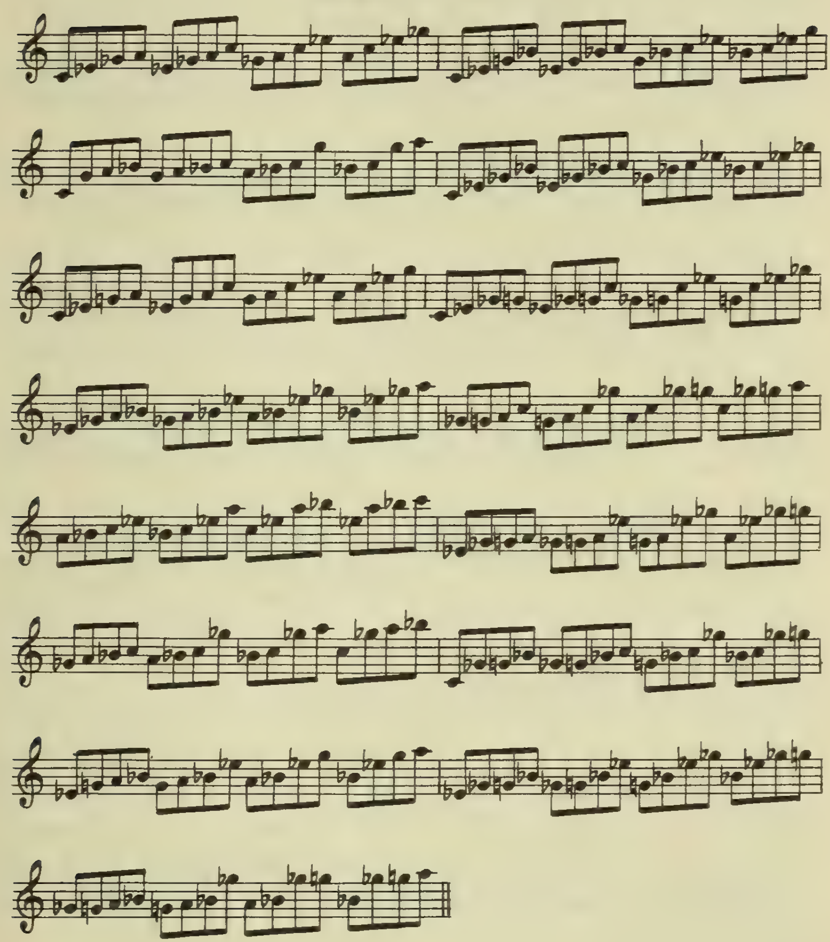
Play each of the triads in the minor-third hexad in each of its three versions, as indicated in Example 11-23. Play each measure several times slowly, with the sustaining pedal held. If you have sufficient pianistic technic, play all of the exercises with both hands in octaves, otherwise the one line will suffice. Now analyze each triad.

EXAMPLE 11-23



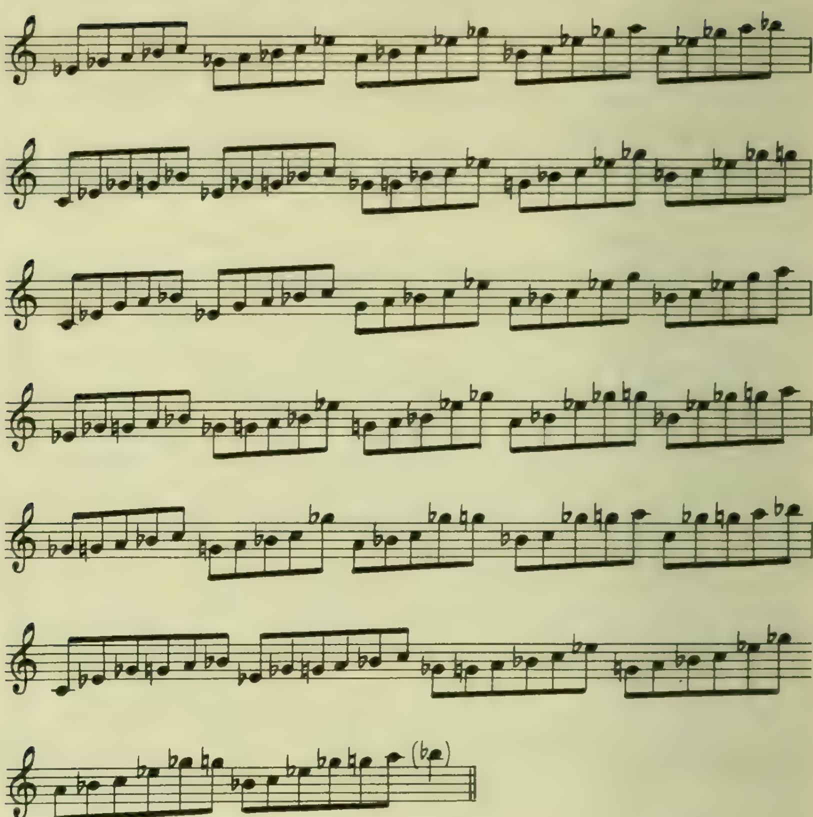
Repeat the same process with the tetrads of the scale:

Example 11-24



Repeat the same process with the six pentads and the hexad:

Example 11-25



One of the most important attributes of any sonority is its degree of consonance or dissonance, because the "tension" induced by the dissonance of one sonority may be increased, reduced, or released by the sonority to which it progresses. An interesting and important study, therefore, is the analysis of the relative degrees of dissonance of different sonorities.

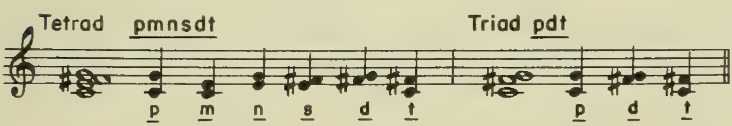
At first glance, this may seem to be an easy matter. The intervals of the perfect octave; the perfect fifth and its inversion, the perfect fourth; the major third and its inversion, the minor sixth; and the minor third and its inversion, the major sixth, are generally considered to perform a consonant function in a sonority. The major second and its inversion, the minor seventh;

the minor second and its inversion, the major seventh; and the tritone (augmented fourth or diminished fifth) are generally considered to perform a dissonant function. When these intervals are mixed together, however, the comparative degree of dissonance in different sonorities is not always clear. Some questions, indeed, cannot be answered with finality.

We may safely assume that the dissonance of the major seventh and minor second is greater than the dissonance of the minor seventh, major second, or tritone. To the ears of many listeners, however, there is not much difference between the dissonance of the minor seventh and the tritone.

Another problem arises when we compare the relative consonance or dissonance of two sonorities containing a different *number* of tones. For example, we might conclude that the sonority C-E-F♯-G is more dissonant than the sonority C-F♯-G, since the second contains two dissonances—the minor second and the tritone, whereas the first contains three dissonances—the minor second, the tritone, and the major second. However, it might also be argued that whereas the sonority C-E-F♯-G contains a larger *number* of dissonant intervals, C-F♯-G contains a greater *proportion* of dissonance. The analysis of the first sonority is *pmnsdt*—one-half of the intervals being dissonant; whereas the analysis of the second sonority is *pdt*—two-thirds of the intervals being dissonant:

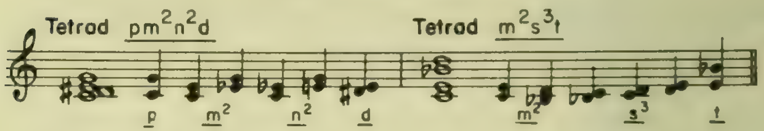
Example 11-26



Finally, it would seem that the presence of one primary dissonance, such as the minor second, renders the sonority more dissonant than the presence of several mild dissonances such as the tritone or minor seventh. For example, the sonority C-D♯-E-G, with only one dissonant interval, the minor second, sounds

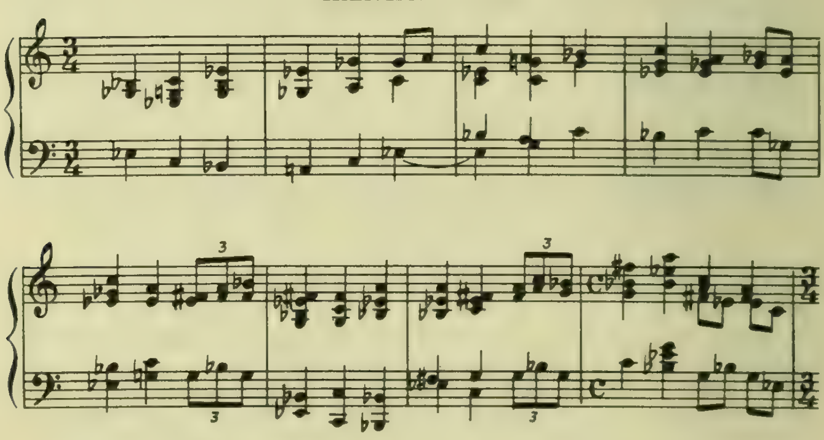
more dissonant than the tetrad C-E-B♭-D, which contains four mild dissonances:

EXAMPLE 11-27



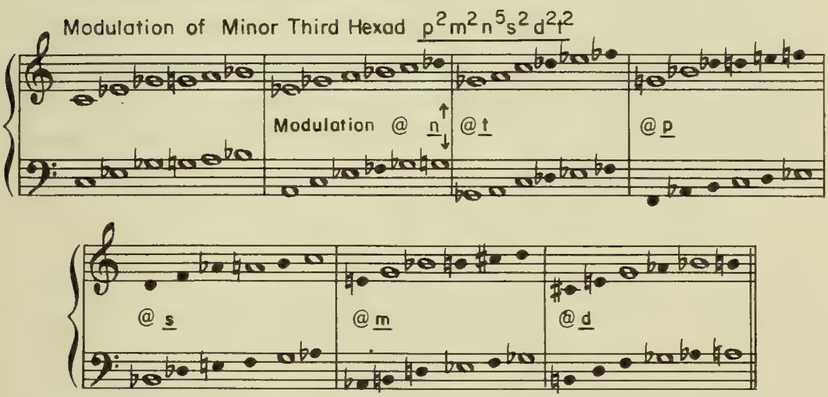
With the above theories in mind, I have tried to arrange all of the sonorities of the minor-third hexad in order of their relative dissonance, beginning with the three most consonant triads—major and minor—and moving progressively to the increasingly dissonant sonorities. Play through Example 11-28 carefully, listening for the increasing tension in successive sonorities. Note where the degree of "tension" seems to remain approximately the same. Analyze all of the sonorities and see if you agree with the order of dissonance in which I have placed them. Have someone play the example for you and take it down from dictation:

EXAMPLE 11-28



Reread Chapters 6 and 7 on modal and key modulation. Since the minor-third hexad has the analysis $p^2m^2n^5s^2d^2t^2$, it is evident that the closest modulatory relationship will be at the interval of the minor third; the next closest will be at the interval of the tritone;* and the third order of relationship will be at the interval of the perfect fifth, major second, major third, or minor second. Modulation at the interval of the minor third will have five common tones; at the tritone, four common tones; at the other intervals two common tones.

Example 11-29



Write a sketch using the material of the minor-third hexad. Begin with C as the key center and modulate *modally* to E♭ as the key center, and back to C. Now perform a *key modulation* to the minor-third hexad a minor third below C (that is, A); modulate to the key a fifth above (E), and then back to the key of C.

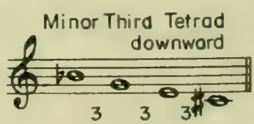
* See Chapter 17, pages 139 and 140.

Involution of the Six-Tone Minor-Third Projection

The first three series of projections, the perfect fifth, minor second, and major second, have all produced *isometric* scales. For example, the perfect-fifth six-tone scale $C_2D_2E_3G_2A_2B$, begun on B and constructed *downward,* produces the identical scale, $B_2A_2G_3E_2D_2C$. This is not true of the six-tone minor-third projection. The same projection downward produces a *different* scale.

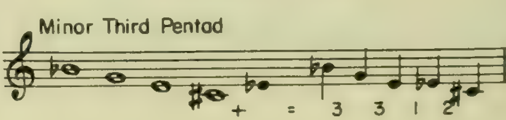
If we take the six-tone minor-third scale discussed in the previous chapter, $C_3E\flat_3G\flat_1G\natural_2A_1B\flat$, and begin it on the final note reached in the minor-third projection, namely, B♭, and produce the same scale *downward,* we add first the minor third below B♭, or G; the minor third below G, or E; and the minor third below E, or C♯.

Example 12-1



We then introduce, as in the previous chapter, the foreign tone a perfect fifth *below* B♭, or E♭, producing the five-tone scale $B\flat_3G_3E\natural_1E\flat_2C\sharp$:

Example 12-2

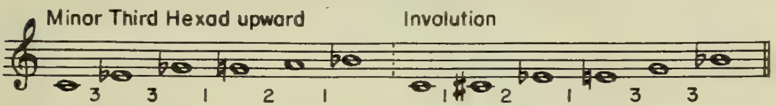


By adding another minor third below E♭, or C, we produce the six-tone involution $B\flat_3G_3E\natural_1E\flat_2C\sharp_1C\natural$:

EXAMPLE 12-3

A simpler method would be to take the configuration of the original minor third hexad, 3 3 1 2 1, beginning on C, but in *reverse,* 1 2 1 3 3 , which produces the same tones, $C_1C\sharp_2E\flat_1E\natural_3$ $G_3B\flat$:

EXAMPLE 12-4

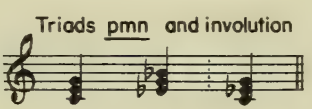


If we examine the components of this scale we shall find them to be the same as those of the scale conceived upward *but in involution.* The analysis of the scale is, of course, the same: $p^2m^2n^5s^2d^2t^2$. We find, again, the four basic diminished triads $C\sharp_3E_3G$, $E_3G_3B\flat$, $G_3B\flat_3D\flat(C\sharp)$, and $A\sharp(B\flat)_3C\sharp_3E$;

EXAMPLE 12-5

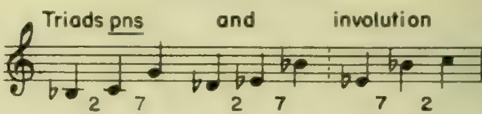
the *major* triads—(where before we had *minor* triads)—C_4E_3G and $E\flat_4G_3B\flat$, with the one involution, the minor triad $C_3E\flat_4G$;

EXAMPLE 12-6



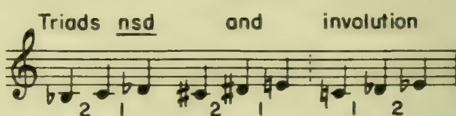
the triads $B\flat_2C_7G$ and $D\flat(C\sharp)_2E\flat_7B\flat$, *pns*, with the one involution, $E\flat_7B\flat_2C$;

EXAMPLE 12-7



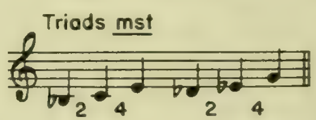
the triads $B\flat_2C_1D\flat(C\sharp)$ and $C\sharp_2D\sharp_1E$, *nsd*, together with the one involution $C_1D\flat_2E\flat$;

EXAMPLE 12-8



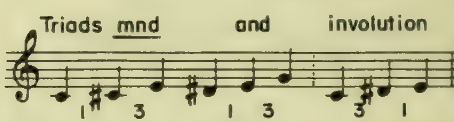
the triads $B\flat_2C_4E$ and $D\flat_2E\flat_4G$, *mst*;

EXAMPLE 12-9



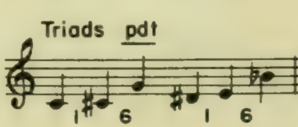
the triads $C_1C\sharp_3E$ and $D\sharp_1E_3G$, *mnd*, with the one involution $C_3D\sharp_1E$;

EXAMPLE 12-10



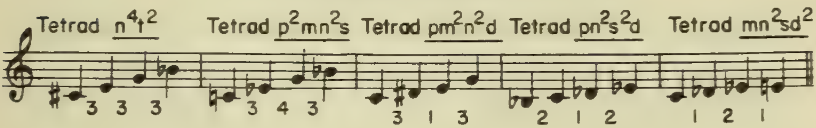
and the triads $C_1C\sharp_6G$ and $D\sharp_1E_6B\flat$, *pdt*:

EXAMPLE 12-11



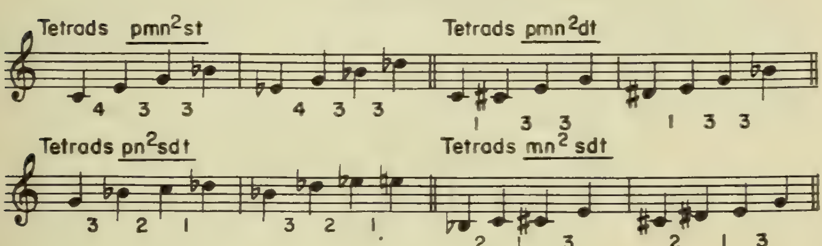
The tetrads consist of the same isometric tetrads found in the first minor-third scale: the diminished-seventh tetrad, $C\sharp_3E_3G_3B\flat$, *n^4t^2*, the other isometric tetrads, $C_3E\flat_4G_3B\flat$, *p^2mn^2s*, $C_3D\sharp_1E_3G$, *pm^2n^2d*, $B\flat_2C_1D\flat_2E\flat$, *$pn^2s^2d$*, and $C_1D\flat_2E\flat_1E\natural$, *$mn^2sd^2$*;

Example 12-12



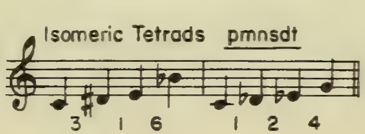
four tetrads consisting of a diminished triad and one foreign tone, each of which will be discovered to be the involution of a similar tetrad in the first minor-third scale: $C_4E_3G_3B\flat$ and $E\flat_4G_3B\flat_3D\flat$, *$pmn^2st$*; $C_1C\sharp_3E_3G$ and $D\sharp_1E_3G_3B\flat$, *pmn^2dt*; $G_3B\flat_2C_1D\flat$ and $B\flat_3D\flat_2E\flat_1E\natural$, *$pn^2sdt$*; and $B\flat_2C_1C\sharp_3E$ and $C\sharp_2D\sharp_1E_3G$, *mn^2sdt*;

Example 12-13



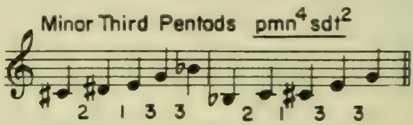
and the "twins", $C_3D\sharp_1E_6B\flat$ and $C_1D\flat_2E\flat_4G$, *pmnsdt,* the involutions of similar tetrads discussed in the previous chapter:

Example 12-14



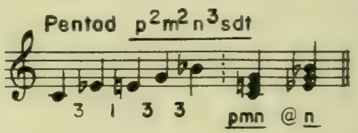
The pentads consist of the basic pentads $C\sharp_2 D\sharp_1 E_3 G_3 B\flat$ and $B\flat_2 C_1 C\sharp_3 E_3 G$, pmn^4sdt^2 (the involutions of the basic pentads in the previous chapter);

Example 12-15



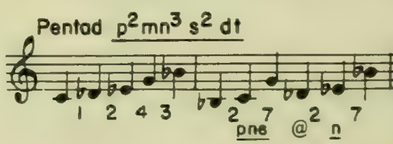
the pentad $C_3 E\flat_1 E\natural_3 G_3 B\flat$, $p^2m^2n^3sdt$, which may be analyzed as a combination of two major triads at the interval of the minor third;

Example 12-16



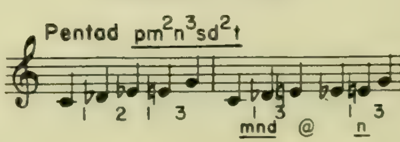
the pentad $C_1 D\flat_2 E\flat_4 G_3 B\flat$, $p^2mn^3s^2dt$, which may be analyzed as the combination of two triads, *pns*, at the interval of the minor third;

Example 12-17



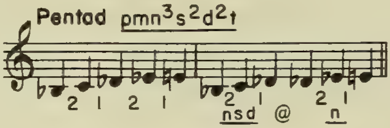
the pentad $C_1 D\flat_2 E\flat_1 E\natural_3 G$, $pm^2n^3sd^2t$, which may be analyzed as the combination of two triads, *mnd*, at the interval of the minor third;

Example 12-18



and the pentad $B\flat_2C_1D\flat_2E\flat_1E\natural$, $pmn^3s^2d^2t$, which may be analyzed as the combination of two triads, *nsd,* at the interval of the minor third:

EXAMPLE 12-19



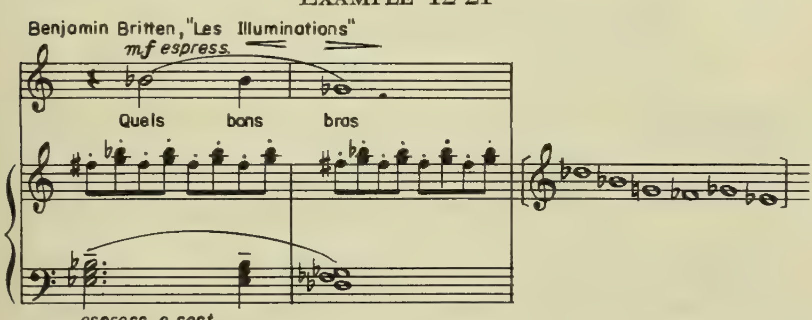
All of the above pentads will be seen to be involutions of similar pentads discussed in the previous chapter.

From the many examples of the involution of the minor-third hexad we may choose two, first from page 13 of the vocal score of Debussy's *Pelléas et Mélisande;*

EXAMPLE 12-20

and from the second movement of Benjamin Britten's *Illuminations* for voice and string orchestra:

EXAMPLE 12-21



Analyze the following two measures which come at the end of a section of Debussy's "*Les fées sont d'exquises danseuses.*" If all of the notes of the two measures are considered as integral parts of *one* scale, we have the rather complex scale ↓C-C♭-B♭-A-A♭-G-G♭-F-E♭-D composed of the two minor-third tetrads, ↓C-A-G♭-E♭ and ↓F-D-C♭-A♭, plus the minor third, B♭-G (forming the ten-tone minor-third projection).

A closer—and also simpler—analysis, however, shows that the first measure contains the notes of the minor-third hexad ↓F-D-C♭-A♭-B♭-G, and the second measure is the identical scale pattern transposed a perfect fifth, to begin on C, ↓ C-A-G♭-E♭-F-D.

This simpler analysis is much to be preferred, for most composers, whose desire is to communicate to their listeners rather than to befuddle them, tend to think in the simplest vocabulary commensurate with their needs.

EXAMPLE 12-22

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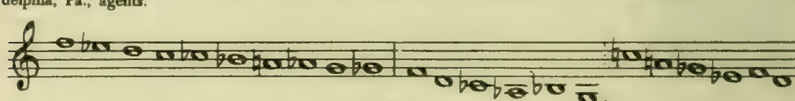


A detailed comparison of the material of the minor-third hexad discussed in Chapter 11 with that of the material in Chapter 12 will indicate that the *isometric* material of the two

hexads is identical, but that where the sonorities have involutions, each sonority of one scale is the *involution* of a similar sonority in the other. For example, the minor-third hexad discussed in Chapter 11 contains two *minor* triads and one major *triad,* whereas the involution of the hexad contains two *major* triads and one *minor* triad. The involution does not, therefore, strictly speaking, add any new *types* of sonorities, but merely substitutes involutions of those sonorities.

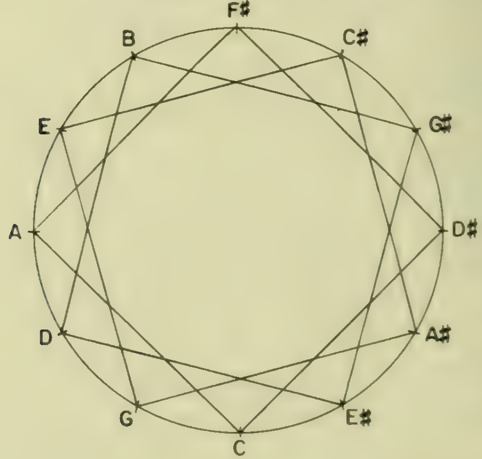
13

Projection of the Minor Third Beyond the Six-Tone Series

We produced the six-tone minor-third scale in Chapter 11 by beginning on any given tone, superimposing three minor thirds above that tone, adding the foreign tone of the perfect fifth, and superimposing another minor third above that tone.

We may now complete the series by superimposing two more minor thirds, thereby completing a second diminished-seventh chord, then adding a second foreign tone a perfect fifth above the first foreign tone, and superimposing three more minor thirds, thereby completing the third diminished-seventh chord. For the student who is "eye-minded" as well as "ear-minded," the following diagram may be helpful:

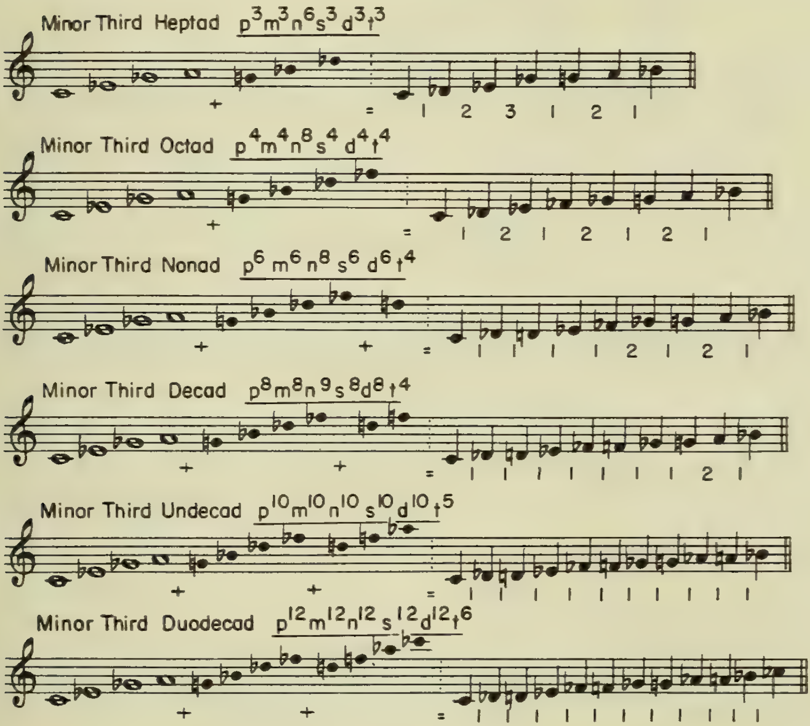
Example 13-1



Here it will be seen that the minor-third projection divides the twelve points in the circle into three squares, the first beginning on C, the second on G, and the third on D. We begin by superimposing E♭, G♭, and A above C, then adding G and superimposing B♭, D♭, and F♭ (E), and then adding D and superimposing F, A♭, and C♭ (B):

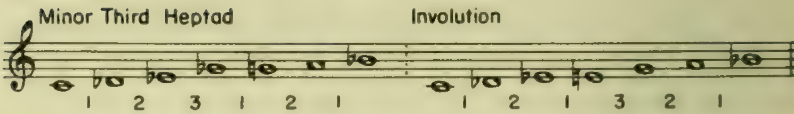
The scales thus produced, with their respective analyses, become:

EXAMPLE 13-2



All of these scales are isometric with the exception of the seven-tone scale, the involution of which produces a different scale:

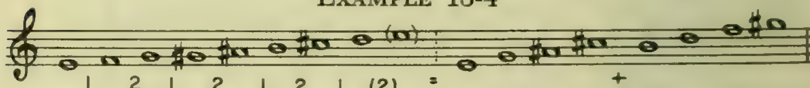
EXAMPLE 13-3

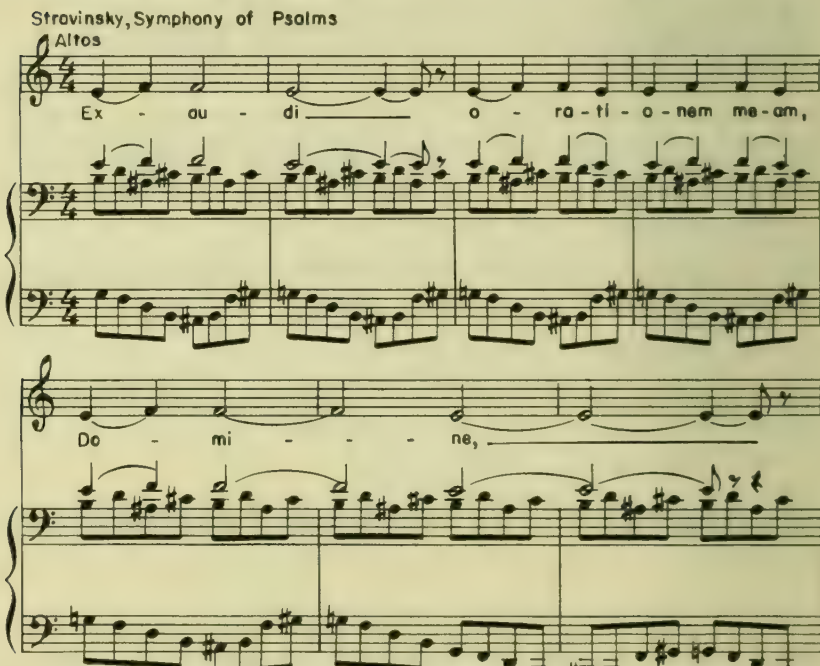


These scales with their rich variety of tonal material and their generally "exotic" quality have made them the favorites of many contemporary composers.

A beautiful example of the eight-tone minor-third scale will be found in the first movement of Stravinsky's *Symphony of Psalms,* Example 13-4, where the first seven measures are consistently in this scale, $E_1F_2G_1G\sharp_2A\sharp_1B_2C\sharp_1D$:

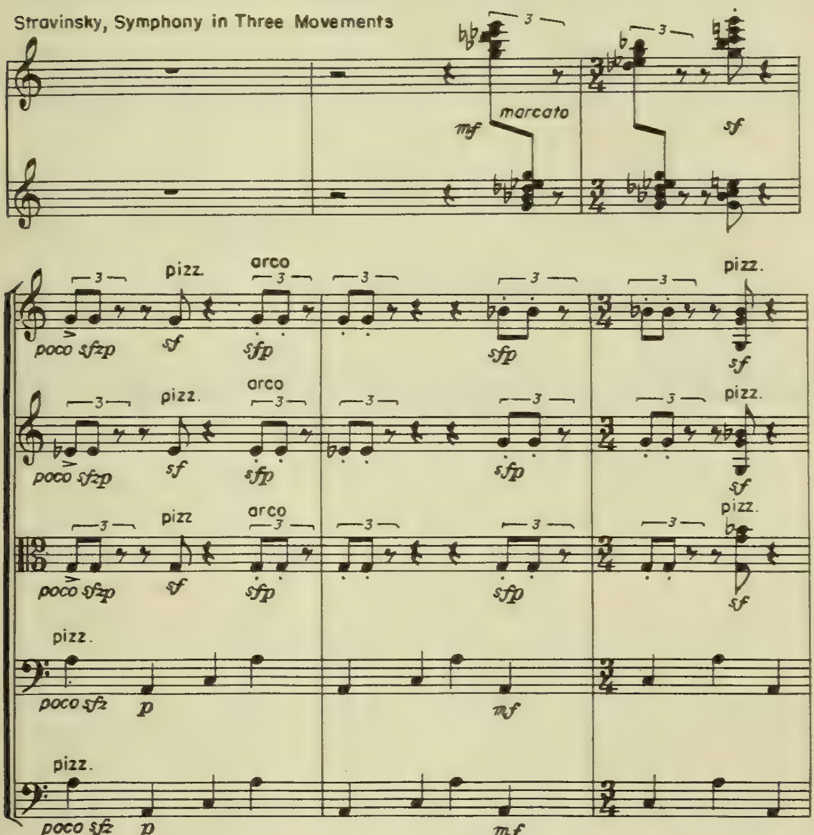
EXAMPLE 13-4

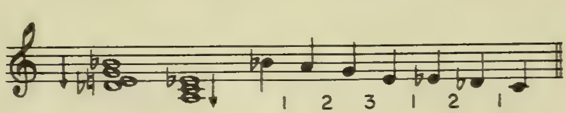




A completely consistent use of the involution of the seven-tone minor-third scale will be found in the first movement of the same composer's *Symphony in Three Movements,* beginning at rehearsal number 7, and continuing without deviation for twenty-three measures:

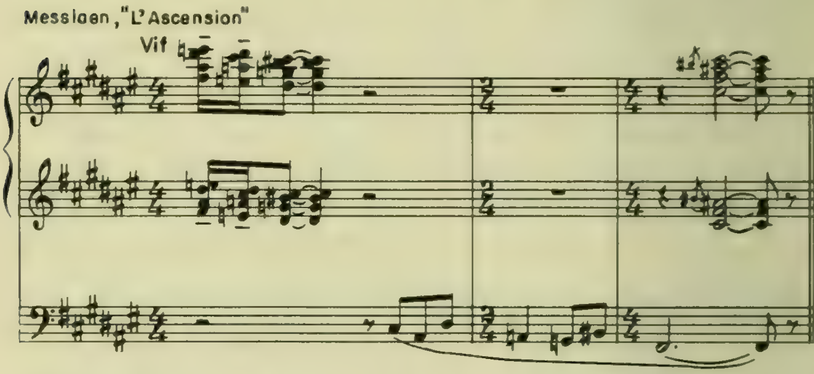
EXAMPLE 13-5

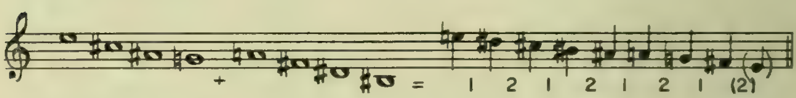




Another interesting example of the eight-tone minor-third scale is found at the opening of the third movement of Messiaen's *L'Ascension*:

EXAMPLE 13-6





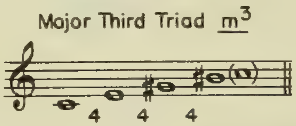
Analyze further the Stravinsky *Symphony of Psalms* and try to find additional examples of the minor-third projection.

Projection of the Major Third

WE HAVE OBSERVED that there are only two intervals which can be projected consistently through the twelve tones, the perfect fifth and the minor second. The major second may be projected through a six-tone series and then must resort to the interjection of a "foreign" tone to continue the projection, while the minor third can be projected in pure form through only four tones.

We come now to the major third, which can be projected only to three tones. Beginning again with the tone C, we superimpose the major third, E, and the second major third, E to G♯, producing the augmented triad C-E-G♯ consisting of the three major thirds, C to E, E to G♯, and G♯ to B♯ (C), m^3:

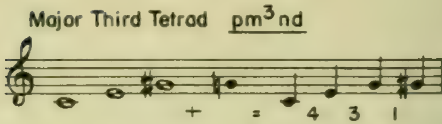
EXAMPLE 14-1



To project the major third beyond these three tones, we again add the foreign tone G♮*, a perfect fifth above C, producing the basic major-third tetrad $C_4E_3G_1G\sharp$ having, in addition to the three major thirds already enumerated, a perfect fifth, from C to G; a minor third, from E to G; and a minor second from G to G♯ (A♭); pm^3nd:

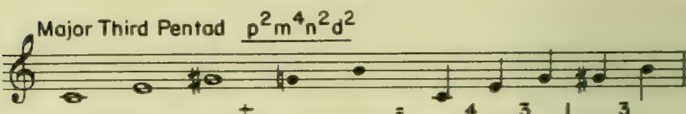
* Here the choice of the foreign tone is more important, since the addition of D, F♯, or A♯ with their superimposed major thirds would duplicate the major-second hexad. The addition of *any other* foreign tone to the augmented triad produces the *same* tetrad in a different version, or in involution.

EXAMPLE 14-2



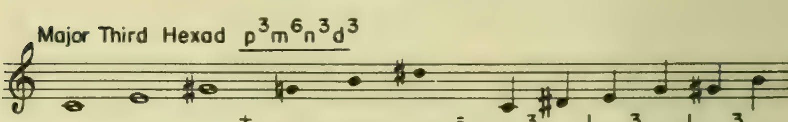
To produce the pentad, we superimpose a major third above G, or B, forming the scale $C_4E_3G_1G\sharp_3B$, and producing, in addition to the major third, G to B, the perfect fifth, E to B; the minor third, G♯ to B; and the minor second, B to C; $p^2m^4n^2d^2$:

EXAMPLE 14-3



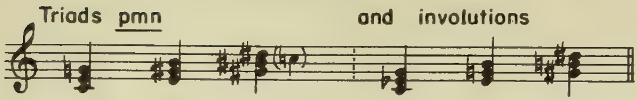
To produce the six-tone major-third scale, we add the major third above B, or D♯, giving the scale $C_3D\sharp_1E_3G_1G\sharp_3B$. The new tone, D♯, in addition to forming the major third, B to D♯, adds an additional major third, from D♯ (E♭) to G. It also adds another perfect fifth, G♯ to D♯; a minor third, C to D♯ (E♭); and a minor second, D♯ to E; $p^3m^6n^3d^3$.

EXAMPLE 14-4



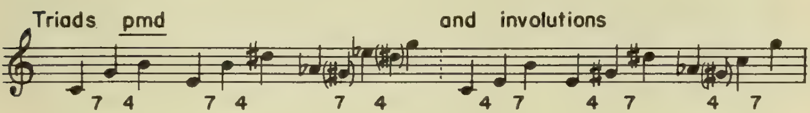
If we proceed to analyze the melodic-harmonic components of this six-tone major-third scale, we find that it contains the augmented triad, which is the basic triad of the major-third scale, m^3, on C and on G. It contains also the major triads C_4E_3G, $E_4G\sharp_3B$ and $G\sharp_4B\sharp_3(C)D\sharp$, *pmn*, with their involutions, the minor triads $C_3E\flat_4(D\sharp)G$, E_3G_4B, and $G\sharp_3B_4D\sharp$;

EXAMPLE 14-5



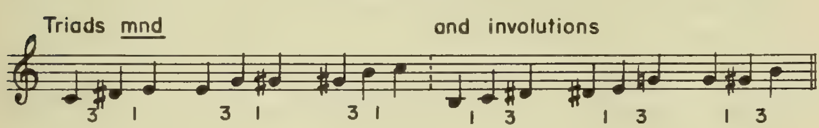
and the triads C_7G_4B, $E_7B_4D\sharp$, and $A\flat(G\sharp)_7E\flat(D\sharp)_4G$, *pmd*, together with their involutions C_4E_7B, $E_4G\sharp_7D\sharp$ and $A\flat(G\sharp)_4C_7G$:

EXAMPLE 14-6



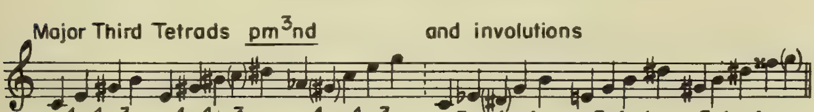
Finally, it contains the triads $C_3D\sharp_1E$, $E_3G_1G\sharp$, and $G\sharp_3B_1C$, *mnd*, with the involutions $B_1C_3D\sharp$, $D\sharp_1E_3G$, and $G_1G\sharp_3B$, which have already been seen as parts of the minor-second and minor-third scales but which would seem to be characteristic of the major-third projection:

EXAMPLE 14-7



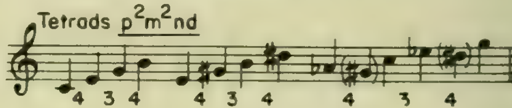
The tetrads consist of the basic tetrads, new to the hexad series, $C_4E_4G\sharp_3B$, $E_4G\sharp_4B\sharp_3(C)D\sharp$, and $A\flat(G\sharp)_4C_4E_3G$, which are a combination of the augmented triad and the major triad, *pm³nd*, together with their involutions $C_3E\flat_4G_4B$, $E\natural_3G_4B_4D\sharp$, and $G\sharp_3B_4D\sharp_4F{\times}(G\natural)$, which consist of the combination of the augmented triad and a minor triad;

EXAMPLE 14-8



the isometric tetrads $C_4E_3G_4B$, $E_4G\sharp_3B_4D\sharp$, and $A\flat_4(G\sharp)C_3E\flat_4(D\sharp)G$, p^2m^2nd, which we first observed in the perfect-fifth projection;

EXAMPLE 14-9

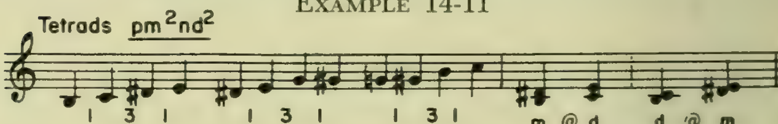


the isometric tetrads $C_3D\sharp_1E_3G$, $E_3G_1G\sharp_3B$, and $G\sharp_3B_1C_3D\sharp$, pm^2n^2d, which we have encountered as parts of the minor-third series;

EXAMPLE 14-10

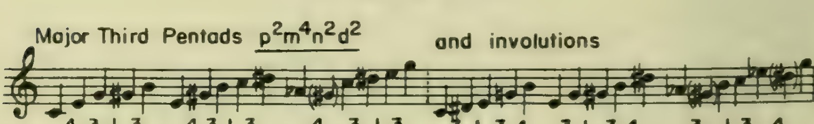
and the isometric tetrads $B_1C_3D\sharp_1E$, $D\sharp_1E_3G_1G\sharp$, and $G_1G\sharp_3B_1C$, pm^2nd^2, which can be analyzed as two major thirds at the interval of the minor second, or two minor seconds at the interval of the major third, previously observed in the minor-second series:

EXAMPLE 14-11



The pentads consist only of the basic pentads $C_4E_3G_1G\sharp_3B$, $E_4G\sharp_3B_1C_3D\sharp$, and $A\flat_4(G\sharp)C_3D\sharp_1E_3G\natural$ $p^2m^4n^2d^2$, together with their involutions $C_3D\sharp_1E_3G_4B$, $E_3G_1G\sharp_3B_4D\sharp$, and $A\flat_3(G\sharp)B_1C_3E\flat_4(D\sharp)G\natural$.

EXAMPLE 14-12

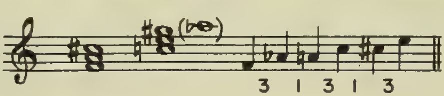


From this analysis it will be seen that the six-tone major-third scale has something of the same homogeneity of material that is characteristic of the six-tone major-second scale. The scale includes only the intervals of the perfect fifth, the major third, the minor third, and the minor second, or their inversions. It does not contain either the major second or the tritone. It is, however, a more striking scale than the whole-tone scale, for it contains a greater variety of material and varies in consonance from the consonant perfect fifth to the dissonant minor second.

The six-tone major-third scale is an isometric scale, because if we begin the scale $C_3D\sharp_1E_3G_1G\sharp_3B$ on B, and project it in reverse, the order of the intervals remains the same. There is, therefore, no involution as was the case in the minor-third scale.

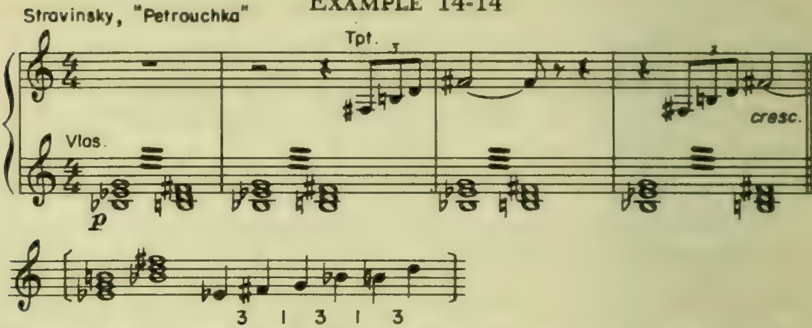
A clear example of the major-third hexad may be found in the sixth Bartok string quartet:

Example 14-13



An harmonic example of the same scale is illustrated by the following example from Stravinsky's *Petrouchka*:

EXAMPLE 14-14

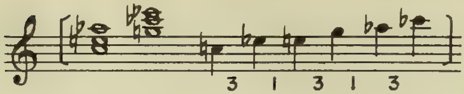


A purely consonant use of this hexad may be found in the opening of the author's Fifth Symphony, *Sinfonia Sacra*:

EXAMPLE 14-15

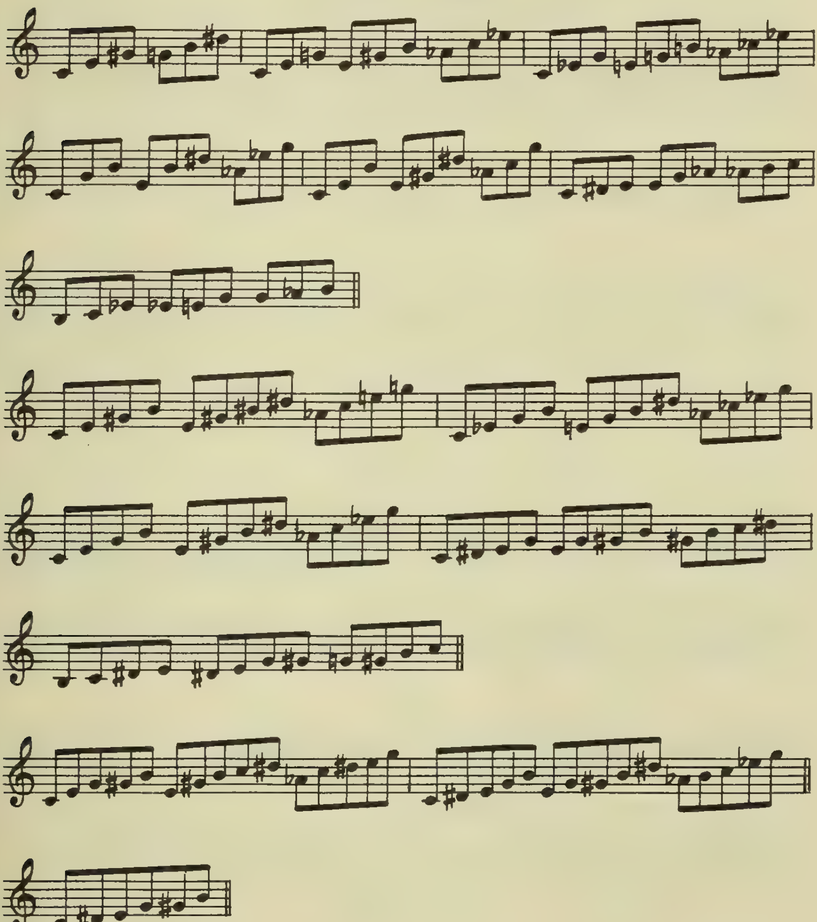
A charming use of this scale is the flute-violin passage from Prokofieff's *Peter and the Wolf*:

EXAMPLE 14-16



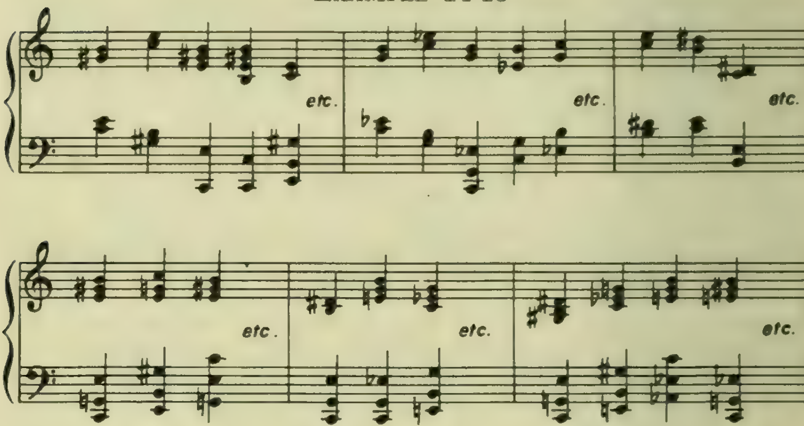
Play the triads, tetrads, pentads, and the hexad in Example 14-17 which constitute the material of the major-third hexad. Play each measure slowly and listen carefully to the fusion of tones in each sonority:

Example 14-17



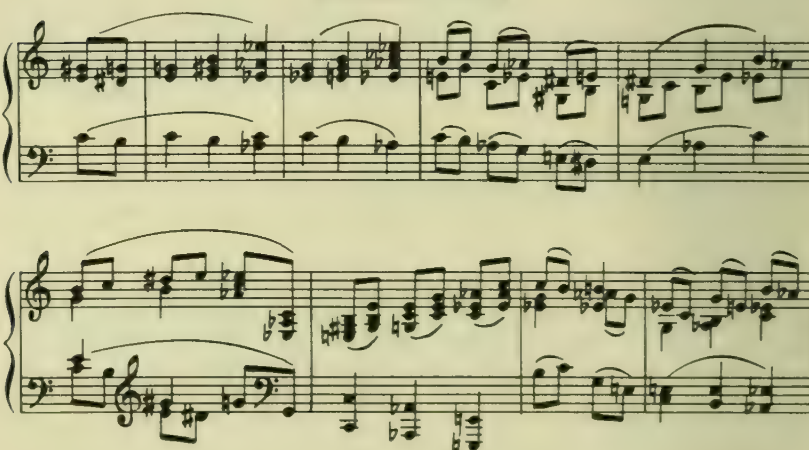
Experiment with different positions and doublings of the characteristic sonorities of this scale, as in Example 14-18:

EXAMPLE 14-18



The following exercise contains all of the sonorities of the major-third hexad. Play it through several times and analyze each sonority. Have someone play through the exercise for you and take it down from dictation:

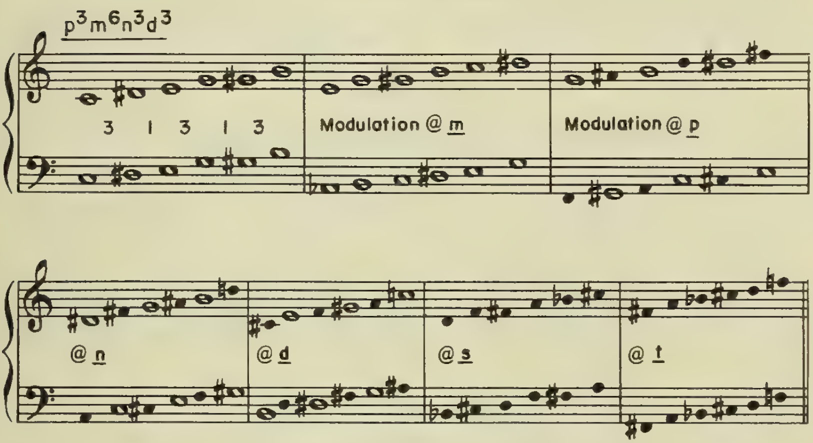
EXAMPLE 14-19



Write a short sketch limited to the material of the major-third hexad on C.

Example 14-20 illustrates the modulatory possibilities of this scale. Modulations at the interval of the major third, up or down, produce no new tones; modulations at the interval of the perfect fifth, minor third, and minor second, up or down, produce three new tones; modulations at the interval of the major second and the tritone produce *all* new tones.

EXAMPLE 14-20



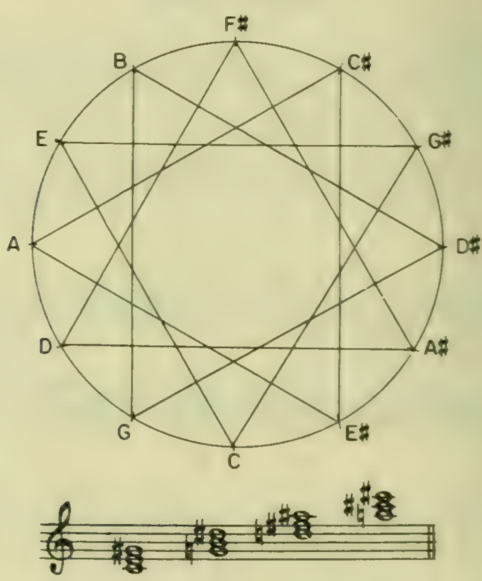
Write a short sketch which modulates from the major-third hexad on C to the major-third hexad on D, but do not "mix" the two keys.

15

Projection of the Major Third Beyond the Six-Tone Series

If we refer to the diagram below we see that the twelve points in the circle may be connected to form four triangles: the first consisting of the tones C-E-G♯; the second of the tones G♮-B-D♯; the third of the tones D♮-F♯-A♯; and the fourth of the tones A♮-C♯-E♯:

Example 15-1

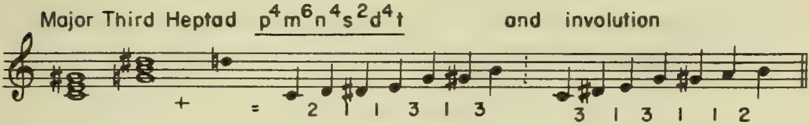


We may, therefore, project the major third beyond the six tones by continuing the process by which we formed the six-tone scale. Beginning on C we form the augmented triad C-E-G♯;

add the foreign tone, G♮, and superimpose the augmented triad G-B-D♯; add the fifth above the foreign tone G, that is, D♮, and superimpose the augmented triad D-F♯-A♯; and, finally, add the fifth above the foreign tone D, or A♮, and superimpose the augmented triad A-C♯-E♯. Rearranged melodically, we find the following projections:

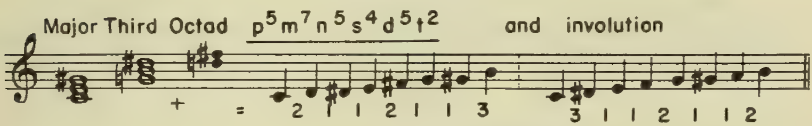
Seven tone: C-E-G♯ + G-B-D♯ + D♮ = $C_2D_1D\sharp_1E_3G_1G\sharp_3B$, $p^4m^6n^4s^2d^4t$, with its involution $C_3D\sharp_1E_3G_1G\sharp_1A_2B$:

Example 15-2



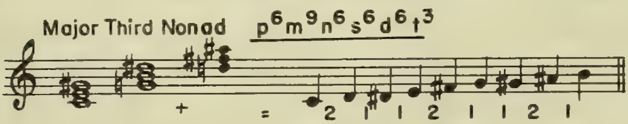
Eight tone: C-E-G♯ + G♮-B-D♯ + D♮-F♯ = $C_2D_1D\sharp_1E_2F\sharp_1G_1G\sharp_3B$, $p^5m^7n^5s^4d^5t^2$, with its involution $C_3D\sharp_1E_1F_2G_1G\sharp_1A_2B$:

Example 15-3



Nine tone: C-E-G♯ + G♮-B-D♯ + D♮-F♯-A♯, = $C_2D_1D\sharp_1E_2F\sharp_1G_1G\sharp_2A\sharp_1B$, $p^6m^9n^6s^6d^6t^3$:

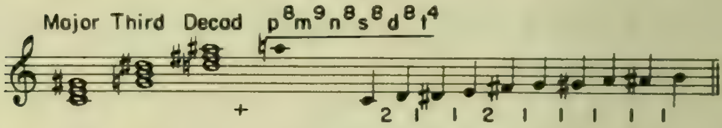
Example 15-4



(This is an isometric scale, for if we begin the scale on A♯ and proceed downward, we have the same order of whole and half steps, 21121121.)

Ten tone: C-E-G♯ + G♮-B-D♯ + D♮-F♯-A♯ + A♮ = $C_2D_1D\sharp_1E_2F\sharp_1G_1G\sharp_1A_1A\sharp_1B$, $p^8m^9n^8s^8d^8t^4$:

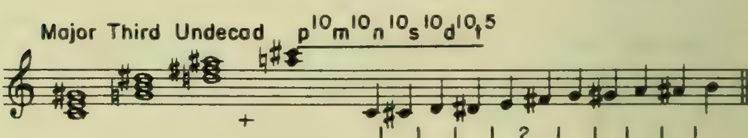
EXAMPLE 15-5



(This scale is also isometric, for if we begin the scale on F♯ and progress downward, we have the same order of whole and half-steps.)

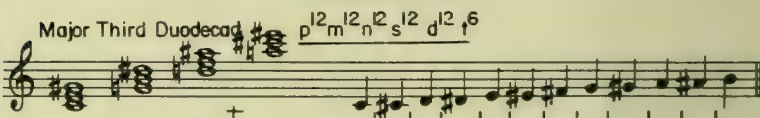
Eleven tone: C-E-G♯ + G♮-B-D♯ + D♮-F♯-A♯ + A♮-C♯ = $C_1C\sharp_1D_1D\sharp_1E_2F\sharp_1G_1G\sharp_1A_1A\sharp_1B$, $p^{10}m^{10}n^{10}s^{10}d^{10}t^5$:

EXAMPLE 15-6



Twelve tone: C-E-G♯ + G♮-B-D♯ + D♮F♯-A♯ + A♮-C♯-E♯ = $C_1C\sharp_1D_1D\sharp_1E_1E\sharp_1F\sharp_1G_1G\sharp_1A_1A\sharp_1B$, $p^{12}m^{12}n^{12}s^{12}d^{12}t^6$:

EXAMPLE 15-7

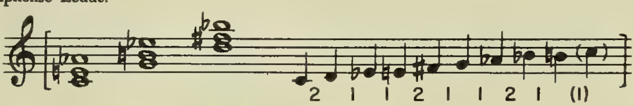


(The eleven- and twelve-tone scales are, of course, also isometric formations.)

The student will observe that the seven-tone scale adds the formerly missing intervals of the major second and the tritone, while still maintaining a preponderance of major thirds and a proportionately greater number of perfect fifths, minor thirds, and minor seconds. The scale gradually loses its basic characteristic as additional tones are added but retains the preponderance of major thirds through the ten-tone projection.

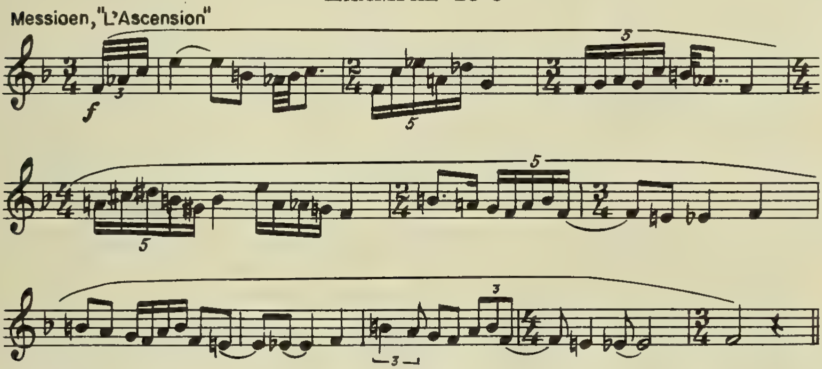
The following measure from *La Nativité du Seigneur* by Messiaen, fourth movement, page 2, illustrates a use of the nine-tone major-third scale:

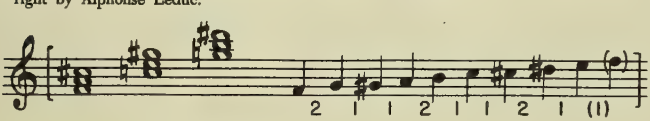
EXAMPLE 15-8



The long melodic line from the second movement of the same composer's *L'Ascension* is a striking example of the melodic use of the same scale:

EXAMPLE 15-9





Analyze further the second movement of Messiaen's *L'Ascension* and try to find other examples of the major-third projection.

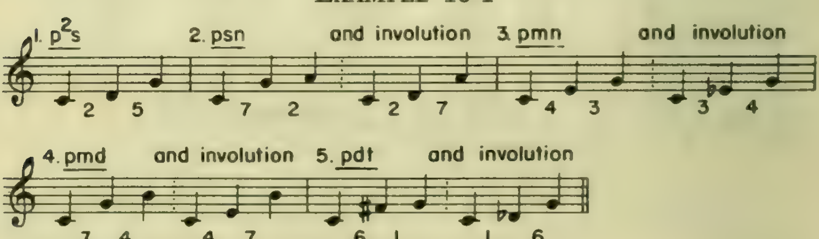
16

Recapitulation of the Triad Forms

INASMUCH AS THE PROJECTIONS that we have discussed contain *all* of the triads possible in twelve-tone equal temperament, it may be helpful to summarize them here. There are only twelve types in all if we include both the triad and its involution as one form, and if we consider inversions to be merely a different arrangement of the same triad.

There are five triads which contain the perfect fifth in their composition: (1) the basic perfect-fifth triad p^2s, consisting of two perfect fifths and the concomitant major second; (2) the triad *pns*, consisting of a perfect fifth, a minor third, and a major second, with its involution; (3) the major triad *pmn*, consisting of a perfect fifth, major third, and minor third, with its involution, the minor triad; (4) the triad *pmd*, consisting of a perfect fifth, a major third, and a major seventh with its involution; and (5) the triad *pdt*, in which the tritone is the characteristic interval, consisting of the perfect fifth, minor second, and tritone with its involution. Here they are with their involutions:

EXAMPLE 16-1



The first, p^2s, has appeared in the perfect-fifth hexad. The second, *pns,* has appeared in the perfect-fifth, minor-second, and minor-third hexads. The third, *pmn,* is found in the perfect-fifth, minor-third, and major-third hexads. The fourth, *pmd,* has been encountered in the perfect-fifth, minor-second, and major-third hexads. The fifth, *pdt,* has appeared only in the minor-third hexad, but will be found as the characteristic triad in the projection to be considered in the next chapter.

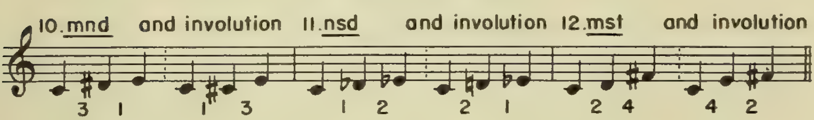
There are, in addition to the perfect-fifth triad p^2s, four other triads, each characteristic of a basic series: ms^2, n^2t, m^3, and sd^2:

EXAMPLE 16-2

The triad ms^2 is the basic triad of the major-second scale, but is also found in the perfect-fifth and minor-second hexads. The triad n^2t, has occurred only in the minor-third hexad. The triad m^3 has been found only in the major-second and major-third hexads. The triad sd^2 is the basic triad of the minor-second projection and is found in none of the other hexads which have been examined.

There remain three other triad types: *mnd, nsd,* and *mst*:

EXAMPLE 16-3

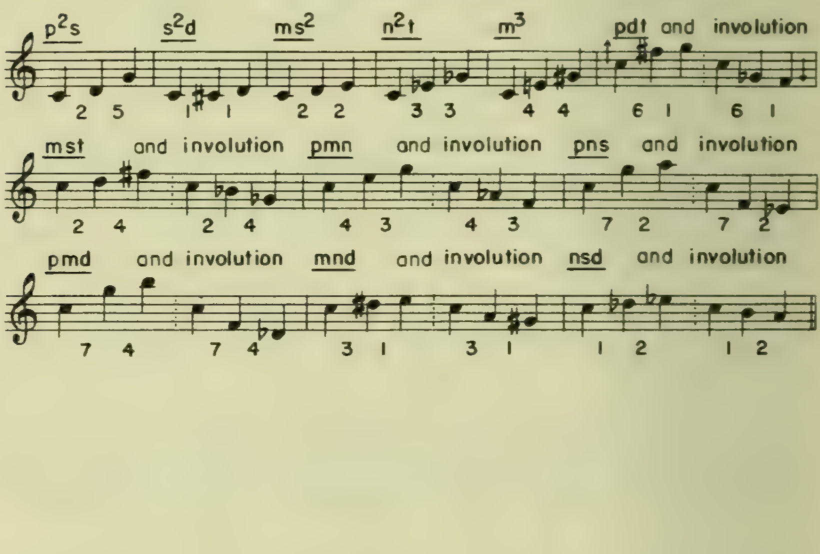


The triad *mnd* is found in the major-third, minor-third, and minor-second hexads. The triad *nsd* is a part of the minor-second hexad and is also found in the perfect-fifth and minor-third hexads. The twelfth, *mst,* has occurred in the major-second and minor-third hexads.

Since these twelve triad types are the basic vocabulary of musical expression, the young composer should study them carefully, listen to them in various inversions and with various doublings, and absorb them as a part of his tonal vocabulary.

If we "spell" all of these triads and their involutions above and below C, instead of relating them to any of the particular series which we have discussed, we have the triads and their involutions as shown in the next example. Notice again that the first five triads—basic triads of the perfect-fifth, minor-second, major-second, minor-third, and major-third series—are all isometric, the involution having the same "shape" as the original triad. The remaining seven triads all have involutions.

EXAMPLE 16-4



17

Projection of the Tritone

THE STUDENT WILL HAVE OBSERVED, in examining the five series which we have discussed, the strategic importance of the tritone. Three of the six-tone series have contained no tritones—the perfect-fifth, minor-second, and major-third series—while in the other two series, the major-second and minor-third series, the tritone is a highly important part of the complex.

It will be observed, further, that the tritone in itself is not useful as a unit of projection, because when one is superimposed upon another, the result is the enharmonic octave of the first tone. For example, if we place an augmented fourth above C we have the tone F♯, and superimposing another augmented fourth above F♯ we have B♯, the enharmonic equivalent of C:

EXAMPLE 17-1

For this very reason, however, the tritone may be said to have twice the *valency* of the other intervals. An example will illustrate this. The complete chromatic scale contains, as we have seen, twelve perfect fifths, twelve minor seconds, twelve major seconds, twelve minor thirds, and twelve major thirds. It contains, however, only *six* tritones: C to F♯, D♭ to G, D♮ to G♯, E♭ to A, E♮ to A♯, and F to B, since the tritones above F♯, G, A♭, A♮, B♭, and B♮ are duplications of the first six. It is necessary,

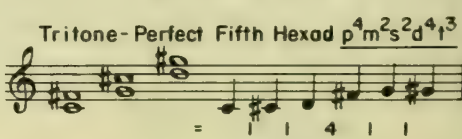
therefore, in judging the *relative importance* of the tritone in any scale to multiply the number of tritones by two.

In the whole-tone scale, for example, we found six major thirds, six major seconds, and three tritones. Since three tritones is the maximum number of tritones which can exist in any six-tone sonority, and since six is the maximum of major seconds or major thirds which can exist in any six-tone sonority, we may say that this scale is *saturated* with major seconds, major thirds, and tritones; and that the three tritones have the same valency as the six major seconds and six major thirds.

Since the tritone cannot be projected upon itself to produce a scale, the tritone projection must be formed by superimposing the tritone upon those scales or sonorities *which do not themselves contain tritones*. We may begin, therefore, by superimposing tritones on the tones of the perfect-fifth series.

Starting with the tone C, we add the tritone F♯; we then add the perfect fifth above C, or G, and superimpose the tritone C♯; and, finally, we add the fifth above G, or D, and superimpose the tritone G♯, forming the projection C-F♯-G-C♯-D-G♯, which arranged melodically produces the six-tone scale $C_1C\sharp_1D_4F\sharp_1G_1G\sharp$:

Example 17-2

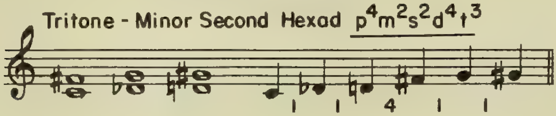


This scale will be seen to consist of four perfect fifths, four minor seconds, two major thirds, two major seconds, and three tritones: $p^4m^2s^2d^4t^3$. Multiplying the number of tritones by two, we find that this scale predominates in tritones, with the intervals of the perfect fifth and the minor second next in importance, and with no minor thirds. This is an isometric scale, since the same order of intervals reversed, 11411, produces the identical scale.

If we superimpose the tritones above the minor-second projec-

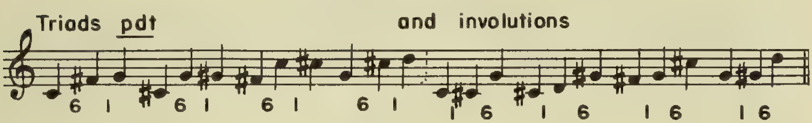
tion we produce the same scale: C to F♯, D♭ to G♮, D♮ to G♯, or arranged melodically, $C_1D♭_1D♮_4F♯_1G_1G♯$:

EXAMPLE 17-3



The components of this perfect-fifth—tritone projection are the characteristic triads $C_6F♯_1G$, $C♯_6G_1G♯$, $F♯_6C_1C♯$, and $G_6C♯_1D$, *pdt,* and their involutions $C_1C♯_6G$, $C♯_1D_6G♯$, $F♯_1G_6C♯$, and $G_1G♯_6D$, which, though they have been encountered in the minor-third scale, are more characteristic of this projection;

EXAMPLE 17-4

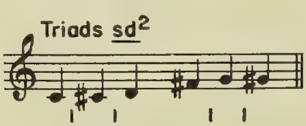


the triads C_2D_5G and $F♯_2G♯_5C♯$, p^2s, the characteristic triads of the perfect-fifth projection;

EXAMPLE 17-5

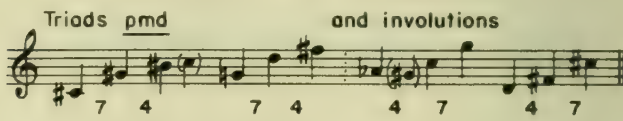
the triads $C_1C♯_1D$ and $F♯_1G_1G♯$, sd^2, the characteristic triads of the minor-second projection;

EXAMPLE 17-6



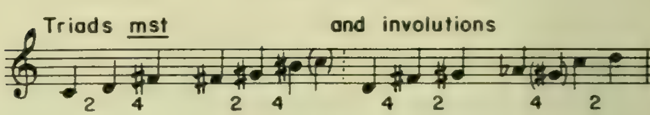
the triads $C\sharp_7G\sharp_4B\sharp$(C) and $G_7D_4F\sharp$, *pmd*, with the involutions $A\flat_4C_7G$ and $D_4F\sharp_7C\sharp$, which have been found in the six-tone perfect-fifth, minor-second, and major-third projections;

Example 17-7



and the triads $C_2D_4F\sharp$ and $F\sharp_2G\sharp_4B\sharp$(C), *mst*, with the involutions $D_4F\sharp_2G\sharp$ and $A\flat(G\sharp)_4C_2D$, which have been met in the major-second and minor-third hexads:

Example 17-8



The series contains five new forms of tetrads which have not appeared in any of the other hexads so far discussed:

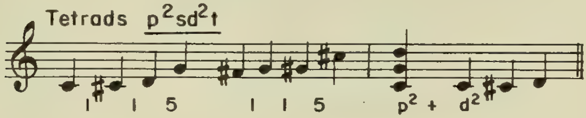
1. The characteristic isometric tetrads of the series, $C_1C\sharp_5F\sharp_1G$ and $C\sharp_1D_5G_1G\sharp$, $p^2d^2t^2$, which contain the maximum number of tritones possible in a tetrad, and which also contain two perfect fifths and two minor seconds. These tetrads may also be considered to be formed of two perfect fifths at the interval of the tritone, of two tritones at the interval of the perfect fifth, of two minor seconds at the interval of the tritone, or of two tritones at the interval of the minor second:

Example 17-9

2. The isometric tetrads $C_1C\sharp_1D_5G$ and $F\sharp_1G_1G\sharp_5C\sharp$, p^2sd^2t,

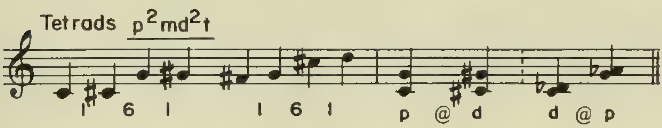
which also contain two perfect fifths and two minor seconds, but which contain only one tritone and one major second. These tetrads may be considered to be formed by the simultaneous projection of two perfect fifths and two minor seconds:

EXAMPLE 17-10



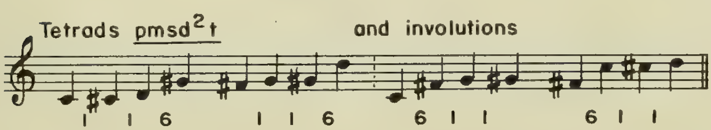
3. The isometric tetrads $C_1C\sharp_6G_1G\sharp$ and $F\sharp_1G_6C\sharp_1D$, p^2md^2t, which contain two perfect fifths, two minor seconds, one major third and one tritone; and which will be seen to embrace two relationships: the relationship of two perfect fifths at the interval of the minor second, and the relationship of two minor seconds at the interval of the perfect fifth:

EXAMPLE 17-11



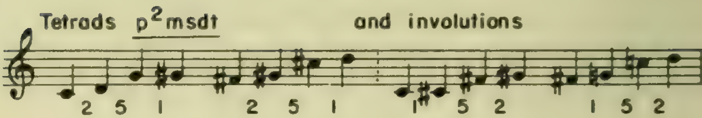
4. The tetrads $C_1C\sharp_1D_6G\sharp$ and $F\sharp_1G_1G\sharp_6D$, $pmsd^2t$, with their involutions $C_6F\sharp_1G_1G\sharp$ and $F\sharp_6C_1C\sharp_1D$:

EXAMPLE 17-12



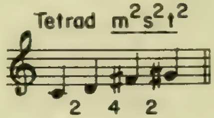
5. The tetrads $C_2D_5G_1G\sharp$, and $F\sharp_2G\sharp_5C\sharp_1D$, p^2msdt, with their involutions $C_1C\sharp_5F\sharp_2G\sharp$ and $F\sharp_1G_5C_2D$:

EXAMPLE 17-13



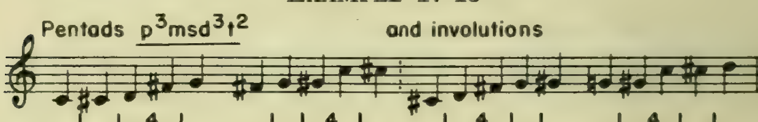
The remaining tetrad is the isometric tetrad $C_2D_4F\sharp_2G\sharp$, $m^2s^2t^2$, which we have already discussed as an important part of the major-second projection:

EXAMPLE 17-14



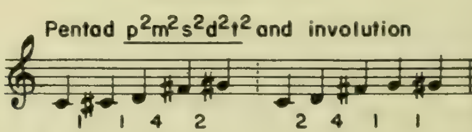
The series contains two new pentad forms and their involutions: the characteristic pentads $C_1C\sharp_1D_4F\sharp_1G$, $p^3msd^3t^2$, and $F\sharp_1G_1G\sharp_4C_1C\sharp$, with the involutions $C\sharp_1D_4F\sharp_1G_1G\sharp$ and $G_1G\sharp_4C_1C\sharp_1D$;

EXAMPLE 17-15



and $C_1C\sharp_1D_4F\sharp_2G\sharp$, $p^2m^2s^2d^2t^2$, and its involution $C_2D_4F\sharp_1G_1G\sharp$, which also predominate in tritones:

EXAMPLE 17-16



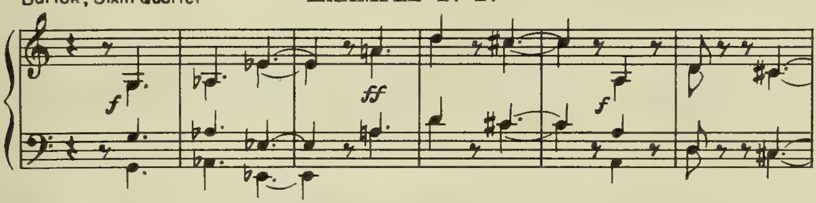
The characteristics of the hexad will be seen to be a predominance of tritones, with the perfect fifths and minor seconds

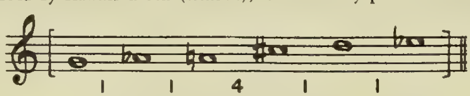
of secondary importance, and with the major third and the major second of tertiary importance. It will be noted, furthermore, that the six-tone scale contains no minor thirds.

Listening to this scale as a whole, and to its component parts, the student will find that it contains highly dissonant but tonally interesting material. The unison theme near the beginning of the Bartok sixth quartet dramatically outlines this scale:

Bartok, Sixth Quartet

EXAMPLE 17-17

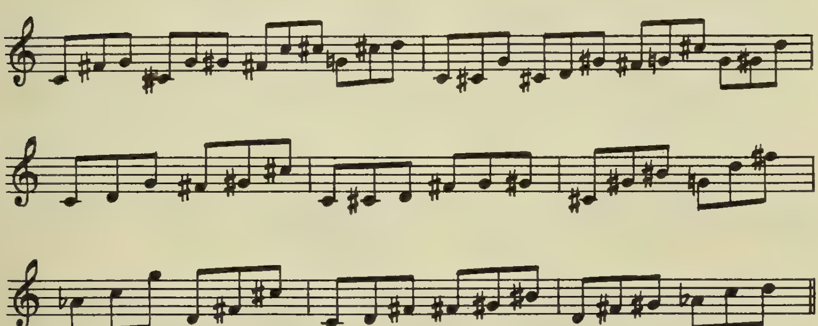


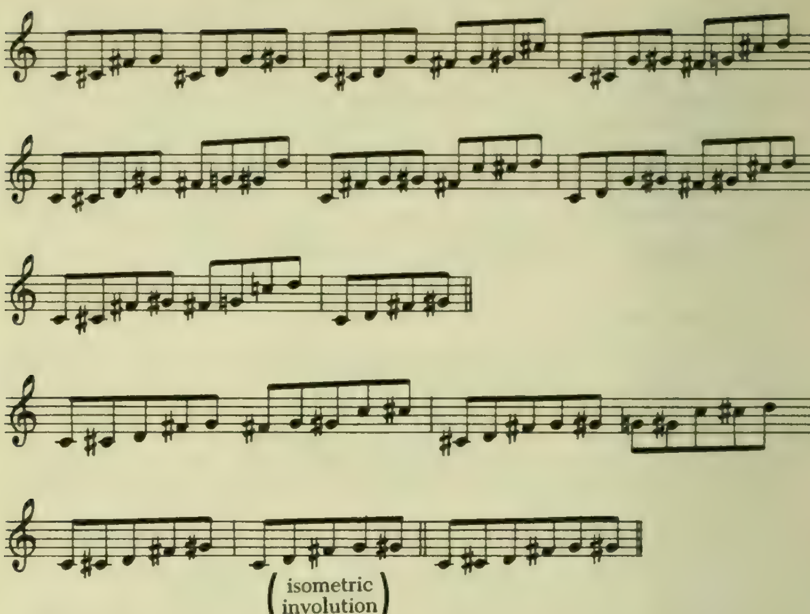


See also the beginning of the fifth movement of the Bartok fourth quartet for the use of the same scale in its five-tone form.

Play several times the triad, tetrad, pentad, and hexad material of this scale as outlined in Example 17-18.

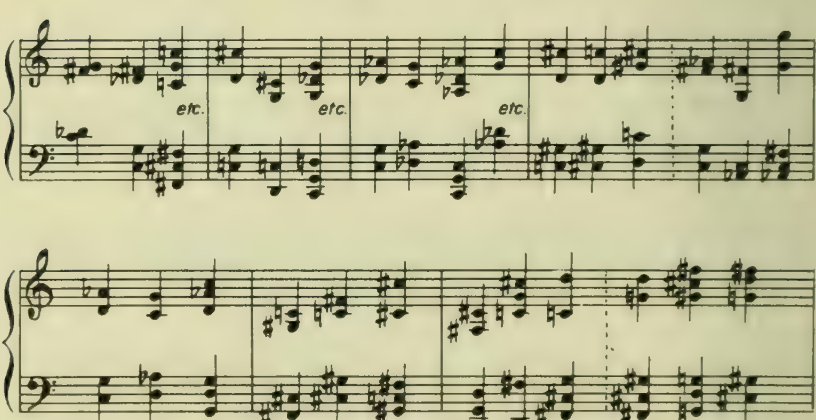
EXAMPLE 17-18





This scale adds five new tetrad forms, two new pentad forms, and, of course, one new hexad form. Experiment with these new sonorities as in Example 17-19, changing the spacing, position, and doublings of the tones of each sonority.

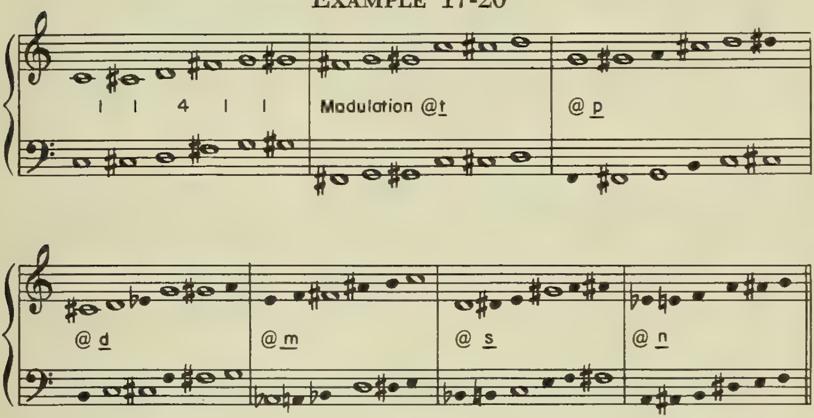
EXAMPLE 17-19



Now write a short sketch based on the material of the perfect-fifth–tritone hexad.

Example 17-20 indicates the modulatory possibilities of the perfect-fifth–tritone hexad. Write a short sketch employing any one of the five possible modulations, up or down.

EXAMPLE 17-20

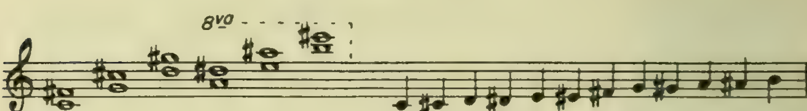


18

Projection of the Perfect-Fifth–Tritone Series Beyond Six Tones

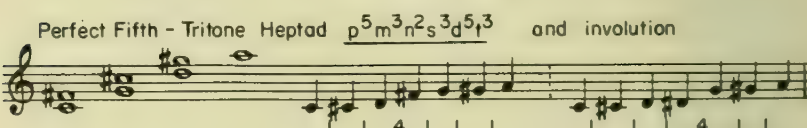
Beginning with the six-tone perfect-fifth–tritone scale $C_1C\sharp_1D_4F\sharp_1G_1G\sharp$, we may now form the remaining scales by continuing the process of superimposing tritones above the remaining tones of the perfect-fifth scale. The order of the projection will, therefore, be C to F♯, G to C♯, D to G♯, A to D♯, E to A♯, B to E♯:

Example 18-1



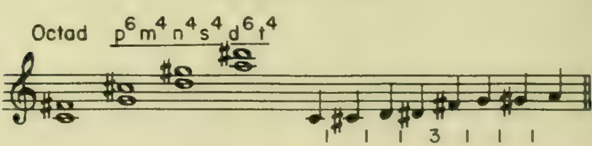
Seven tone: $C_1C\sharp_1D_4F\sharp_1G_1G\sharp_1A$, $p^5m^3n^2s^3d^5t^3$, with its involution $C_1C\sharp_1D_1D\sharp_4G_1G\sharp_1A$:

Example 18-2



Eight tone: $C_1C\sharp_1D_1D\sharp_3F\sharp_1G_1G\sharp_1A$ (isometric), $p^6m^4n^4s^4d^6t^4$:

Example 18-3



Nine tone: $C_1C\sharp_1D_1D\sharp_1E_2F\sharp_1G_1G\sharp_1A$, $p^7m^6n^6s^6d^7t^4$, with its involution $C_1C\sharp_1D_1D\sharp_2F\natural_1F\sharp_1G_1G\sharp_1A$:

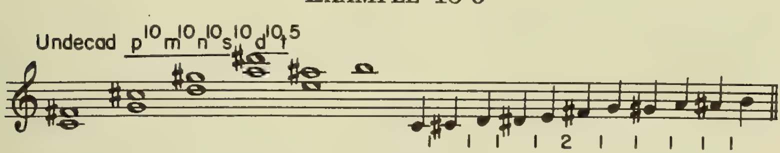
EXAMPLE 18-4

Ten tone: $C_1C\sharp_1D_1D\sharp_1E_2F\sharp_1G_1G\sharp_1A_1A\sharp$ (isometric), $p^8m^8n^8s^8d^8t^5$:

EXAMPLE 18-5

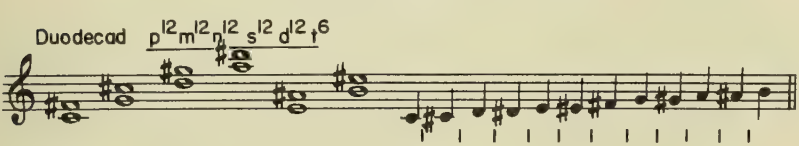
Eleven tone: $C_1C\sharp_1D_1D\sharp_1E_2F\sharp_1G_1G\sharp_1A_1A\sharp_1B$ (isometric), $p^{10}m^{10}n^{10}s^{10}d^{10}t^5$:

EXAMPLE 18-6



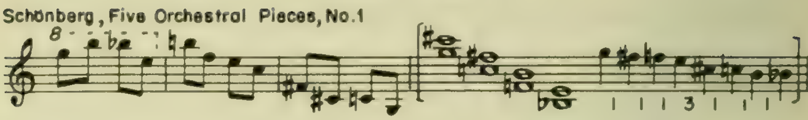
Twelve tone: $C_1C\sharp_1D_1D\sharp_1E_1E\sharp_1F\sharp_1G_1G\sharp_1A_1A\sharp_1B$, $p^{12}m^{12}n^{12}s^{12}d^{12}t^6$:

EXAMPLE 18-7



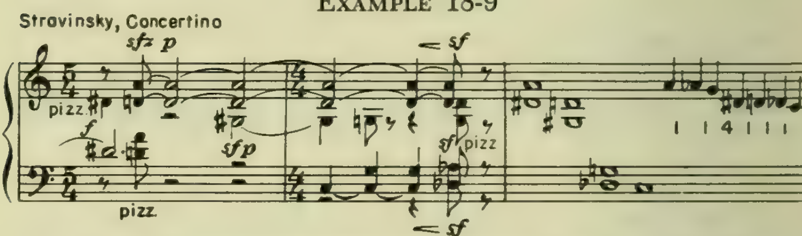
The melodic line in the violins in measures 60 to 62 of the first of the Schönberg *Five Orchestral Pieces,* is an excellent example of the eight-tone perfect-fifth–tritone projection:

Example 18-8



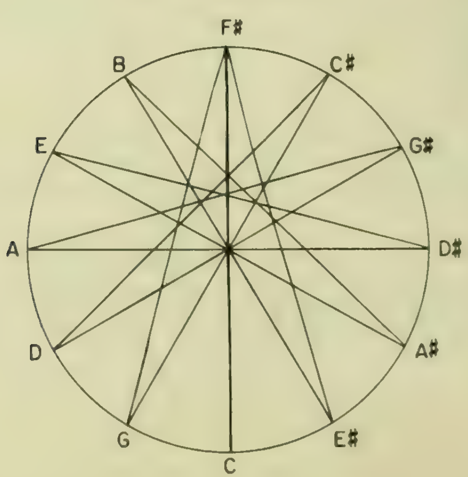
Measures 3 and 4 of the Stravinsky *Concertino* for string quartet are a striking example of the seven-tone perfect-fifth–tritone projection in involution:

Example 18-9



The following diagram is a graphic representation of the perfect-fifth–tritone projection.

Example 18-10

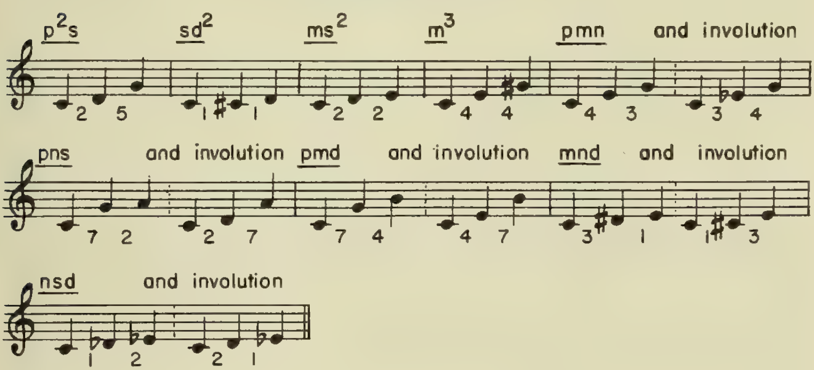


19

The pmn-Tritone Projection

There are nine triads which contain no tritones, the triads already described by the symbols *p²s, sd², ms², m³, pmn, pns, pmd, mnd,* and *nsd.*

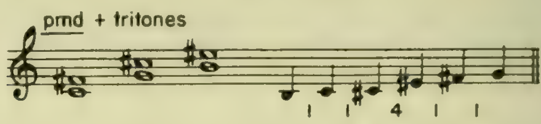
Example 19-1



It would seem, therefore, logical to assume that we might produce a six-tone tritone projection using each of these triads. However, if we use each of the above triads as a basis for the projection of the tritone, we find that *only one new scale is produced.* The projection of tritones upon the triads *p²s* and *sd²*, as we have already seen, produces the same scale, $C_1C\sharp_1D_4F\sharp_1G_1G\sharp$. The projection of tritones on the triad *pmd*

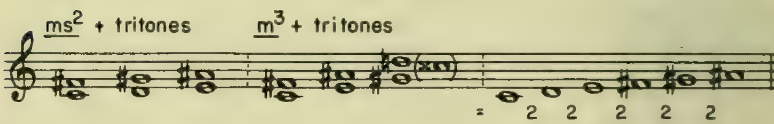
also produces the same scale, C-G-B + F♯-C♯-E♯ = $B_1C_1C\sharp_4E\sharp_1F\sharp_1G$:

EXAMPLE 19-2



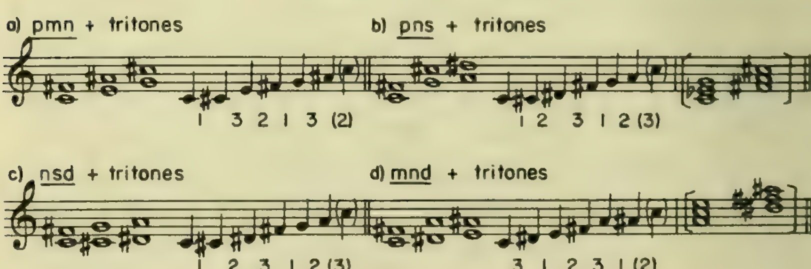
The projection of tritones above the triads *ms*² and *m*³ produces the major-second scale, C-D-E + F♯-G♯-A♯ = $C_2D_2E_2F\sharp_2G\sharp_2A\sharp$; and C-E-G♯ + F♯-A♯-C𝄪-(D) = $C_2D_2E_2F\sharp_2G\sharp_2A\sharp$:

EXAMPLE 19-3



The projection of tritones above the major triad, however, produces a new six-tone scale (Example 19-4*a*). The projection of tritones above the triads *pns* and *nsd* produces the involution of the same scale, that is, two minor triads, C-E♭-G and F♯-A-C♯, at the interval of the tritone (Example 19-4*b*, *c*). The projection of the tritone above the triad *mnd* also produces the involution of the first scale: two minor triads, A-C-E and D♯-F♯-A♯, at the interval of the tritone (Example 19-4*d*).

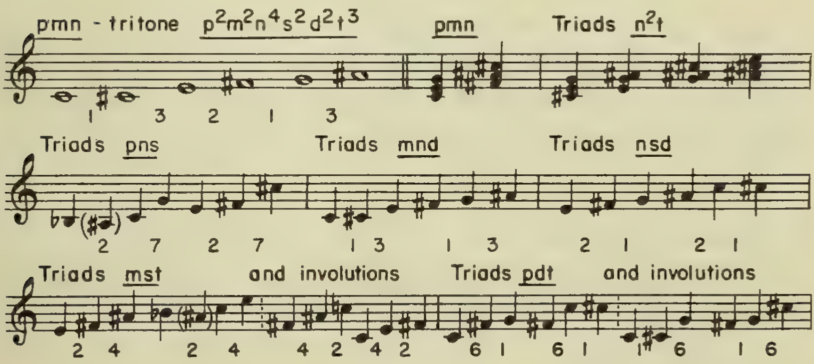
EXAMPLE 19-4



Beginning with the major triad C-E-G, we project a tritone above each of the tones of the triad: C to F♯; E to A♯, and G to C♯, producing the six-tone scale $C_1C\sharp_3E_2F\sharp_1G_3A\sharp$. This scale has two perfect fifths, two major thirds, four minor thirds, two major seconds, two minor seconds, and three tritones: $p^2m^2n^4s^2d^2t^3$. It predominates, therefore, in tritones, but also contains a large number of minor thirds and only two each of the remaining intervals. Its sound, is, therefore, somewhat similar to that of the six-tone minor-third scale which predominates in minor thirds but also has two of the possible three tritones.

The components of this scale are the two major triads C_4E_3G and $F\sharp_4A\sharp_3C\sharp$, *pmn*; the diminished triads $C\sharp_3E_3G$, $E_3G_3B\flat(A\sharp)$, $G_3B\flat(A\sharp)_3D\flat(C\sharp)$, and $A\sharp_3C\sharp_3E$, n^2t; the triads $(A\sharp)B\flat_2C_7G$ and $E_2F\sharp_7C\sharp$, *pns*; the triads $C_1C\sharp_3E$ and $F\sharp_1G_3A\sharp$, *mnd*; the triads $E_2F\sharp_1G$ and $A\sharp_2C_1C\sharp$, *nsd*; the triads $E_2F\sharp_4A\sharp$ and $B\flat_2(A\sharp)C_4E$, *mst*, with the involutions $F\sharp_4A\sharp_2C$ and $C_4E_2F\sharp$; and triads $C_6F\sharp_1G$ and $F\sharp_6C_1C\sharp$, *pdt*, with their involutions $C_1C\sharp_6G$ and $F\sharp_1G_6C\sharp$; all of which we have already met:

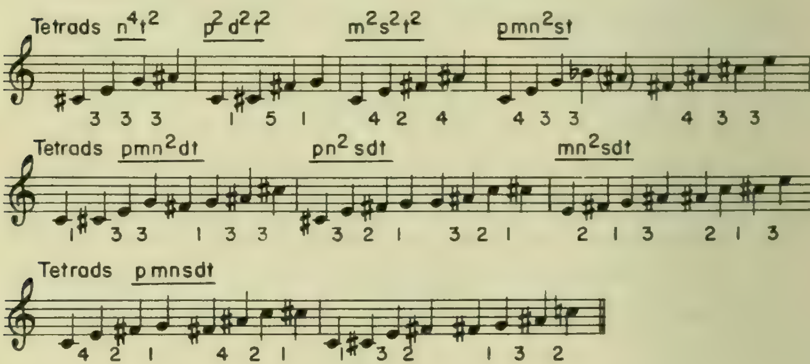
EXAMPLE 19-5



It contains the isometric tetrads $C\sharp_3E_3G_3A\sharp$, n^4t^2, $C_1C\sharp_5F\sharp_1G$, $p^2d^2t^2$ (which will be recalled as the characteristic tetrad of the previous projection), and $C_4E_2F\sharp_4A\sharp$, $m^2s^2t^2$; the tetrads $C_4E_3G_3B\flat(A\sharp)$ and $F\sharp_4A\sharp_3C\sharp_3E$, pmn^2st; $C_1C\sharp_3E_3G$ and $F\sharp_1G_3$

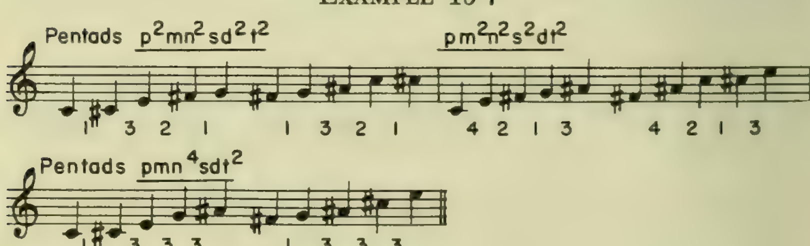
$A\sharp_3C\sharp$, *pmn²dt;* $C\sharp_3E_2F\sharp_1G$ and $G_3A\sharp_2C_1C\sharp$, *pn²sdt;* and $E_2F\sharp_1G_3A\sharp$ and $A\sharp_2C_1C\sharp_3E$, *mn²sdt* (which will be recalled as forming important parts of the six-tone minor-third scale); and the two pairs of "twins," *pmnsdt,* $C_4E_2F\sharp_1G$ and $F\sharp_4A\sharp_2C_1C\sharp$, and $C_1C\sharp_3E_2F\sharp$ and $F\sharp_1G_3A\sharp_2C$, both of which have the same analysis, but neither of which is the involution of the other. None of these tetrads is a new form, as all have been encountered in previous chapters.

Example 19-6



Finally, we find the characteristic pentads $C_1C\sharp_3E_2F\sharp_1G$ and $F\sharp_1G_3A\sharp_2C_1C\sharp$, *p²mn²sd²t²*, and $C_4E_2F\sharp_1G_3A\sharp$ and $F\sharp_4A\sharp_2C_1C\sharp_3E$, *pm²n²s²dt²*; and the characteristic pentads of the minor-third scale, $C_1C\sharp_3E_3G_3A\sharp$ and $F\sharp_1G_3A\sharp_3C\sharp_3E$, *pmn⁴sdt²*:

Example 19-7



Of these pentads, only the first two are new forms, the third

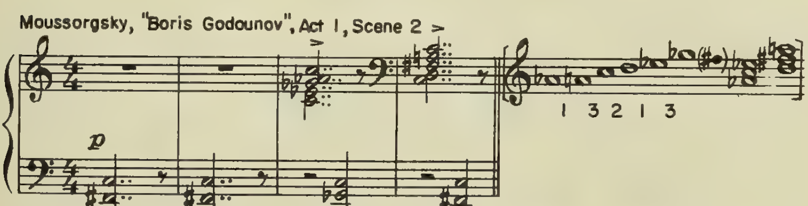
having already appeared as part of the minor-third projection.

This projection has been a favorite of contemporary composers since early Stravinsky, particularly observable in *Petrouchka.*

EXAMPLE 19-8

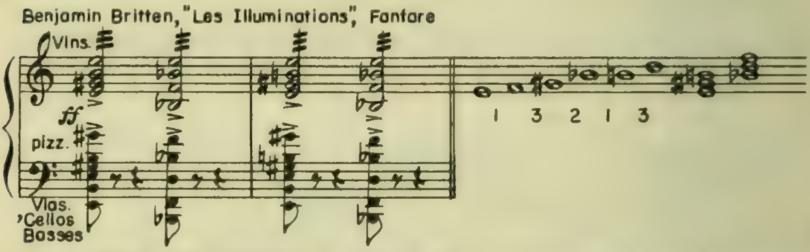
A striking earlier use is found in the coronation scene from *Boris Goudonov* by Moussorgsky:

EXAMPLE 19-9



A more recent example may be found in Benjamin Britten's *Les Illuminations*, the entire first movement of which is written in this scale:

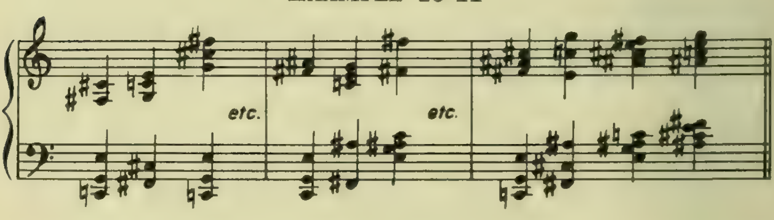
EXAMPLE 19-10



Play over several times Examples 19-5, 6, and 7; then play the entire six-tone scale until you have the sound of the scale firmly established.

Play the two characteristic pentads and their involutions, and the six-tone scale, in block harmony, experimenting with spacing, position, and doubling as in Example 19-11.

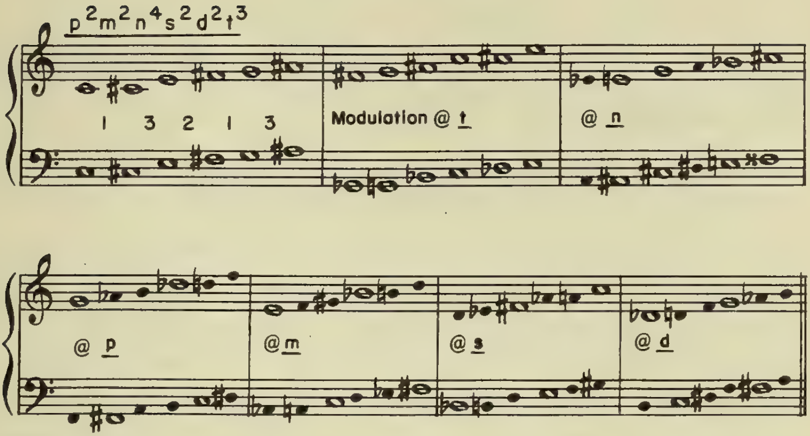
EXAMPLE 19-11



Write a short sketch using the material of the six-tone *pmn*-tritone projection.

Example 19-12 indicates the possible modulations of this scale. It will be noted that the modulation at the tritone changes *no* tones; modulation at the minor third, up or down, changes two tones; modulation at the perfect fifth, major third, major second, and minor second changes four of the six tones.

EXAMPLE 19-12



Write a short sketch employing any one of the possible modulations.

Analyze the third movement of Messiaen's *L'Ascension* for the projection of the major triad at the interval of the tritone.

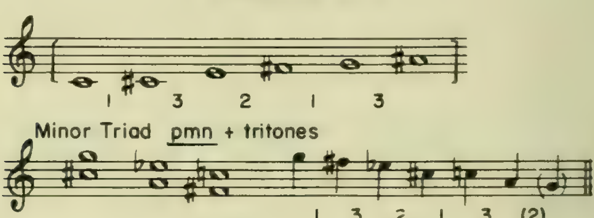
20

Involution of the pmn-Tritone Projection

IF, instead of taking the major triad C-E-G, we take its involution, the minor triad ↓G E♭ C, and project a tritone below each tone of the triad—G to C♯, E♭ to A, C to F♯—we will produce the six-tone scale $\downarrow G_1F\sharp_3E\flat_2C\sharp_1C\natural_3A_{(2)}(G)$ having the same intervallic analysis, $p^2m^2n^4s^2d^2t^3$.

This scale will be seen to be the involution of the major triad-tritone scale of the previous chapter.

EXAMPLE 20-1



The components of this scale are the involutions of the components of the major triad-tritone projection. They consist of the two minor triads $C_3E\flat_4G$ and $F\sharp_3A_4C\sharp$, *pmn*; the diminished triads $C_3E\flat_3G\flat(F\sharp)$, $D\sharp_3(E\flat)F\sharp_3A$, $F\sharp_3A_3C$ and $A_3C_3E\flat$, n^2t; the triads C_7G_2A and $F\sharp_7C\sharp_2D\sharp(E\flat)$, *pns;* the triads $E\flat_3F\sharp_1G$ and $A_3C_1C\sharp$, *mnd*; the triads $C_1D\flat_2E\flat$ and $F\sharp_1G_2A$, *nsd*; the triads $E\flat_4G_2A$ and $A_4C\sharp_2D\sharp(E\flat)$, *mst,* with the involutions $G_2A_4C\sharp$ and $D\flat(C\sharp)_2E\flat_4G$; and the triads $C_1C\sharp_6G$ and $F\sharp_1G_6C\sharp$, *pdt,* with their involutions $C_6F\sharp_1G$ and $F\sharp_6C_1C\sharp$.

Example 20-2

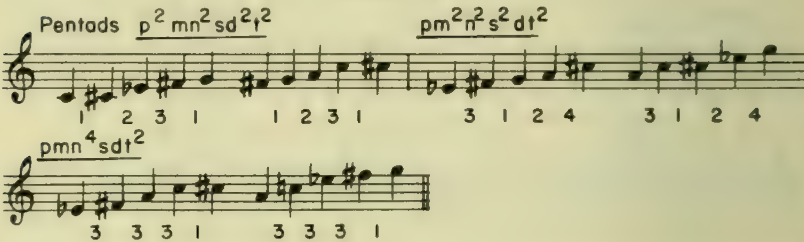
It contains the isometric tetrads $C_3E\flat_3F\sharp_3A$, n^4t^2, $C_1C\sharp_5F\sharp_1G$, $p^2d^2t^2$, and $E\flat_4G_2A_4C\sharp$, $m^2s^2t^2$; the tetrads $D\sharp_3(E\flat)F\sharp_3A_4C\sharp$ and $A_3C_3E\flat_4G$, *pmn*²*st;* $C_3E\flat_3F\sharp_1G$ and $F\sharp_3A_3C_1C\sharp$, *pmn*²*dt;* $C_1C\sharp_2D\sharp(E\flat)_3F\sharp$ and $F\sharp_1G_2A_3C$, *pn*²*sdt;* $E\flat_3F\sharp_1G_2A$ and $A_3C_1C\sharp_2D\sharp(E\flat)$, *mn*²*sdt* (all of which will be seen to be involutions of the tetrads in the major triad-tritone projection); and the involutions of the two pairs of the "twins," $C_1C\sharp_2E\flat_4G$ and $F\sharp_1G_2A_4C\sharp$, and $C\sharp_2D\sharp(E\flat)_3F\sharp_1G$ and $G_2A_3C_1C\sharp$, *pmnsdt.*

Example 20-3

Finally, we have the characteristic pentads $C_1C\sharp_2E\flat_3F\sharp_1G$ and $F\sharp_1G_2A_3C_1C\sharp$, $p^2mn^2sd^2t^2$; and $E\flat_3F\sharp_1G_2A_4C\sharp$ and $A_3C_1C\sharp_2E\flat_4G$,

$pm^2n^2s^2dt^2$; and the characteristic pentads of the minor-third scale, $E\flat_3F\sharp_3A_3C_1C\sharp$ and $A_3C_3E\flat_3F\sharp_1G$, pmn^4sdt^2, all of which are involutions of the pentads of the major triad-tritone projection:

EXAMPLE 20-4



Since the triad has only three tones, it is clear that the resultant scale formed by adding tritones above the original triad cannot be projected beyond six tones. The complementary scales beyond the six-tone projection will be discussed in a later chapter.

Write a short exercise, without modulation, employing the involution of the *pmn*-tritone hexad.

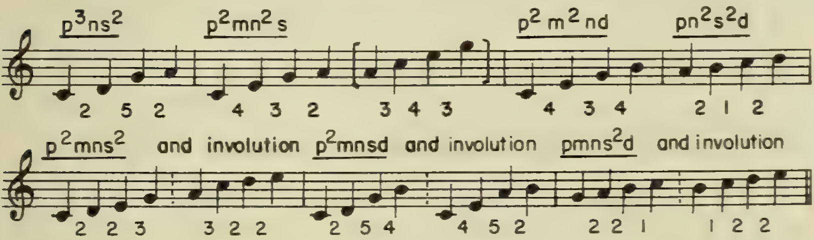
21

Recapitulation of the Tetrad Forms

We have now encountered all of the tetrad forms possible in the twelve-tone scale, twenty-nine in all, with their respective involutions. The young composer should review them carefully, listen to them in various inversions, experiment with different types of doubling and spacing of tones, until they gradually become a part of his tonal material.

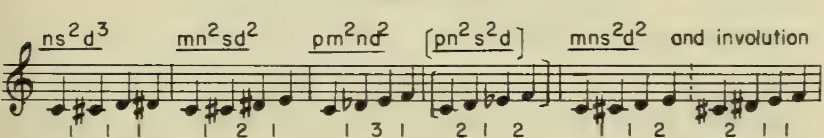
The six-tone perfect-fifth projection introduces the following tetrad types with their involutions (where the tetrad is not isometric):

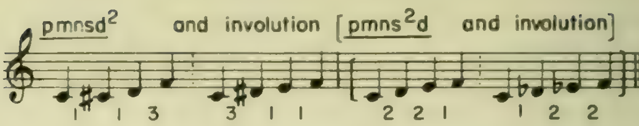
Example 21-1



The six-tone minor-second projection adds five new tetrad types:

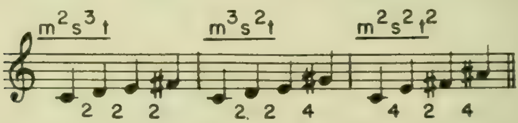
Example 21-2





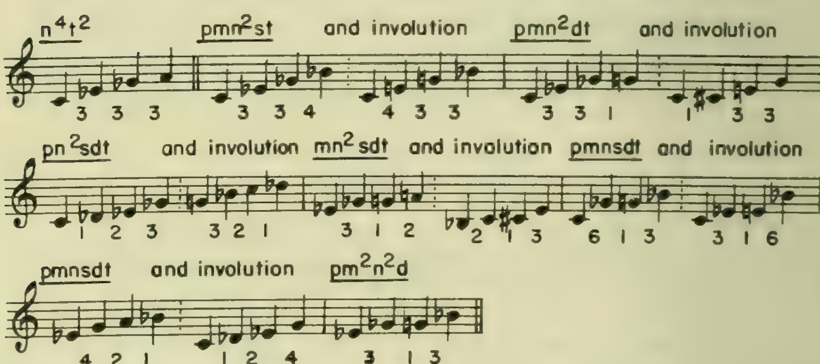
The six-tone major-second scale adds three new tetrad types:

EXAMPLE 21-3



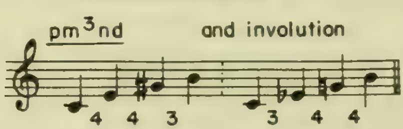
The six-tone minor-third scale adds eight new tetrad types.

EXAMPLE 21-4



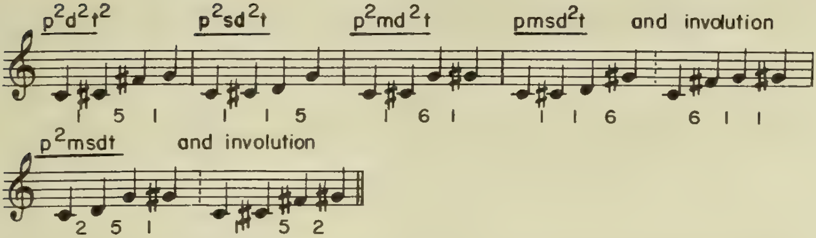
The six-tone major-third scale adds one new tetrad:

EXAMPLE 21-5



The tritone-perfect-fifth scale adds five new tetrads:

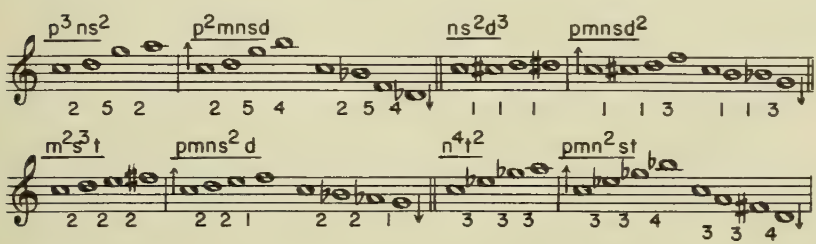
EXAMPLE 21-6



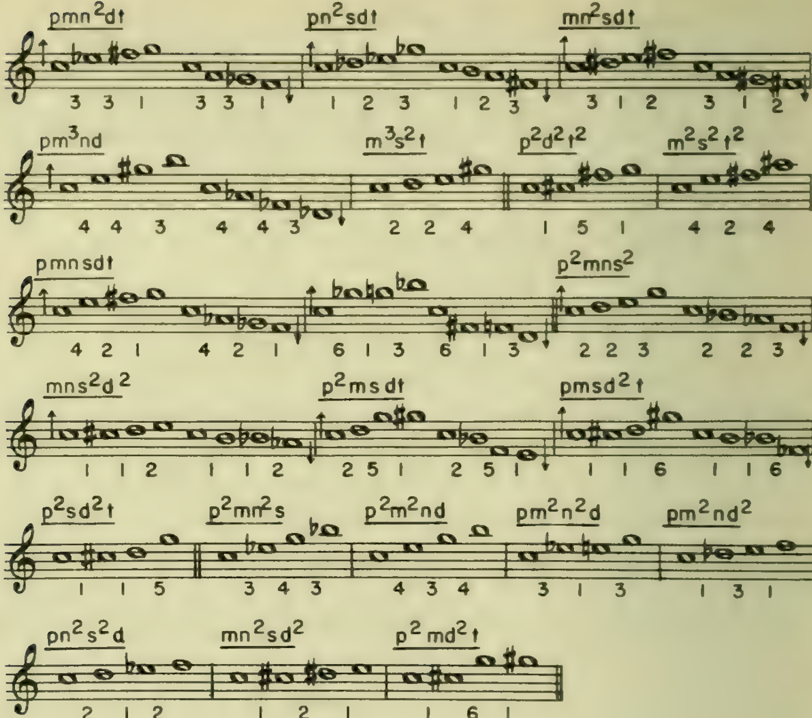
The *pmn*-tritone projection adds no new tetrads.

If we build all of the tetrads on the tone C and construct their involutions—where the tetrads are not isometric—below C, we have the sonorities as in Example 21-7. The sonorities are arranged in the following order: first, those in which the perfect fifth predominates, then those in which the minor second predominates, then the major second, minor third,* major third, and finally, those in which the tritone predominates. These are followed by the tetrads which are the result of the simultaneous projection of two intervals: the perfect-fifth and major second; the major second and minor second; two perfect fifths plus the tritone; two minor seconds plus the tritone; and finally the simultaneous projection of two perfect fifths and two minor seconds. These are followed by the tetrads which consist of two similar intervals related at a foreign interval.

EXAMPLE 21-7



* In the case of the minor-third tetrads it would be more accurate to say that they are dominated equally by the minor third and the tritone because of the latter's double valency.



Play the tetrads of Example 21-7 as indicated in previous chapters, listening to each carefully and experimenting with different positions and doublings.

Part II

CONSTRUCTION OF HEXADS BY THE SUPERPOSITION OF TRIAD FORMS

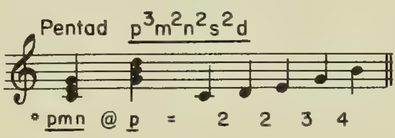
Projection of the Triad pmn

HAVING EXHAUSTED the possibilities of projection in terms of single intervals we may now turn to the formation of sonorities —or scales—by the superposition of triad forms. For reasons which will later become apparent, we shall not project these triads beyond six-tone chords or scales, leaving the discussion of the scales involving more than six tones to a later section.

We have found that there are five triads which consist of three *different* intervals and which *exclude* the tritone: *pmn, pns, pmd, mnd,* and *nsd.* Each of these triads projected upon its own tones will produce a distinctive six-tone scale in which the three intervals of the original triad predominate.

Beginning with the projection of the major triad, we form the major triad upon C—C-E-G—and superimpose another major triad upon its fifth, producing the second major triad, G-B-D. This gives the pentad $C_2D_2E_3G_4B$, $p^3m^2n^2s^2d$, which has already appeared in Chapter 5, page 47, as a part of the perfect-fifth projection:

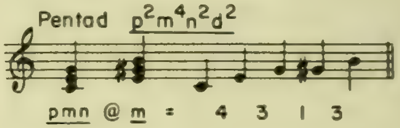
EXAMPLE 22-1



* The symbol *pmn* @ *p* should be translated as "the triad *pmn* projected at the interval of the perfect fifth."

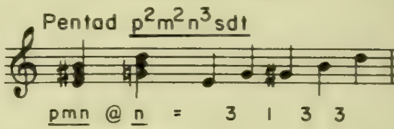
We then superimpose a major triad on the major third of the original triad, that is, E-G♯-B, producing in combination with the first triad, the pentad $C_4E_3G_1G\sharp_3B$, $p^2m^4n^2d^2$ (which we have already observed as a part of the major-third projection):

EXAMPLE 22-2



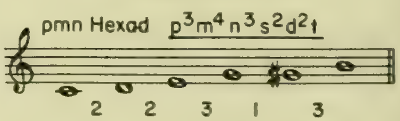
The triad on E and the triad on G together form the pentad $E_3G_1G\sharp_3B_3D$, $p^2m^2n^3sdt$ (which we have observed as a part of the minor-third projection):

EXAMPLE 22-3



The combined triads on C, E, and G form the six-tone major-triad projection $C_2D_2E_3G_1G\sharp_3B$, $p^3m^4n^3s^2d^2t$:

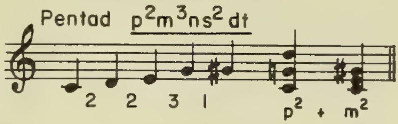
EXAMPLE 22-4



The chief characteristic of this scale is that it contains the maximum number of major triads. Since these triads are related at the intervals of the perfect fifth, the major third, and the minor third, the scale as a whole is a mixture of the materials from the perfect-fifth, major-third, and minor-third projections and has a preponderance of intervals of the perfect fifth, major third, and minor third.

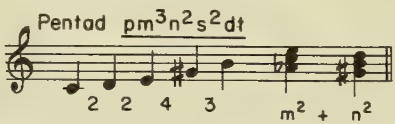
The major-triad projection adds no new triads or tetrads. It contains, in addition to the pentads already mentioned (combinations of two major triads at the intervals of the perfect fifth, major third, and minor third, respectively), three new pentads: the pentad $C_2D_2E_3G_1G\sharp$, $p^2m^3ns^2dt$, which may be analyzed as the simultaneous projection of two perfect fifths and two major thirds;

Example 22-5



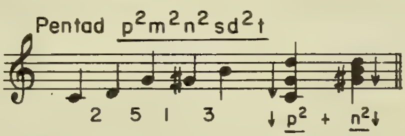
the pentad $C_2D_2E_4G\sharp_3B$, $pm^3n^2s^2dt$, which may be analyzed as the simultaneous projection of two major thirds and two minor thirds above G♯ (A♭);

Example 22-6



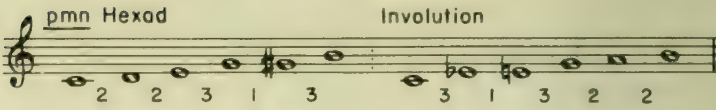
and the pentad $C_2D_5G_1G\sharp_3B$, $p^2m^2n^2sd^2t$, which may be analyzed as the simultaneous projection of two perfect fifths and two minor thirds, *downward*:

Example 22-7



The involution of the projection of the major triad $C_2D_2E_3G_1G\sharp_3B$ will be the same order of half-steps in reverse, that is, 31322, producing the scale $C_3E\flat_1E\natural_3G_2A_2B$:

EXAMPLE 22-8

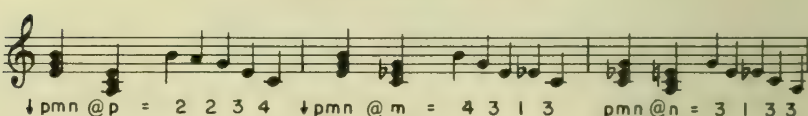


This will seem to be the same formation as that of the previous chapter, if begun on the tone B and constructed *downward*:

EXAMPLE 22-9

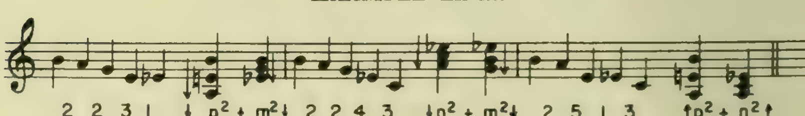
If we think the scale upward rather than downward, it becomes the projection of three minor *triads*: A-C-E, C-E♭-G, and E♮-G-B. The scale contains six pentads, the first three of which are formed of two minor triads at the interval of the perfect fifth, major third, and minor third, respectively:

EXAMPLE 22-10



The remaining pentads are:

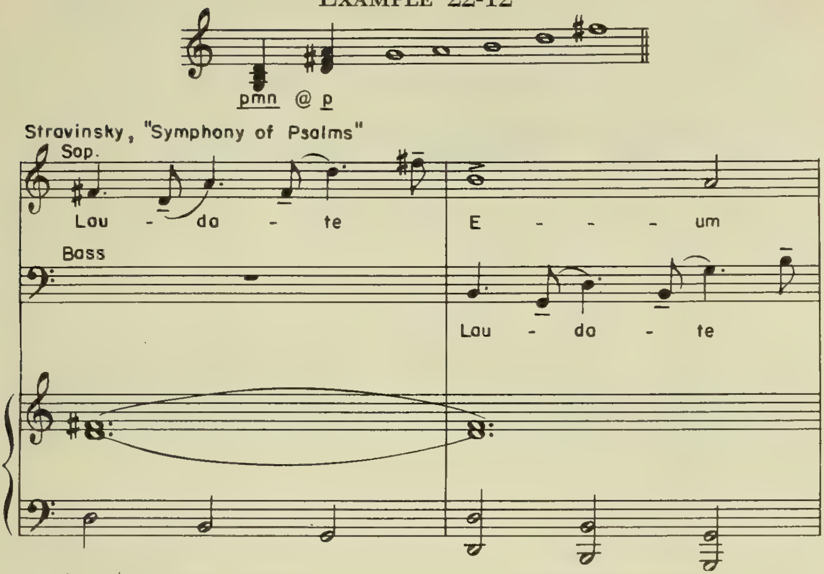
EXAMPLE 22-11



All of these will be seen to be involutions of the pentads discussed in the first part of this chapter.

A short but clear exposition of the mixture of two triads *pmn* at the interval of the perfect fifth may be found in Stravinsky's *Symphony of Psalms*:

EXAMPLE 22-12



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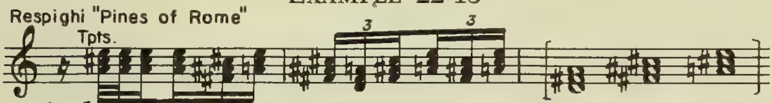
The short trumpet fanfare from Respighi's *Pines of Rome*, first movement, constitutes another very clear example of the projection of the triad *pmn*:

EXAMPLE 22-13



By permission of G. Ricordi & Co., Inc.

An exposition of the complete projection of the triad *pmn* in involution is found in the opening of the seventh movement, *Neptune*, from Gustav Holst's suite, *The Planets*:

EXAMPLE 22-14

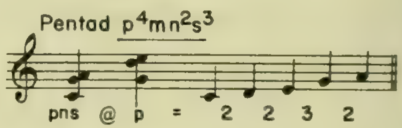
By permission of J. Curwen & Sons, Ltd.

23

Projection of the Triad pns

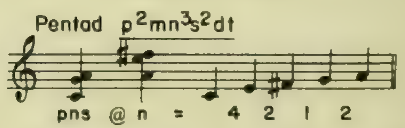
To project the triad *pns*, we may begin with the triad on C—C-G-A—and superimpose similar triads on G and A. We produce first the pentad $C_7G_2A + G_7D_2E$, or $C_2D_2E_3G_2A$, $p^4mn^2s^3$, which we recognize as the perfect-fifth pentad:

Example 23-1



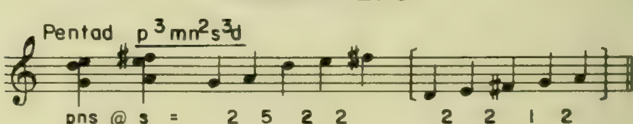
Next we superimpose upon C_7G_2A the triad $A_7E_2F\sharp$, producing the pentad $C_4E_2F\sharp_1G_2A$, $p^2mn^3s^2dt$;

Example 23-2



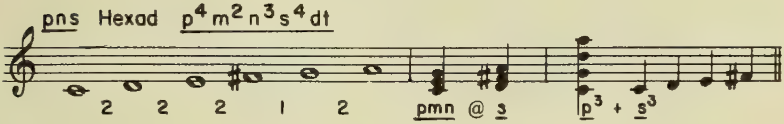
and, finally, the pentad formed by the combination of G_7D_2E and $A_7E_2F\sharp$, or $G_2A_5D_2E_2F\sharp$, $p^3mn^2s^3d$:

Example 23-3



Together with the original triad C-G-A, they produce the six-tone scale $C_2D_2E_2F\sharp_1G_2A$, $p^4m^2n^3s^4dt$. This scale has two other equally logical analyses. It may be considered to consist of two major triads at the interval of the major second, that is, C-E-G + D-F♯-A; and it may also be formed by the simultaneous projection of three perfect fifths and three major seconds above the first tone, that is, C-G-D-A-(E) + C-D-E-F♯ = C-D-E-F♯-G-A:

EXAMPLE 23-4

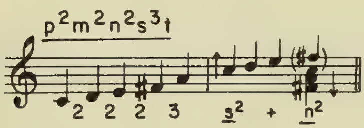


It is a graceful scale in which to write, deriving a certain pastoral quality from its equal combination of perfect fifths and major seconds and having among its intervals one strong dissonance of the minor second, and one tritone.

This scale contains, in addition to the pentads already discussed, three more pentads, none of which has appeared before.

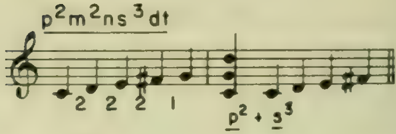
1. The isometric pentad $C_2D_2E_2F\sharp_3A$, $p^2m^2n^2s^3t$, formed by the projection of two major seconds *above* and two minor thirds *below* C, which we shall consider in a later chapter:

EXAMPLE 23-5



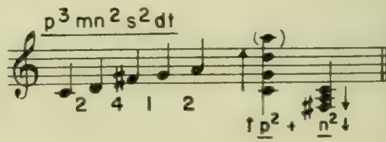
2. The pentad $C_2D_2E_2F\sharp_1G$, $p^2m^2ns^3dt$, which may be analyzed as the simultaneous projection of two perfect fifths and three major seconds:

EXAMPLE 23-6



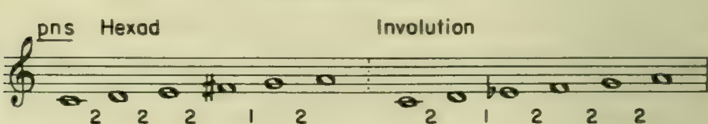
3. The pentad $C_2D_4F\sharp_1G_2A$, $p^3mn^2s^2dt$, which may be analyzed as the projection of two (or three) perfect fifths above and two minor thirds below C:

EXAMPLE 23-7



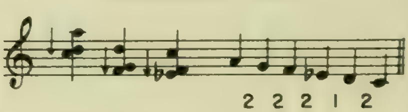
The involution of the projection $C_2D_2E_2F\sharp_1G_2A$, *pns*, will have the same order of half-steps *in reverse*, 21222, forming the scale $C_2D_1E\flat_2F_2G_2A$:

EXAMPLE 23-8



This scale will be seen to be the same formation as the original *pns* hexad if begun on the tone A and constructed *downward*:

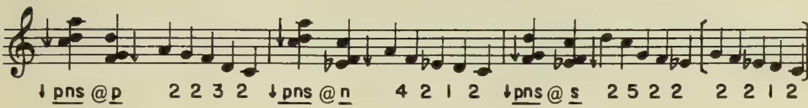
EXAMPLE 23-9



The scale contains six pentads, all of which are involutions of those found in the original hexad, except, the first and fourth pentad, which are isometric. The first pentad contains the involu-

tion of two triads *pns* at the interval of the perfect fifth; the second at the interval of the major sixth; and the third at the interval of the major second:

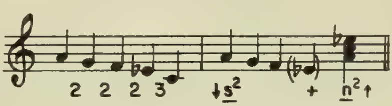
EXAMPLE 23-10



This scale contains, in addition to the pentads already discussed, three more pentad forms, all of which will be found to be involutions of the pentads discussed in the first part of this chapter:

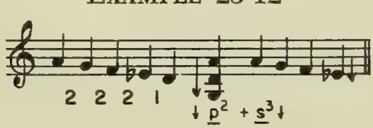
1. The isometric pentad $A_2G_2F_2E\flat_3C$, $p^2m^2n^2s^3t$, which may be analyzed as the projection of two major seconds below, and two minor thirds above, A:

EXAMPLE 23-11



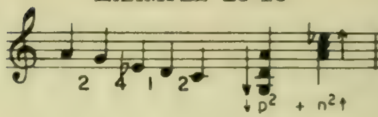
2. The pentad $A_2G_2F_2E\flat_1D$, $p^2m^2ns^3dt$, which may be analyzed as the simultaneous projection of two perfect fifths and three major seconds *below* A:

EXAMPLE 23-12



3. The pentad $A_2G_4E\flat_1D_2C$, $p^3mn^2s^2dt$, which may be analyzed as the projection of two perfect fifths below A and two minor thirds above A:

EXAMPLE 23-13

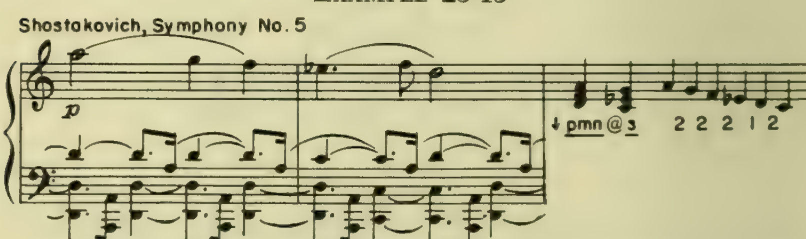


The smooth, pastoral qualities of this scale are beautifully illustrated by the following excerpt from Vaughn-Williams' *The Shepherds of the Delectable Mountains*:

EXAMPLE 23-14

The involution of this scale is clearly projected in the theme from the Shostakovich Fifth Symphony, first movement:

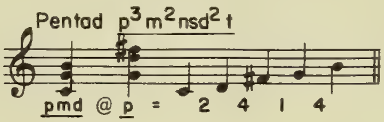
EXAMPLE 23-15



Projection of the Triad pmd

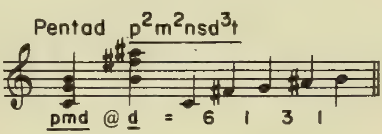
THE PROJECTION of the triad C-G-B, *pmd,* produces the pentad, *pmd* @ *p*, $C_7G_4B + G_7D_4F\sharp$, or $C_2D_4F\sharp_1G_4B$, $p^3m^2nsd^2t$;

EXAMPLE 24-1



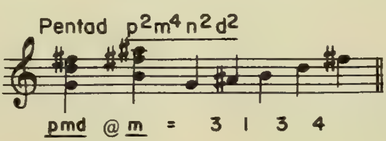
the pentad, *pmd* @ *d*, $C_7G_4B + B_7F\sharp_4A\sharp$, or $C_6F\sharp_1G_3A\sharp_1B$, $p^2m^2nsd^3t$;

EXAMPLE 24-2



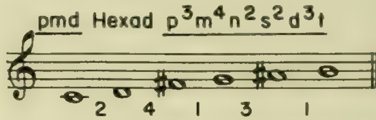
and the pentad, *pmd* @ *m*, $G_7D_4F\sharp + B_7F\sharp_4A\sharp$, or $G_3A\sharp_1B_3D_4F\sharp$, $p^2m^4n^2d^2$;

EXAMPLE 24-3



which we have already observed as the involution of the characteristic pentad of the major-third series. The triad *pmd* and the two projections together form the six-tone scale $C_2D_4F\sharp_1G_3A\sharp_1B$, $p^3m^4n^2s^2d^3t$:

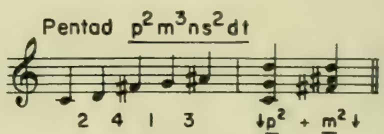
EXAMPLE 24-4



In addition to the three pentads already described, the *pmd* projection contains three other pentads:

1. The pentad $C_2D_4F\sharp_1G_3A\sharp$, $p^2m^3ns^2dt$, the projection of two perfect fifths and two major thirds below D, already found in the involution of the projection of the triad *pmn*:

EXAMPLE 24-5

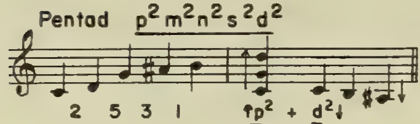


2. The pentad $C_2D_4F\sharp_4A\sharp_1B$, $pm^3ns^2d^2t$, which, if begun on A♯, may be analyzed as the simultaneous projection of two major thirds and two minor seconds above A♯ (or B♭):

EXAMPLE 24-6

3. The pentad $C_2D_5G_3A\sharp_1B$, $p^2m^2n^2s^2d^2$, which may be analyzed as the projection of two perfect fifths *above* C and two minor seconds *below* C:

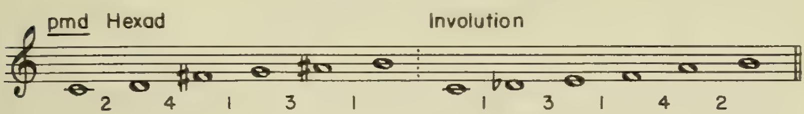
EXAMPLE 24-7



This scale has one major and two minor triads which may serve as key centers if the scale is begun on G or on B. It bears the closest affinity to the major-third scale but contains both major seconds and a tritone, which the major-third scale lacks.

The involution of the projection *pmd* will have the same order of half-steps in reverse. Since the order of the original *pmd* projection was 24131, the order of the involution will be 13142, or $C_1D\flat_3E_1F_4A_2B$:

EXAMPLE 24-8

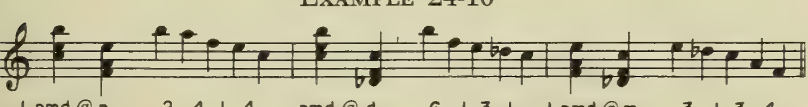


If we begin on B and project the original triad *pmd downward,* we produce the same scale:

EXAMPLE 24-9

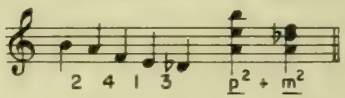
The scale contains six pentads, the first three of which are formed by the relationship of the involution of *pmd* at the intervals of the perfect fifth, major seventh, and major third, respectively;

EXAMPLE 24-10



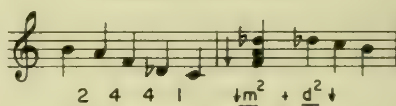
the pentad $B_2A_4F_1E_3D\flat$, $p^2m^3ns^2dt$, the projection of two perfect fifths and two major thirds above A, already found in the major-triad projection;

EXAMPLE 24-11



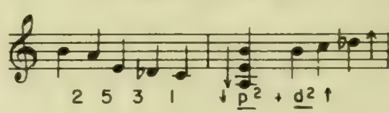
the pentad $B_2A_4F_4D\flat_1C$, $pm^3ns^2d^2t$, which, if begun on D♭, may be analyzed as the simultaneous projection of two major thirds and two minor seconds *downward*;

EXAMPLE 24-12



and the pentad $B_2A_5E_3D\flat_1C$, $p^2m^2n^2s^2d^2$, which may be analyzed as the projection of two perfect fifths *below* B and two minor seconds *above* B:

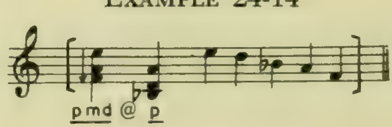
EXAMPLE 24-13



All of the above pentads will be observed to be involutions of the pentads in the first part of this chapter.

An illustration of the use of the triad *pmd* at the interval of the perfect fifth, used as harmonic background, in the *Danse Sacrale* from Stravinsky's *Le Sacre du Printemps*, follows:

EXAMPLE 24-14



Stravinsky, "Danse Sacrale"

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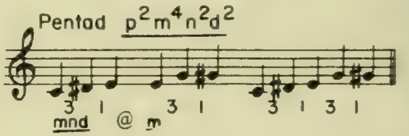
All of the above pentads will be observed to be involutions of the pentads in the first part of this chapter.

An illustration of the use of the triad *pmd* at the interval of the perfect fifth, used as harmonic background, in the *Danse Sacrale* from Stravinsky's *Le Sacre du Printemps,* follows:

Projection of the Triad *mnd*

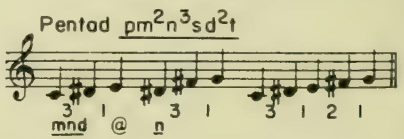
THE PROJECTION of the triad $C_3D\sharp_1E$, *mnd*, forms the pentad *mnd* @ *m*, $C_3D\sharp_1E + E_3G_1G\sharp$, or $C_3D\sharp_1E_3G_1G\sharp$, $p^2m^4n^2d^2$, which, if begun on G♯, or A♭, will be seen to be the characteristic pentad of the major-third series;

EXAMPLE 25-1



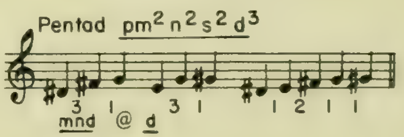
the pentad *mnd* @ *n*, C-D♯-E + D♯-F♯-G, or $C_3D\sharp_1E_2F\sharp_1G$, $pm^2n^3sd^2t$;

EXAMPLE 25-2



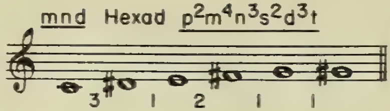
the pentad *mnd* @ *d*, $D\sharp_3F\sharp_1G + E_3G_1G\sharp$, or $D\sharp_1E_2F\sharp_1G_1G\sharp$, $pm^2n^2s^2d^3$:

EXAMPLE 25-3



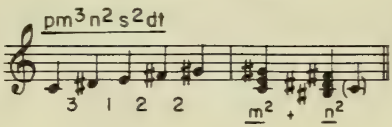
Together they form the six-tone scale $C_3D\sharp_1E_2F\sharp_1G_1G\sharp$, $p^2m^4n^3s^2d^3t$:

EXAMPLE 25-4



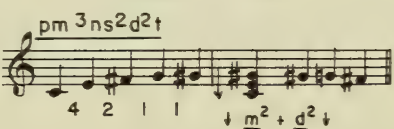
The remaining pentads are the pentad $C_3D\sharp_1E_2F\sharp_2G\sharp$, $pm^3n^2s^2dt$, which may be analyzed as the simultaneous projection of two major thirds and two minor thirds, and which has already appeared as a part of the *pmn* projection;

EXAMPLE 25-5



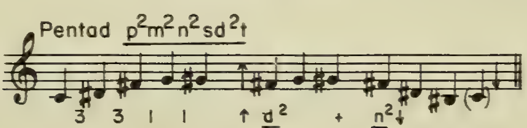
the pentad $C_4E_2F\sharp_1G_1G\sharp$, $pm^3ns^2d^2t$; which has already been observed as a part of the *pmd* projection, and which may be analyzed as the combination of two major thirds and two minor seconds below G♯;

EXAMPLE 25-6



and the new pentad $C_3D\sharp_3F\sharp_1G_1G\sharp$, $p^2m^2n^2sd^2t$, which may be analyzed as a combination of two minor seconds *above*, and two minor thirds *below* F♯:

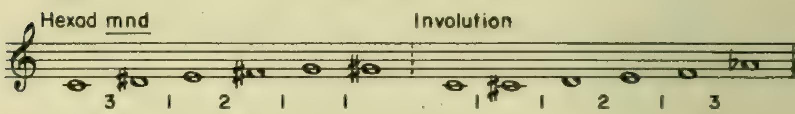
EXAMPLE 25-7



This hexad has a close affinity to the six-tone major-third scale C-D♯-E-G-G♯-B. The presence of the tritone and two major seconds destroys the homogeneity of the major-third hexad but produces a greater variety of material.

Since the projection of the triad *mnd* has the order 31211, the involution of the projection will have the same order in reverse, 11213, or $C_1C\sharp_1D_2E_1F_3A\flat$:

EXAMPLE 25-8

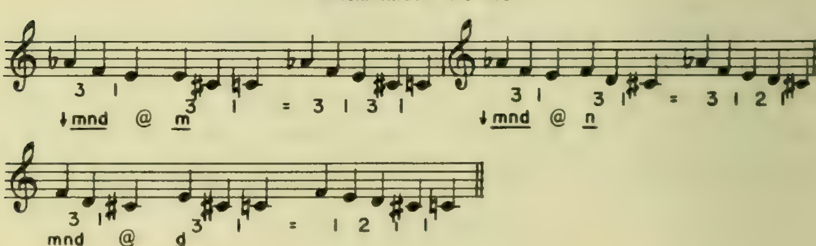


If we begin with the tone A♭ and project the triad *mnd downward,* we obtain the same results:

EXAMPLE 25-9

This scale has six pentads, the first three of which are formed by combinations of the involution of the triad *mnd* at the intervals of the major third, the minor third, and the minor second:

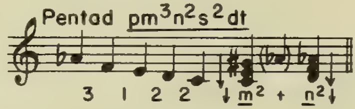
EXAMPLE 25-10



The others are the pentad $A\flat_3F_1E_2D_2C$, $pm^3n^2s^2dt$, which may be analyzed as the simultaneous projection of two major thirds

and two minor thirds *below* A♭ (or G♯);

EXAMPLE 25-11

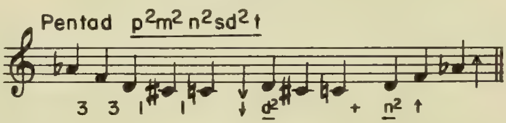


the pentad A♭$_4$E$_2$D$_1$C♯$_1$C♮, $pm^3ns^2d^2t$, which may be analyzed as the simultaneous projection of two minor seconds and two major thirds *above* C;

EXAMPLE 25-12

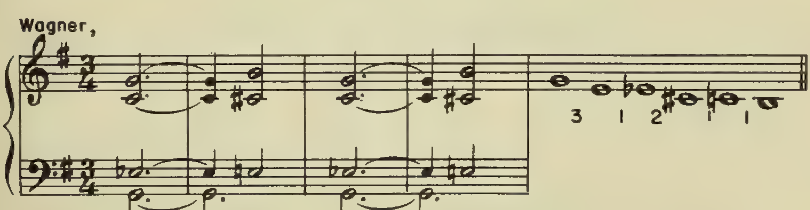
and the pentad A♭$_3$F$_3$D$_1$C♯$_1$C, $p^2m^2n^2sd^2t$, which may be analyzed as being composed of two minor seconds *below* and two minor thirds *above* D:

EXAMPLE 25-13



A nineteenth-century example of the involution of this scale may be found in the following phrase from Wagner's *Ring des Nibelungen*:

EXAMPLE 25-14



Another simple but effective example of the involution of this projection from Debussy's *Pelléas et Mélisande* follows:

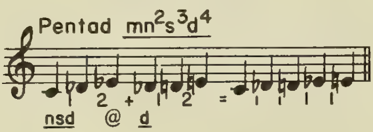
EXAMPLE 25-15

Projection of the Triad nsd

FINALLY, we come to the last of the triad projections, the projection of the triad *nsd*. Beginning with the triad C-D♭-E♭, we form the three pentads:

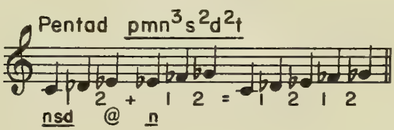
1. $C_1D\flat_2E\flat + D\flat_1D\natural_2E\natural = C_1D\flat_1D\natural_1E\flat_1E\natural$, $mn^2s^3d^4$, which is the basic pentad of the minor second series:

EXAMPLE 26-1



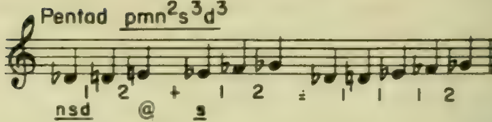
2. The pentad *nsd* @ *n*,
$C_1D\flat_2E\flat + E\flat_1F\flat_2G\flat = C_1D\flat_2E\flat_1F\flat_2G\flat$, $pmn^3s^2d^2t$:

EXAMPLE 26-2



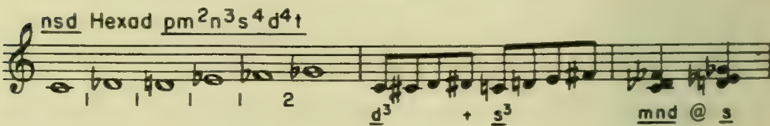
3. The pentad *nsd* @ *s*,
$D\flat_1D\natural_2E + E\flat_1F\flat_2G\flat = D\flat_1D\natural_1E\flat_1F\flat_2G\flat$, $pmn^2s^3d^3$:

EXAMPLE 26-3



The three together produce the scale $C_1D\flat_1D\natural_1E\flat_1F\flat_2G\flat$, $pm^2n^3s^4d^4t$, which may also be analyzed as the simultaneous projection of three minor seconds and three major seconds above C; or as two triads *mnd* at the interval of the major second:

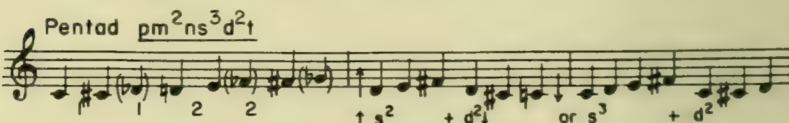
EXAMPLE 26-4



This scale contains three other pentads:

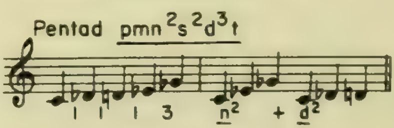
1. $C_1C\sharp_1D\natural_2E_2F\sharp$, $pm^2ns^3d^2t$, which may be analyzed as the projection of two major seconds *above* D and two minor seconds *below* D; or as the simultaneous projection of three major seconds and two minor seconds *above* C:

EXAMPLE 26-5



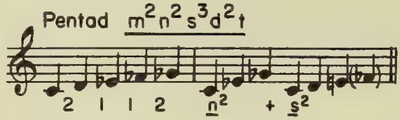
2. The pentad $C_1D\flat_1D\natural_1E\flat_3G\flat$, $pmn^2s^2d^3t$, which may be analyzed as the simultaneous projection of two minor thirds and two minor seconds *above* C:

EXAMPLE 26-6



3. The isometric pentad $\mathrm{C_2D_1E\flat_1F\flat_2G\flat}$, $m^2n^2s^3d^2t$, which may be analyzed as the simultaneous projection of two minor thirds and two major seconds *above* C:

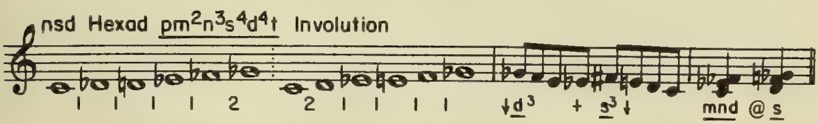
EXAMPLE 26-7



This hexad will be seen to have a strong affinity to the minor second six-tone scale. It does, however, have somewhat more variety with the addition of the tritone.

Since the projection of the triad *nsd* has the order 11112, the involution of the projection will have the same order in reverse: 21111, or $\mathrm{C_2D_1E\flat_1E\natural_1F_1G\flat}$. This hexad may be analyzed as the simultaneous projection of three minor seconds and three major seconds below G♭ (F♯), or as two triads *mnd* at the interval of the major second:

EXAMPLE 26-8

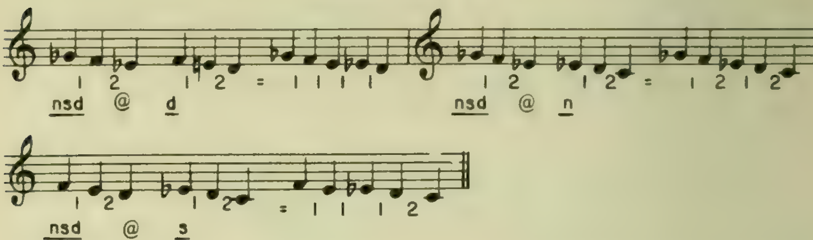


If we begin with the tone G♭ and project the triad *nsd downward,* we obtain the same result:

EXAMPLE 26-9

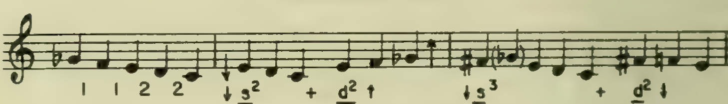
This scale has six pentads, three of which are formed by combinations of the involution of the triad *nsd* at the interval of the minor second, minor third, and major second:

EXAMPLE 26-10



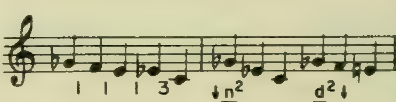
It contains also the pentad $G\flat_1F_1E_2D_2C$, $pm^2ns^3d^2t$, which may be analyzed as the projection of two major seconds *below* E and two minor seconds *above* E; or as the projection of three major seconds and two minor seconds below G♭ (F♯);

EXAMPLE 26-11



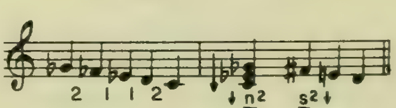
the pentad $G\flat_1F_1E_1E\flat_3C$, $pmn^2s^2d^3t$, which may be analyzed as the simultaneous projection of two minor thirds and two minor seconds below G♭;

EXAMPLE 26-12



and the isometric pentad $G\flat_2F\flat_1E\flat_1D_2C$, $m^2n^2s^3d^2t$, which may be analyzed as two minor thirds and two major seconds below G♭ (F♯):

EXAMPLE 26-13



All of these pentads are, again, involutions of the pentads discussed in the first part of this chapter.

The remaining triads add no further possibilities. The superposition of the triads p^2s, ms^2, and sd^2 form the perfect-fifth, major-second, and minor-second scales, already discussed.

The superposition of the augmented triad, m^3, upon its own tones duplicates itself:

EXAMPLE 26-14

The superposition of the diminished triad, n^2t, produces only one new tone:

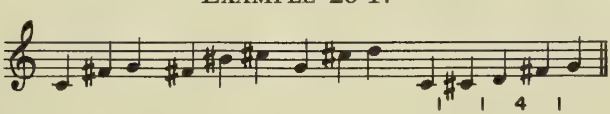
EXAMPLE 26-15

The projection of the triad *mst* merges with the five-tone major-second scale:

EXAMPLE 26-16

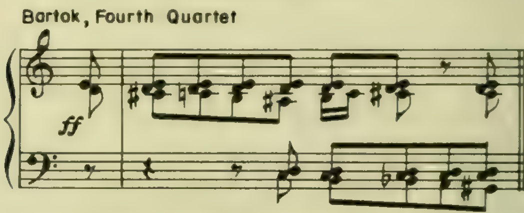
The projection of the triad *pdt* merges with the five-tone tritone–perfect-fifth projection:

EXAMPLE 26-17



An excellent example of the projection of the triad *nsd*, with its characteristic combination of four half-steps plus a whole step, is found in the first movement of the fourth Bartok string quartet where the first and second violins project the scale with a *stretto* imitation at the major ninth below in the viola and cello:

EXAMPLE 26-18



Review the material of the projections of the triads *pmn, pns, pmd, mnd,* and *nsd*. Choose the one which seems best suited to your taste and write a short sketch based exclusively on the six tones of the scale which you select.

Part III

SIX-TONE SCALES FORMED BY THE SIMULTANEOUS PROJECTION OF TWO INTERVALS

27

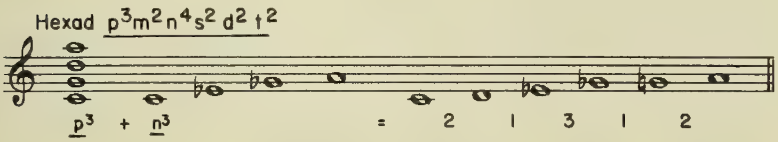
Simultaneous Projection of the Minor Third and Perfect Fifth

WE HAVE ALREADY SEEN that some of the six-tone scales formed by the projection of triads (see Example 23-4) may also be explained as the result of the simultaneous projection of two different intervals. We may now explore further this method of scale structure.

We shall begin with the consideration of the simultaneous projection of the minor third with each of the other basic intervals, since these combinations offer the greatest variety of possibilities. Let us consider first the combination of the minor third and perfect fifth.

If we project three perfect fifths above C, we form the tetrad C-G-D-A. Three minor thirds above C produce the tetrad C-E♭-G♭-A. Combining the two, we form the isometric hexad, $C_2D_1E\flat_3G\flat_1G\natural_2A$, $p^3m^2n^4s^2d^2t^2$:

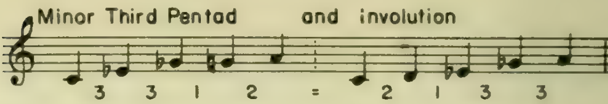
EXAMPLE 27-1



This scale, with its predominance of minor thirds and perfect fifths, is closely related to the minor-third hexad (see Example 11-3) except for the relatively greater importance of the perfect fifth.

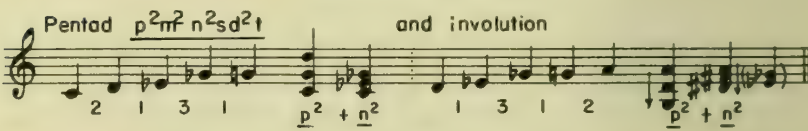
It contains three pentads, each with its own involution:

EXAMPLE 27-2



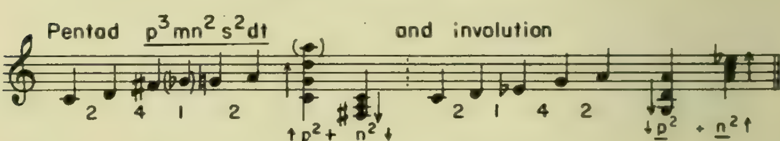
which are the characteristic pentads of the minor third scale; and

EXAMPLE 27-3



which we have already encountered as a part of the *pmn* projection (Chapter 22); and which is formed by the simultaneous projection of two perfect fifths and two minor thirds; and

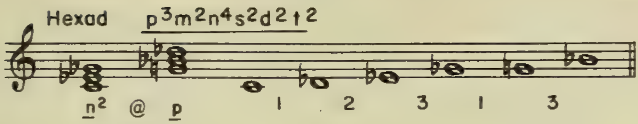
EXAMPLE 27-4



which we have met as a part of the *pns* projection (Chapter 23), and which is formed by the projection of two perfect fifths above and two minor thirds below C.

One interesting fact that should be pointed out here is that every *isometric* six-tone scale formed by the simultaneous projection of two intervals has an *isomeric* "twin" having the identical intervallic analysis. For example, if, instead of superimposing three perfect fifths and three minor thirds above C, we form the relationship of two minor thirds *at the interval* of the perfect fifth we derive the scale C-E♭-G♭ + G♮-B♭-D♭, or $C_1D\flat_2E\flat_3G\flat_1G\natural_3B\flat$, $p^3m^2n^4s^2d^2t^2$:

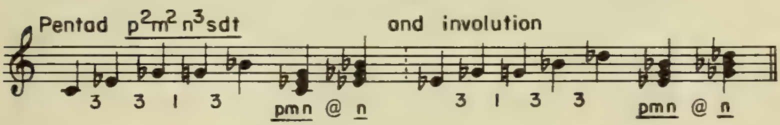
EXAMPLE 27-5



Analyzing this scale we find it to contain three perfect fifths, C to G, E♭ to B♭, and G♭ to D♭; two major thirds, E♭ to G, and G♭ to B♭; four minor thirds, C to E♭, E♭ to G♭, G♮ to B♭, and B♭ to D♭; two major seconds, D♭ to E♭, and B♭ to C; two minor seconds, C to D♭ and G♭ to G♮; and two tritones, C to G♭, and D♭ to G♮; $p^3m^2n^4s^2d^2t^2$, the same interval combinations that existed in the scale formed by simultaneous projection of three perfect fifths and three minor thirds. It will be observed that neither scale is the involution of the other.

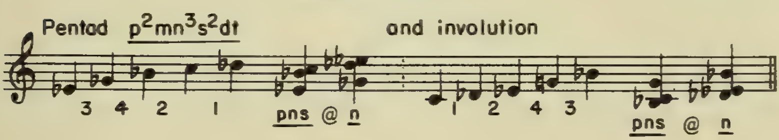
This scale also contains three pentads and their involutions:

EXAMPLE 27-6



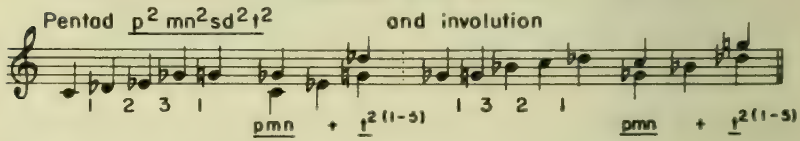
which were found in the projection of the triad *pmn* as the combination of two major or two minor triads at the interval of the minor third; and

EXAMPLE 27-7



which were found in the projection of the triad *pns* at the minor third; and

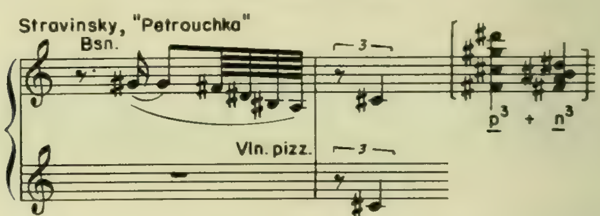
EXAMPLE 27-8



which was found in the *pmn* tritone projection (Chapter 19), as a major or minor triad with added tritones above the root and the fifth.

An example of the six-tone scale formed by the simultaneous projection of three perfect fifths and three minor thirds is found in the following excerpt from Stravinsky's *Petrouchka,* which can, of course, also be analyzed as a dominant ninth in C♯ minor followed by the tonic:

EXAMPLE 27-9

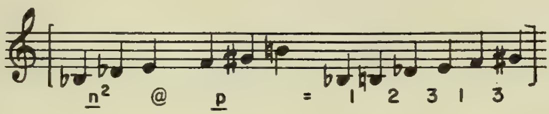


Its "twin" sonority, formed of two minor thirds at the interval of the perfect fifth, is illustrated by the excerpt from Gustav Holst's *Hymn of Jesus,* where the sonority is divided into two triads *pmn,* one major and one minor, at the interval of the tritone:

EXAMPLE 27-10

Holst, "Hymn of Jesus"

By permission of Galaxy Music Corporation, publishers.

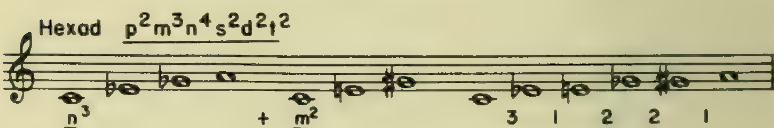


28

Simultaneous Projection of the Minor Third and Major Third

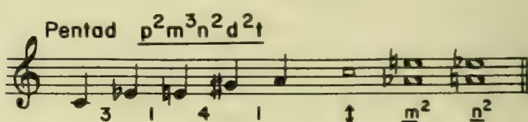
PROJECTING three minor thirds above C and two major thirds above C, we form the isometric six-tone scale C-E♭-G♭-A + C-E♮-G♯, or $C_3E\flat_1E\natural_2G\flat_2G\sharp_1A$, having the analysis $p^2m^3n^4s^2d^2t^2$. This scale bears a close relationship to the minor-third series but with a greater number of major thirds:

EXAMPLE 28-1



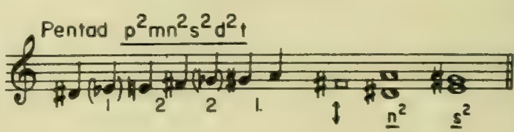
This scale contains two new isometric pentads:

EXAMPLE 28-2



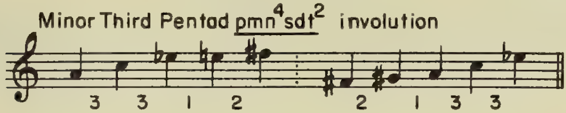
which is formed of a major third and a minor third *above and below* C, $\updownarrow m^2n^2$; and

EXAMPLE 28-3



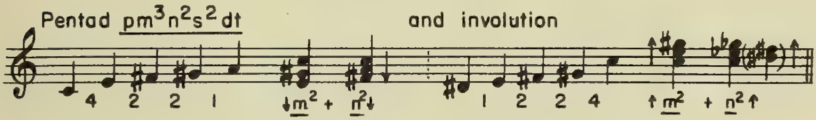
which is formed of a minor third and a major second above and below F♯; and two pentads with their involutions,

EXAMPLE 28-4



which are the basic pentads of the minor-third series: and

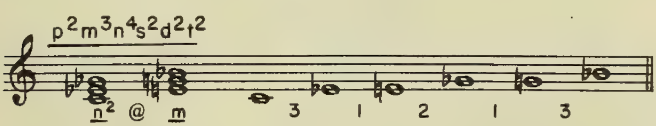
EXAMPLE 28-5



which is a part of the *pmn* and the *mnd* projection, and which may be analyzed as the simultaneous projection of two major thirds and two minor thirds.

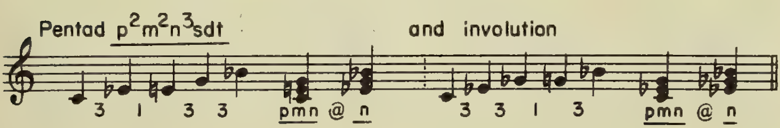
If we now project two minor thirds *at the interval* of the major third, we form the isomeric twin having the same intervallic analysis, $p^2m^3n^4s^2d^2t^2$:

EXAMPLE 28-6



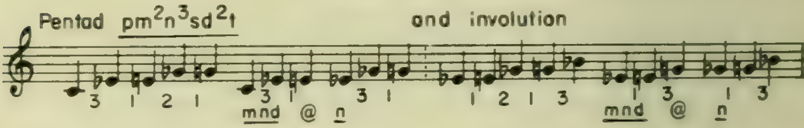
This scale contains three pentads, each with its involution:

EXAMPLE 28-7



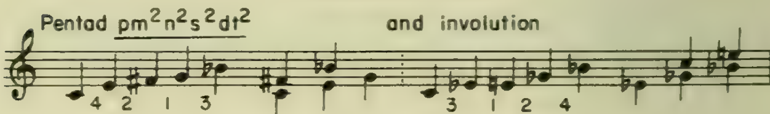
which has already appeared in the *pmn* projection as two triads, *pmn*, at the interval of the minor third; and

EXAMPLE 28-8



which has already appeared in the projection *mnd* as two triads *mnd* at the interval of the minor third, and

EXAMPLE 28-9



which has already been found in the tritone-*pmn* projection.

Two quotations from Debussy's *Pelléas et Mélisande* illustrate the use of the two hexads. The first uses the scale formed by the simultaneous projection of minor thirds and major thirds:

EXAMPLE 28-10

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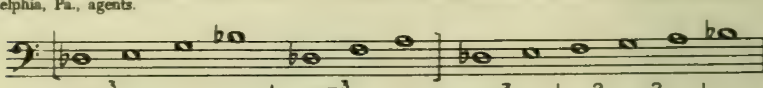
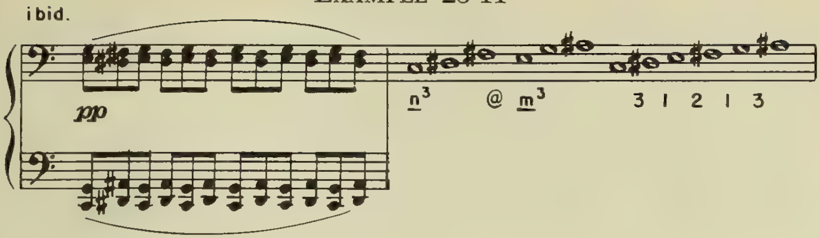


The second employs the hexad formed of two minor thirds at the interval of the major third:

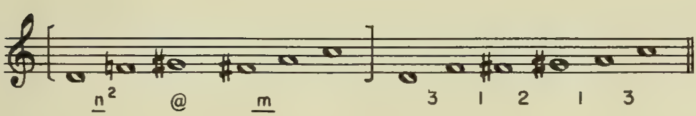
EXAMPLE 28-11



The following interesting example of the second hexad is found in the second of Schönberg's *Five Orchestral Pieces*:

EXAMPLE 28-12

By permission of C. F. Peters Corporation, music publishers.

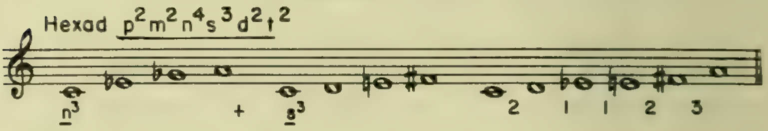


29

Simultaneous Projection of the Minor Third and Major Second

PROJECTING three minor thirds and three major seconds above C, we form the six-tone scale C-E♭-G♭-A + C-D-E♮-F♯, or $C_2D_1E♭_1E♮_2F♯_3A$, with the analysis $p^2m^2n^4s^3d^2t^2$:

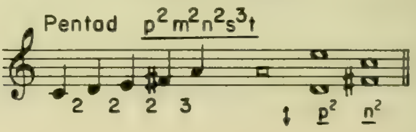
EXAMPLE 29-1



which will be seen to be similar to the minor-third series, but with a greater number of major seconds.

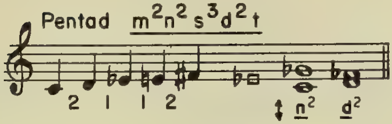
This scale contains two isometric pentads:

EXAMPLE 29-2



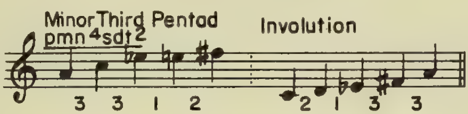
which has appeared in the projection *pns* (see Example 23-5), and may also be considered as the projection of a perfect fifth and a minor third above and below A; and

EXAMPLE 29-3



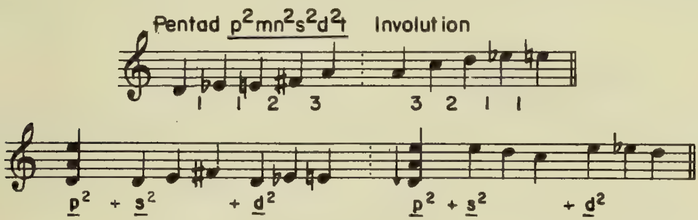
which has been found in the projection *nsd* and may also be considered as the projection of a minor third and minor second above and below E♭. There are also two pentads, each with its involution:

EXAMPLE 29-4



which are basic pentads of the minor third series; and

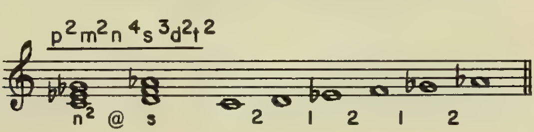
EXAMPLE 29-5



which appears here for the first time and may be analyzed as the simultaneous projection of two perfect fifths, two major seconds, and two minor seconds above D or below E.

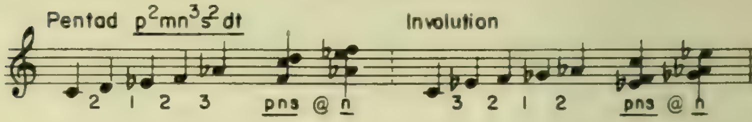
If we now project two minor thirds *at the interval* of the major second, we produce the isomeric twin C-E♭-G♭ + D-F-A♭, or $C_2D_1E\flat_2F_1G\flat_2A\flat$, with the same analysis, $p^2m^2n^4s^3d^2t^2$:

EXAMPLE 29-6



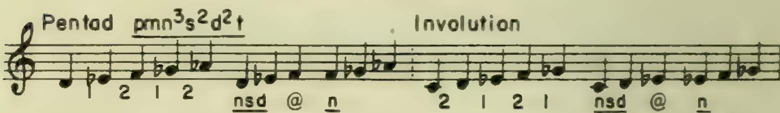
This scale contains three pentads, each with its involution:

EXAMPLE 29-7



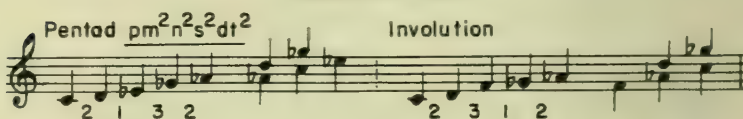
which has already appeared in the *pns* projection as two triads *pns* at the interval of the minor third; and

EXAMPLE 29-8



which has appeared in the projection *nsd* as a combination of two triads *nsd* at the interval of the minor third; and

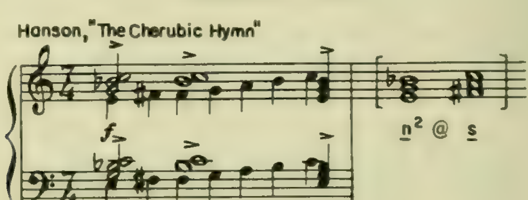
EXAMPLE 29-9



which has appeared in the *pmn*-tritone projection.

The climactic section of the author's *Cherubic Hymn* begins with the projection of two minor thirds at the interval of the major second and gradually expands to the eight-tone minor-third scale:

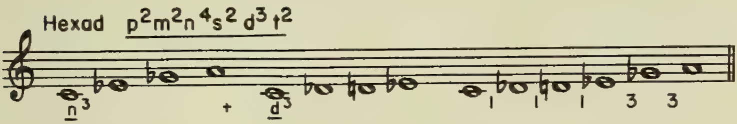
EXAMPLE 29-10



Simultaneous Projection of the Minor Third and Minor Second

PROJECTING three minor thirds and three minor seconds above C, we form the six-tone scale C-E♭-G♭-A + C-D♭-D♮-E♭, or $C_1D\flat_1D\natural_1E\flat_3G\flat_3A$, with the analysis $p^2m^2n^4s^2d^3t^2$:

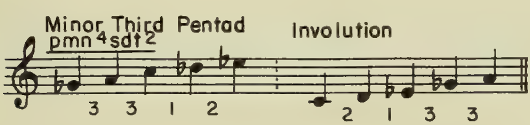
EXAMPLE 30-1



This scale is, again, similar to the minor-third series, but with greater emphasis on the minor second.

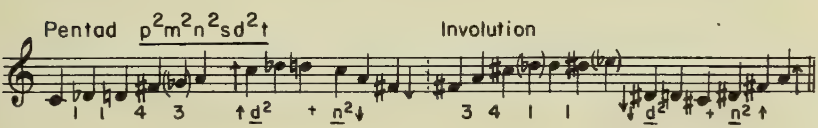
This scale contains three pentads, each with its involution:

EXAMPLE 30-2



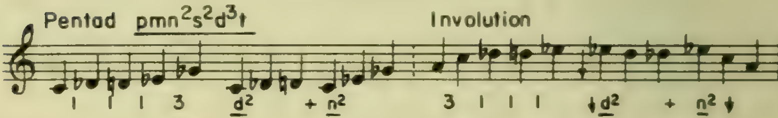
which is the basic pentad of the minor third series; and

EXAMPLE 30-3



which has occurred in the projection *mnd* and appears here as the projection of two minor seconds *above* and two minor thirds *below* C; or, in involution, as two minor seconds *below* and two minor thirds *above* D♯; and

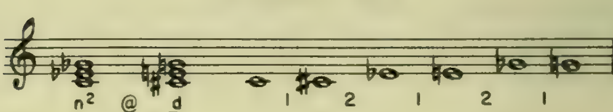
EXAMPLE 30-4



which has occurred in the projection *nsd*. This may be analyzed as the simultaneous projection of two minor seconds and two minor thirds above C or below E♭.

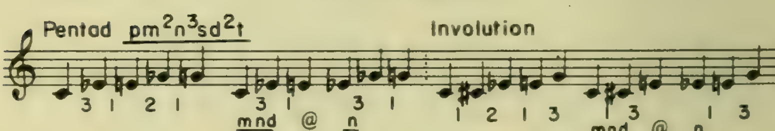
If we project two minor thirds *at the interval* of the minor second, we produce the isomeric twin C-E♭-G♭ + C♯-E♮-G♮, or $C_1C\sharp_2E\flat_1E\natural_2G\flat_1G\natural$, with the same analysis, $p^2m^2n^4s^2d^3t^2$:

EXAMPLE 30-5



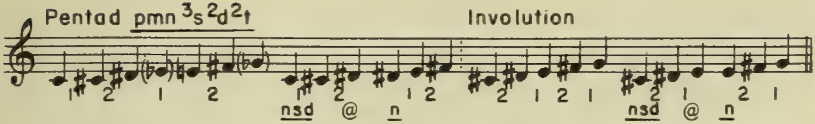
This scale contains three pentads, each with its involution:

EXAMPLE 30-6



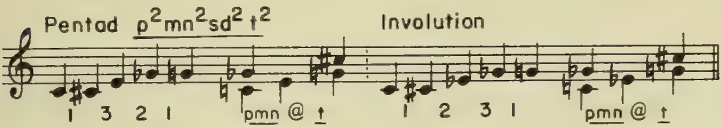
which has appeared in the projection *mnd* as a combination of two triads *mnd* at the interval of the minor third; and

Example 30-7



which has appeared in the projection *nsd* as a combination of two triads *nsd* at the interval of the minor third; and

Example 30-8

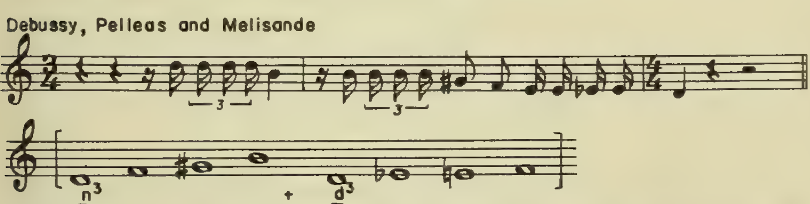


which has already occurred in the *pmn*-tritone projection.

A review of Chapters 27 to 30, which have presented the simultaneous projection of the minor third with the intervals of the perfect fifth, major third, major second, and minor second respectively, will show that all of the hexads so formed fall naturally into the minor-third series, since all of them contain a preponderance of minor thirds with their concomitant tritones.

The short recitative from Debussy's *Pelléas et Mélisande* adequately illustrates the hexad formed by the simultaneous projection of minor thirds and minor seconds:

Example 30-9



The quotation from Stravinsky's *Petrouchka* is an excellent example of the projection of two minor thirds of the interval of the minor second:

EXAMPLE 30-10

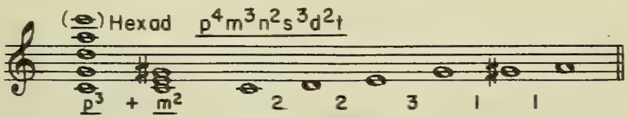
Review the projections of Chapters 27 to 30, inclusive. Select the hexad which most appeals to you and write a short sketch based *exclusively* on the material of the scale which you select.

31

Simultaneous Projection of the Perfect Fifth and Major Third

IF WE PROJECT three perfect fifths above C, C-G-D-A, and two major thirds above C, C-E-G♯, we produce the six-tone isometric scale $C_2D_2E_3G_1G\sharp_1A$, $p^4m^3n^2s^3d^2t$:

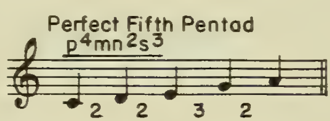
EXAMPLE 31-1



It bears a close relationship to the perfect-fifth series because it is the perfect-fifth pentad above C with the addition of the chromatic tone G♯.

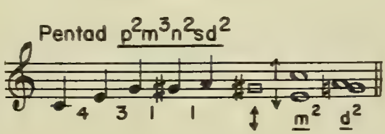
It contains two isometric pentads:

EXAMPLE 31-2



already described as the basic perfect-fifth pentad; and

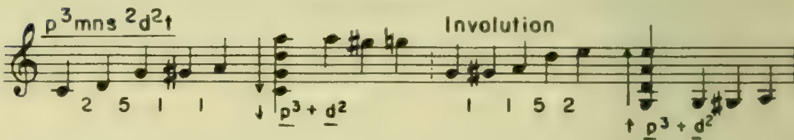
EXAMPLE 31-3



which is a new isometric pentad, and which may be analyzed as the formation of a major third and a minor second above and below G♯, $\updownarrow m^2d^2$.

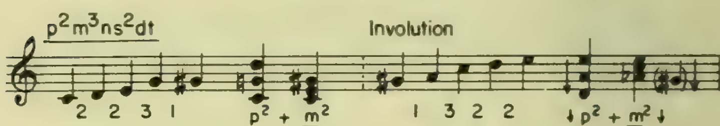
It also contains two pentads, each with its involution:

EXAMPLE 31-4



which may be analyzed as the simultaneous projection of three perfect fifths and two minor seconds, and which has not before been encountered; and

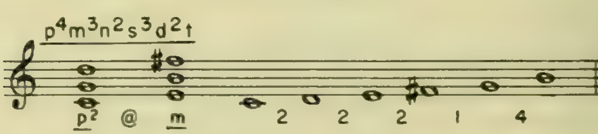
EXAMPLE 31-5



which we have met before as a part of the projection of both the triads *pmn* (Chapter 22) and *pmd* (Chapter 24) and is formed by the simultaneous projection of two perfect fifths and two major thirds.

If we now project two perfect fifths *at the interval* of the major third, we form another isomeric twin having the same intervallic analysis as the previous scale, but not constituting an involution of the first scale. The scale thus formed is C-G-D + E-B-F♯, or $C_2D_2E_2F\sharp_1G_4B$, which also has the intervallic formation $p^4m^3n^2s^3d^2t$:

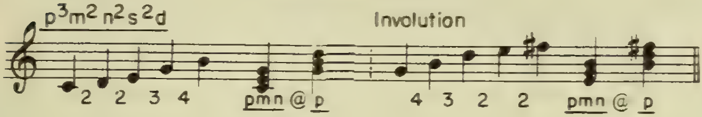
EXAMPLE 31-6



This scale will be seen also to have a close resemblance to the perfect-fifth series, for it consists of the tones of the seven-tone perfect-fifth scale with the tone A omitted.

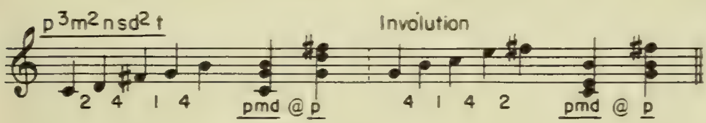
It contains three pentads, each with its involution:

EXAMPLE 31-7



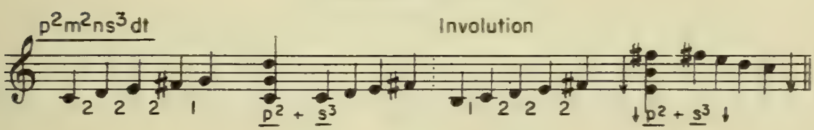
which has already occurred in the *pmn* projection as the relationship of two triads *pmn* at the interval of the perfect fifth; and

EXAMPLE 31-8



which has already occurred as the projection of two triads *pmd* at the interval of the perfect fifth; and

EXAMPLE 31-9



which we have met in the projection of the triad *pns* as the simultaneous projection of two perfect fifths and three major seconds.

A striking example of the projection of two perfect fifths at the interval of a major third is found in the opening of the Stravinsky *Symphony in C*:

EXAMPLE 31-10

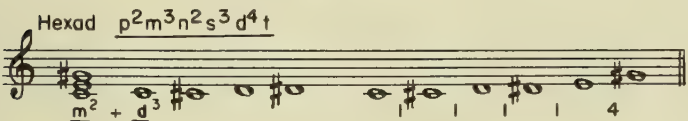
An excellent example of the simultaneous projection of two perfect fifths and two major thirds, giving the pentatonic scale C D E G A♭, may be found in Copland's *A Lincoln Portrait*:

EXAMPLE 31-11

Simultaneous Projection of the Major Third and Minor Second

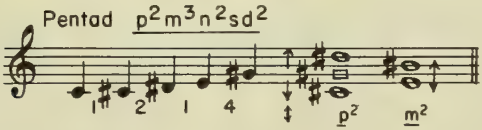
PROJECTING major thirds and minor seconds simultaneously, we form the six-tone scale C-E-G♯ + C-C♯-D-D♯, or $C_1C♯_1D_1D♯_1E_4G♯$, with the analysis $p^2m^3n^2s^3d^4t$. This scale is very similar to the six-tone minor-second series with the exception of the addition of the tritone and greater emphasis on the major third:

EXAMPLE 32-1



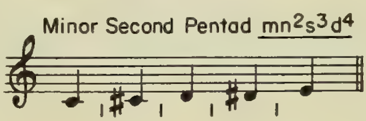
This scale contains two isometric pentads:

EXAMPLE 32-2



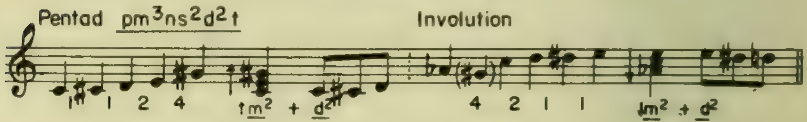
which is formed of a perfect fifth and a major third above and below G♯; and

EXAMPLE 32-3



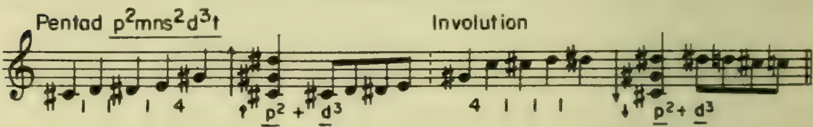
which is the basic minor-second pentad. There are two additional pentads, each with its involution:

EXAMPLE 32-4



which has been found as a part of the projection *pmd* and *mnd*, and is analyzed as the simultaneous projection of two major thirds and two minor seconds; and

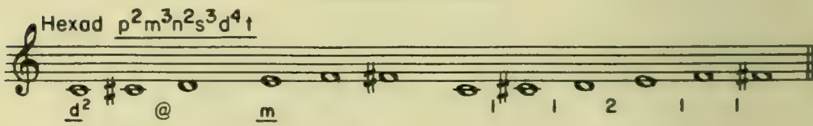
EXAMPLE 32-5



which consists of the simultaneous projection of two perfect fifths and three minor seconds, and which appears here for the first time.

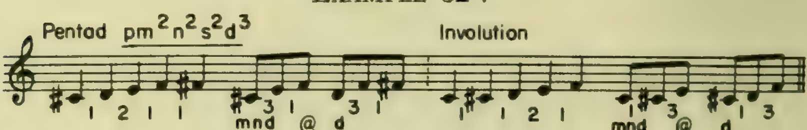
If we project two minor seconds *at the interval* of the major third, we form the isomeric twin C-C♯-D + E-F-F♯, or $C_1C\sharp_1D_2E_1F_1F\sharp$, having the same analysis, $p^2m^3n^2s^3d^4t$:

EXAMPLE 32-6



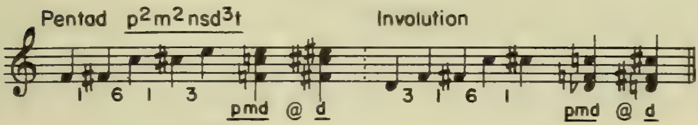
This scale contains three pentads, each with its involution:

EXAMPLE 32-7



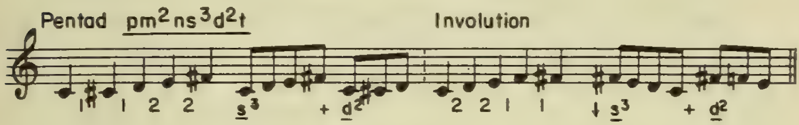
which is a part of the projection *mnd,* being formed of two triads *mnd* at the interval of the minor second; and

EXAMPLE 32-8



which is a part of the projection *pmd,* being formed of two triads *pmd* at the interval of the minor second; and

EXAMPLE 32-9



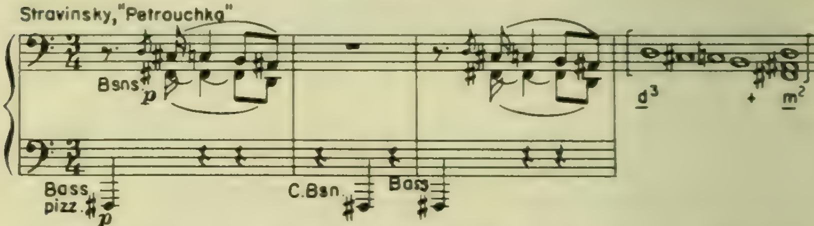
which is a part of the *nsd* projection and may be considered as the simultaneous projection of three major seconds and two minor seconds.

Copland's *A Lincoln Portrait* contains the following example of the projection of two minor seconds and two major thirds, producing the pentad ↓A♭-G-F♯-E-C:

EXAMPLE 32-10

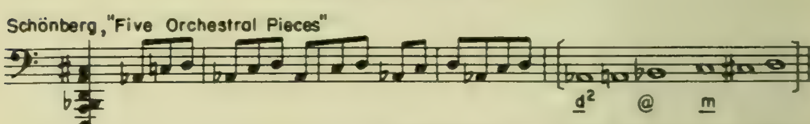
An example of the hexad formed by the simultaneous projection of three minor seconds and major thirds will be found at the beginning of *Le Tour de Passe-Passe* from Stravinsky's *Petrouchka*:

EXAMPLE 32-11



An unusual example of the projection of two minor seconds at the interval of the major third is found in the cadence at the end of the first of the *Five Orchestral Pieces* of Schönberg:

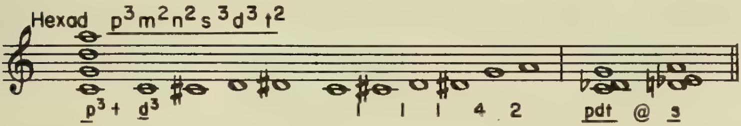
EXAMPLE 32-12



Simultaneous Projection of the Perfect Fifth and Minor Second

THE SIMULTANEOUS PROJECTION of three perfect fifths and three minor seconds produces the scale C-D-G-A + C-C♯-D-D♯, or $C_1C\sharp_1D_1D\sharp_4G_2A$, $p^3m^2n^2s^3d^3t^2$, which may also be analyzed as the triad *pdt* at the interval of the major second:

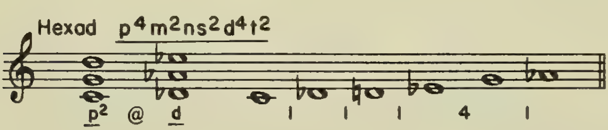
EXAMPLE 33-1



This does *not* form an isometric six-tone scale but a more complex pattern, a scale which *has its own involution and also has its isomeric "twin"* which in turn has its own involution. This type of formation will be discussed in detail in Chapter 39.

If we project two perfect fifths at the interval of the minor second, we form the six-tone scale C-G-D + D♭-A♭-E♭, or $C_1D\flat_1D\natural_1E\flat_4G_1A\flat$, with the analysis $p^4m^2ns^2d^4t^2$:

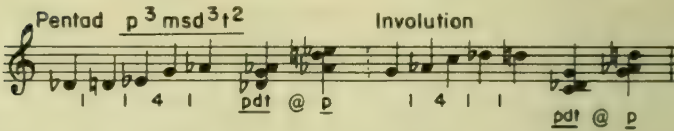
EXAMPLE 33-2



This scale is most closely related to the projection of the tritone discussed in Chapter 17.

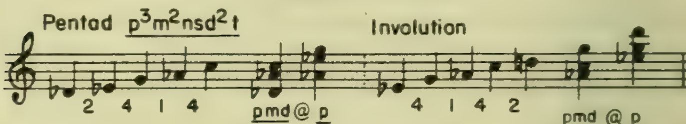
It contains three pentads, each with its involution:

EXAMPLE 33-3



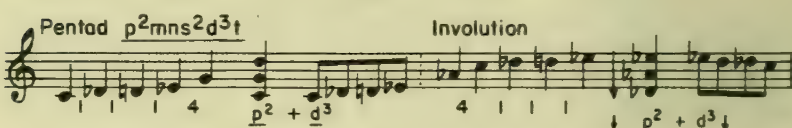
which is a part of the tritone--perfect-fifth projection and may be analyzed as the triad *pdt* at the interval of the perfect fifth; and

EXAMPLE 33-4



which has appeared previously as the triad *pmd* at the interval of the perfect fifth; and

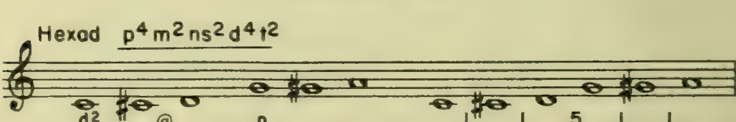
EXAMPLE 33-5



which may be analyzed as the simultaneous projection of two perfect fifths and three minor seconds.

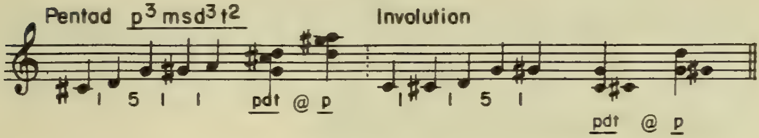
If we now reverse the projection and form two minor seconds at the interval of the perfect fifth, we form the scale C-C♯-D + G-G♯-A, or $C_1C\sharp_1D_5G_1G\sharp_1A$, having the same analysis, $p^4m^2ns^2d^4t^2$:

EXAMPLE 33-6



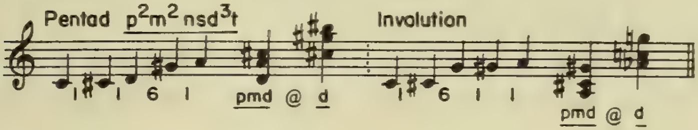
This scale contains three pentads, each with its involution:

EXAMPLE 33-7



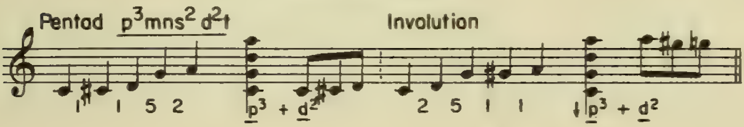
which is a part of the tritone–perfect-fifth projection, being a combination of two triads *pdt* at the interval of the perfect fifth; and

EXAMPLE 33-8



which has occurred in the projection *pmd* as the combination of two triads *pmd* at the interval of the major seventh; and

EXAMPLE 33-9



which may be analyzed as the simultaneous projection of three perfect fifths and two minor seconds.

The first of the hexads discussed in this chapter has a predominance of tritones, while the second and third have an equal strength of tritones, perfect fifths, and minor seconds. This means that all three scales have a close resemblance to the tritone–perfect-fifth projection. The following measure from the Stravinsky *Concertino* illustrates the simultaneous projection of three minor seconds and three perfect fifths. It will be seen to be a variant of the illustration of the tritone projection of Example 18-9.

EXAMPLE 33-10

This concludes the discussion of the simultaneous projection of two intervals, since the only pair remaining is the combination of the major second and the major third, the projection of which forms the major-second pentad.

Review the hexads of Chapters 31 to 33, inclusive. Select one and write a short sketch confined entirely to the material of the scale you select.

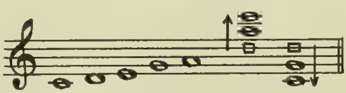
Part IV

PROJECTION BY INVOLUTION AND AT FOREIGN INTERVALS

Projection by Involution

If we examine again the perfect-fifth pentad C-D-E-G-A, formed of the four superimposed fifths, C-G-D-A-E, we shall observe that this combination may be formed with equal logic by beginning with the tone D and projecting two perfect fifths *above and below* the starting tone:

Example 34-1



All such sonorities will obviously be isometric.

Using this principle, we can form a number of characteristic pentads by superimposing two intervals above the first tone and also projecting the same two intervals below the starting tone. Referring again to the twelve-tone circle of fifths, we note that we have six tones clockwise from C: G-D-A-E-B-F♯, and six tones counterclockwise from C: F-B♭-E♭-A♭-D♭-G♭, the G♭ duplicating the F♯. The following visual arrangement may be of aid:

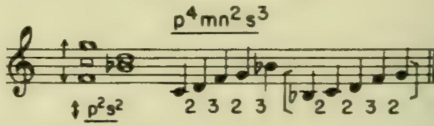
	1	2	3	4	5	
	G	D	A	E	B	
C						F♯ (G♭)
	F	B♭	E♭	A♭	D♭	

G and F form the perfect fifth above and below C; D and B♭

form the major second above and below C; A and E♭ form the major sixth above and below C; E and A♭ form the major third above and below C; and B and D♭ form the major seventh above and below C.

Taking the combination of 1 and 2, $\updownarrow p^2s^2$, we duplicate the perfect-fifth pentad:

EXAMPLE 34-2

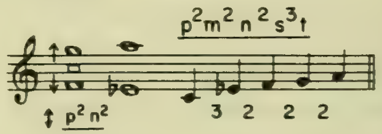


The combination of 1 and 3 forms the pentad $\uparrow s^2n^2\downarrow$ (Example 23-5):

G A
C , $\updownarrow p^2n^2$,
F E♭

or, arranged melodically $C_3E\flat_2F_2G_2A$, $p^2m^2n^2s^3t$:

EXAMPLE 34-3

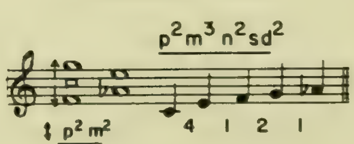


The combination of 1 and 4 forms the pentad

G E
C , $\updownarrow p^2m^2$, or
F A♭

$C_4E_1F_2G_1A\flat$, $p^2m^3n^2sd^2$:

EXAMPLE 34-4

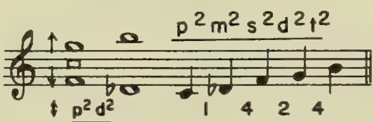


The combination of 1 and 5 forms the pentad

$$C\begin{matrix} G\ B \\ \\ F\ D\flat \end{matrix}, \updownarrow p^2d^2,$$

or $C_1D\flat_4F_2G_4B$, $p^2m^2s^2d^2t^2$:

EXAMPLE 34-5

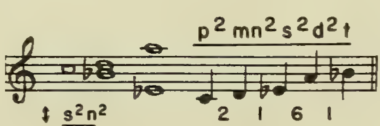


The combination of 2 and 3 forms the pentad

$$C\begin{matrix} D\ \ A \\ \\ B\flat\ E\flat \end{matrix}, \updownarrow s^2n^2,$$

or $C_2D_1E\flat_6A_1B\flat$, $p^2mn^2s^2d^2t$:

EXAMPLE 34-6

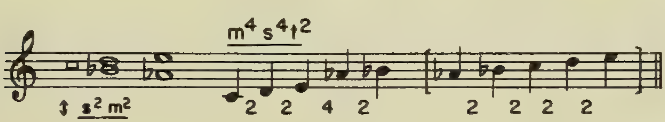


The combination of 2 and 4 duplicates the major-second pentad

$$C\begin{matrix} D\ \ E \\ \\ B\flat\ A\flat \end{matrix}, \updownarrow s^2m^2,$$

or $C_2D_2E_4A\flat_2B\flat$, $m^4s^4t^2$:

EXAMPLE 34-7

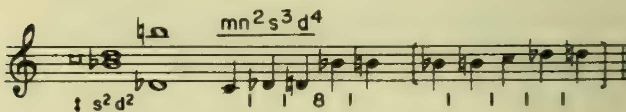


The combination of 2 and 5 duplicates the minor-second pentad

$$\begin{array}{ccc} & D \quad B\natural & \\ C & & ,\ \updownarrow s^2d^2, \\ & B\flat \quad D\flat & \end{array}$$

or $C_1D\flat_1D\natural_8B\flat_1B\natural$, $mn^2s^3d^4$:

Example 34-8

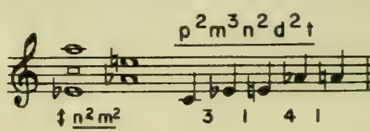


The combination of 3 and 4 forms the pentad

$$\begin{array}{ccc} & A \quad E & \\ C & & ,\ \updownarrow n^2m^2, \\ & E\flat \quad A\flat & \end{array}$$

or $C_3E\flat_1E\natural_4A\flat_1A\natural$, $p^2m^3n^2d^2t$:

Example 34-9

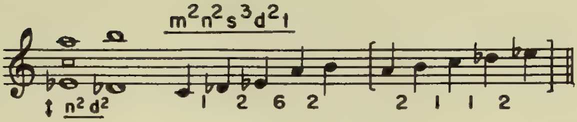


The combination of 3 and 5 forms the pentad

$$\begin{array}{ccc} & A \quad B & \\ C & & ,\ \updownarrow n^2d^2, \\ & E\flat \quad D\flat & \end{array}$$

or $C_1D\flat_2E\flat_6A_2B$, $m^2n^2s^3d^2t$, which has also been analyzed in Example 26-7 as the projection of two major seconds and two minor thirds, A-B-C♯ + A-C♮-E♭:

EXAMPLE 34-10



And finally, the combination of 4 and 5 forms the pentad

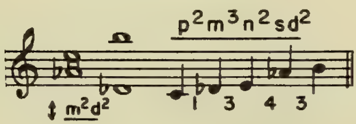
E B

C , ↕m^2d^2,

A♭ D♭

or $C_1D\flat_3E_4A\flat_3B$, $p^2m^3n^2sd^2$:

EXAMPLE 34-11

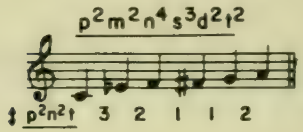


The only way in which an isometric six-tone scale can be formed from the above pentads is by the addition of the tritone F♯ (or G♭). For example, if we take the first of these pentads and add the tritone above and below C, we produce the six-tone scale $C_2D_3F_1F\sharp(G\flat)_1G\natural_3B\flat$, $p^4m^3n^2s^3d^2t$:

EXAMPLE 34-12

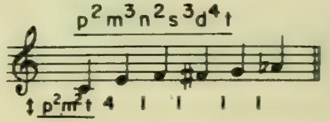
The remaining pentads with the tritone added become $C_3E\flat_2F_1F\sharp_1G_2A$, $p^2m^2n^4s^3d^2t^2$:

EXAMPLE 34-13



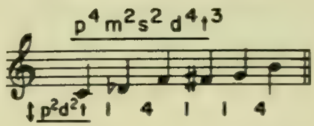
$C_4E_1F_1F\sharp_1G_1A\flat$, $p^2m^3n^2s^3d^4t$:

EXAMPLE 34-14



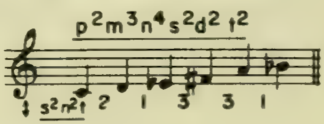
$C_1D\flat_4F_1F\sharp_1G_4B$, $p^4m^2s^2d^4t^3$:

EXAMPLE 34-15



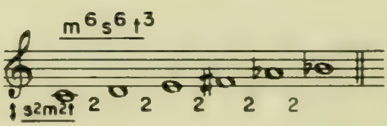
$C_2D_1E\flat_3F\sharp_3A_1B\flat$, $p^2m^3n^4s^2d^2t^2$:

EXAMPLE 34-16



$C_2D_2E_2F\sharp_2A\flat_2B\flat$, $m^6s^6t^3$:

EXAMPLE 34-17



$C_1D\flat_1D\natural_4F\sharp_4B\flat_1B\natural$, $p^2m^3n^2s^3d^4t$; $\updownarrow$ s^2d^2t (duplicating 34-14)
$C_3E\flat_1E_2F\sharp_2A\flat_1A\natural$, $p^2m^3n^4s^2d^2t^2$; $\updownarrow$ n^2m^2t (duplicating 34-16)
$C_1D\flat_2E\flat_3F\sharp_3A_2B$, $p^2m^2n^4s^3d^2t^2$; $\updownarrow$ n^2d^2t (duplicating 34-13)
$C_1D\flat_3E_2F\sharp_2A\flat_3B$, $p^4m^3n^2s^3d^2t$; m^2d^2t (duplicating 34-12)

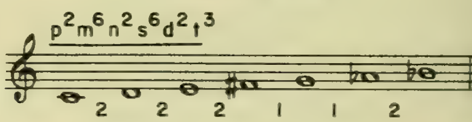
Since all of the six-tone scales produced by the addition of the tritone have already been discussed in previous chapters, we need not analyze them further.

35

Major-Second Hexads with Foreign Tone

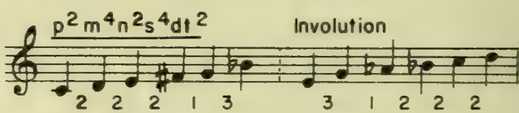
EXAMINING the seven-tone major-second scale C-D-E-F♯-G-A♭-B♭, we find that it contains the whole-tone scale C-D-E-F♯-G♯-A♯: and three other six-tone scales, each with its involution:

EXAMPLE 35-1



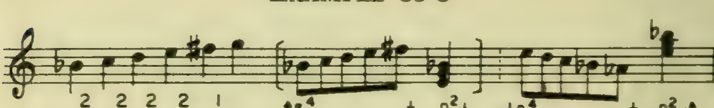
1. $C_2D_2E_2F\sharp_1G_3B\flat$ with the involution $E_3G_1A\flat_2B\flat_2C_2D$, $p^2m^4n^2s^4dt^2$:

EXAMPLE 35-2



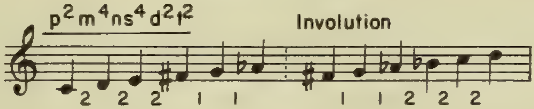
which may also be considered to be formed of four major seconds *above*, and two minor thirds *below* B♭ or, in involution, four major seconds below and two minor thirds above E;

EXAMPLE 35-3



2. $C_2D_2E_2F\sharp_1G_1A\flat$ with the involution $F\sharp_1G_1A\flat_2B\flat_2C_2D$, $p^2m^4ns^4d^2t^2$:

EXAMPLE 35-4

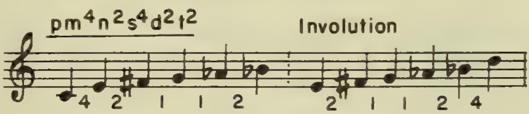


which may also be considered as the projection of four major seconds and two perfect fifths above C, or below D;

EXAMPLE 35-5

3. $C_4E_2F\sharp_1G_1A\flat_2B\flat$ with the involution $E_2F\sharp_1G_1A\flat_2B\flat_4D$, $pm^4n^2s^4d^2t^2$:

EXAMPLE 35-6



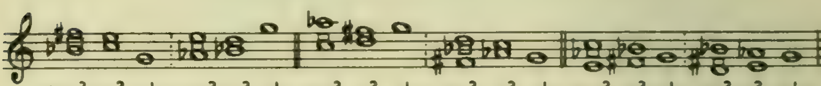
which may also be considered as the projection of four major seconds and two minor thirds above E, or below B♭:

EXAMPLE 35-7

The theory of involution provides an even simpler analysis. Example 35-2 becomes the projection of two major thirds and two major seconds *above and below* D, and *one* perfect fifth *below* D; and the involution becomes two major thirds and two major

seconds *above and below* C, and one perfect fifth *above* C—that is $\updownarrow m^2s^2p\downarrow$ or $\updownarrow m^2s^2p\uparrow$. Similarly, Example 35-4 becomes $\updownarrow m^2s^2n\uparrow$ or $\updownarrow m^2s^2n\downarrow$. Example 35-6 becomes $\updownarrow m^2s^2d\downarrow$ or $\updownarrow m^2s^2d\downarrow$:

EXAMPLE 35-8

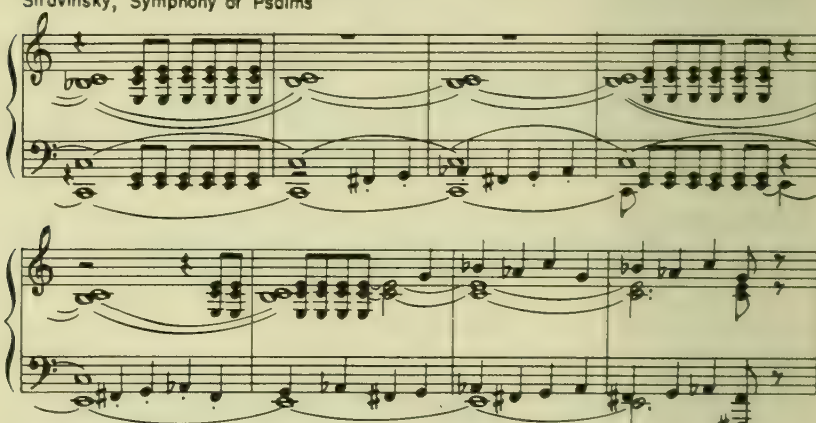


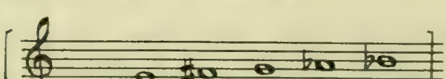
All of these impure major-second scales will be seen to have the characteristic predominance of the major second, major third, and tritone.

A striking use of the impure major-second scale of Example 35-6, where one might not expect to find it, will be seen in the following excerpt from Stravinsky's *Symphony of Psalms*:

EXAMPLE 35-9

Stravinsky, "Symphony of Psalms"





An earlier use of the scale illustrated in Example 35-2 will be

found in the excerpt from Scriabine's *Prometheus*:

EXAMPLE 35-10

A more familiar example is found at the beginning of the same composer's *Le Poeme de l'Extase*:

EXAMPLE 35-11

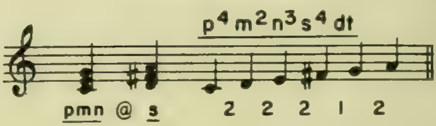
Write a short sketch using the material of the hexads of Examples 35-2, 35-4, or 35-6.

36

Projection of Triads at Foreign Intervals

In Part II we discussed the projection of triads upon the intervals which were a part of their own composition, for example, *pmn* @ *p*, *pmn* @ *m*, *pmn* @ *n*, each of which forms a pentad, and the three together forming the six-tone *pmn* projection. It is obvious that we may form a six-tone scale directly from a triad by projecting it at a *foreign* interval, that is, at an interval which is *not* in the original triad. For example, *pmn* at the interval of the major second produces the six-tone scale which we have already discussed in Chapter 23, C-E-G + D-F♯-A = $C_2D_2E_2F\sharp_1G_2A$, which has been analyzed both as the projection of the triad *pns* and as the simultaneous projection of three perfect fifths and three major seconds:

Example 36-1



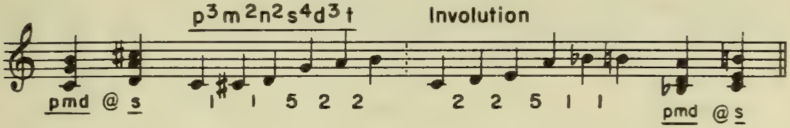
We have noticed, also, that the six-tone scale formed by the projection of the triad *nsd* may be analyzed as the relationship of two triads *mnd* at the major second (see Example 26-4).

Certain of these projections, however, form new hexads which have not so far appeared.

The triad *pmd* at the interval of the major second produces the scale C-G-B + D-A-C♯, or $C_1C\sharp_1D_5G_2A_2B$, $p^3m^2n^2s^4d^3t$, with

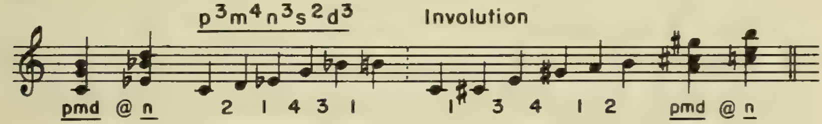
its involution $C_2D_2E_5A_1B\flat_1B\natural$:

EXAMPLE 36-2



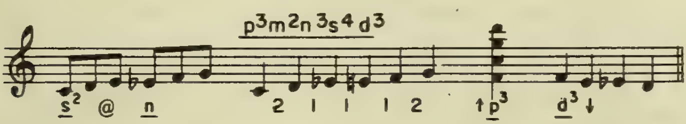
The same triad *pmd* at the interval of the minor third forms the scale C-G-B + E$\flat$-B$\flat$-D, or $C_2D_1E\flat_4G_3B\flat_1B\natural$, with its involution $C_1C\sharp_3E_4G\sharp_1A_2B$, $p^3m^4n^3s^2d^3$:

EXAMPLE 36-3



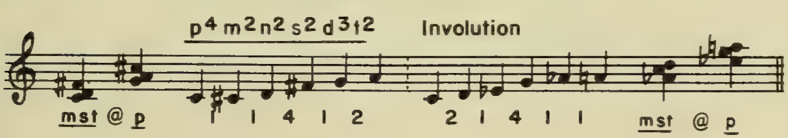
The triad ms^2 at the interval of the minor third forms the new isometric six-tone scale, C-D-E + E$\flat$-F-G, or $C_2D_1E\flat_1E\natural_1F_2G$, $p^3m^2n^3s^4d^3$, which predominates in major seconds, but which also may be analyzed as a projection of three perfect fifths above, and three minor seconds below F (F-C-G-D + F-E-E$\flat$-D):

EXAMPLE 36-4



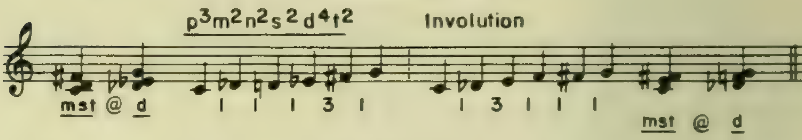
The triad *mst* at the interval of the perfect fifth forms the scale $C_2D_4F\sharp$ + $G_2A_4C\sharp$, or $C_1C\sharp_1D_4F\sharp_1G_2A$ with its involution $C_2D_1E\flat_4G_1A\flat_1A\natural$, $p^4m^2n^2s^2d^3t^2$, which is most closely related to the tritone–perfect-fifth series:

EXAMPLE 36-5



The same triad, *mst*, at the interval of the minor second forms the scale $C_2D_4F\sharp + D\flat_2E\flat_4G = C_1D\flat_1D\natural_1E\flat_3F\sharp_1G$, with its involution $C_1D\flat_3E_1F_1F\sharp_1G$, $p^3m^2n^2s^2d^4t^2$, which also resembles the tritone–perfect-fifth projection:

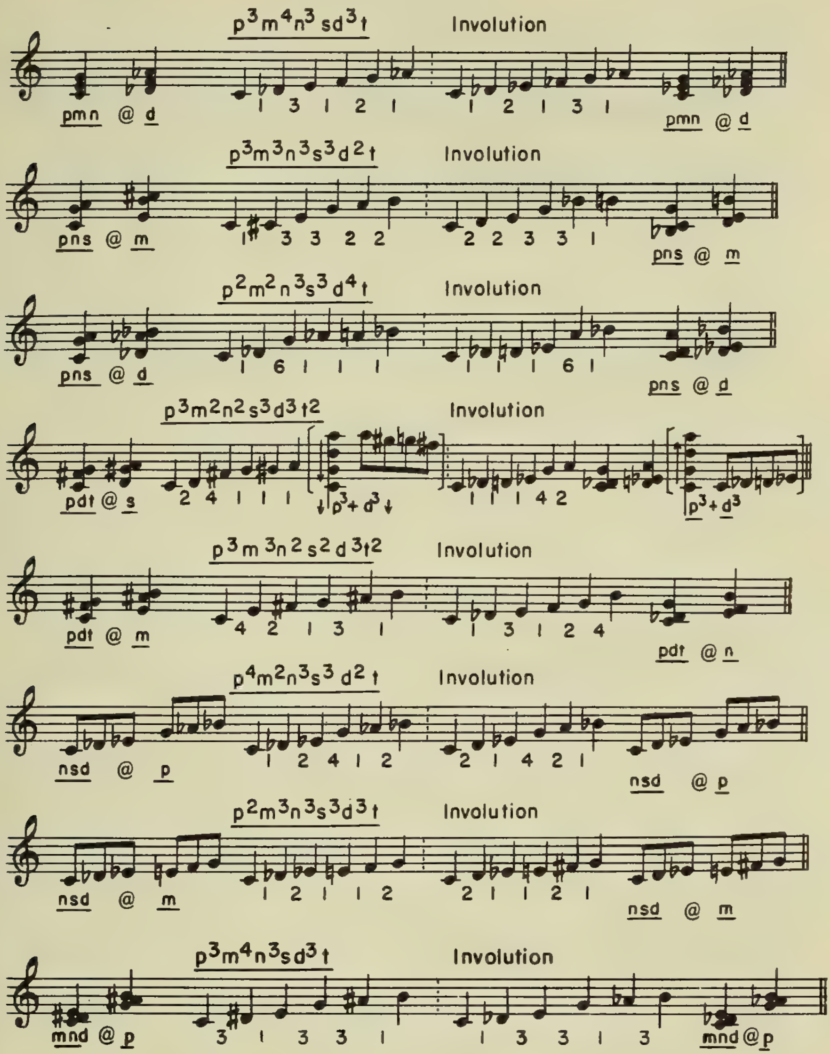
EXAMPLE 36-6



There are, finally, eight projections of triads at foreign tones, in which the scales and their involutions follow a pattern somewhat similar to the projections discussed in Chapters 27 to 33. They should, for the sake of completeness be mentioned here, but will be discussed in detail in a later chapter. They are:

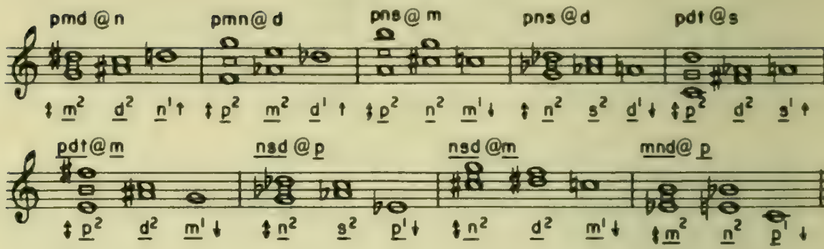
The projection of the triad *pmn* at the interval of the minor second, which forms the scale $C_1D\flat_3E_1F_2G_1A\flat$, with its involution $C_1D\flat_2E\flat_1F\flat_3G_1A\flat$, $p^3m^4n^3sd^3t$; the triad *pns* at the major third, $C_1C\sharp_3E_3G_2A_2B$, with its involution $C_2D_2E_3G_3B\flat_1B\natural$, $p^3m^3n^3s^3d^2t$; the triad *pns* at the minor second, $C_1D\flat_6G_1A\flat_1A\natural_1$ $B\flat$, with its involution $C_1D\flat_1D\natural_1E\flat_6A_1B\flat$, $p^2m^2n^3s^3d^4t$; the triad *pdt* at the major second, $C_2D_4F\sharp_1G_1G\sharp_1A$, with its involution $C_1D\flat_1D\natural_1E\flat_4G_2A$, $p^3m^2n^2s^3d^3t^2$, which may also be analyzed as the simultaneous projection of three perfect fifths and three minor seconds; the triad *pdt* at the major third, $C_4E_2F\sharp_1G_3A\sharp_1B$, with its involution $C_1D\flat_3E_1F_2G_4B$, $p^3m^3n^2s^2d^3t^2$; the triad *nsd* at the perfect fifth, $C_1D\flat_2E\flat_4G_1A\flat_2B\flat$, with its involution $C_2D_1E\flat_4G_2A_1B\flat$, $p^4m^2n^3s^3d^2t$; the triad *nsd* at the major third, $C_1D\flat_2E\flat_1E\natural_1F_2G$, with its involution $C_2D_1E\flat_1E\natural_2F\sharp_1G$, $p^2m^3n^3s^3d^3t$; and the triad *mnd* at the perfect fifth, $C_3D\sharp_1E_3G_3$ $A\sharp_1B$, with its involution $C_1D\flat_3E_3G_1A\flat_3B$, $p^3m^4n^3sd^3t$.

EXAMPLE 36-7



Of the thirteen new hexads discussed in this chapter, all but four may also be explained as projection by involution, as illustrated in Example 36-8.

EXAMPLE 36-8



The four new hexads which cannot be arranged in similar manner are: *pmd* @ *s*, s^2 @ *n*, *mst* @ *p*, and *mst* @ *d*.

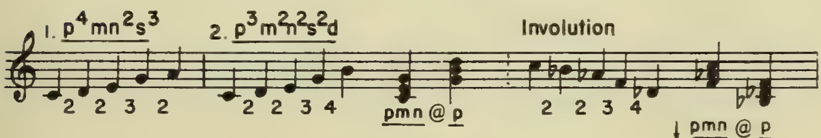
37

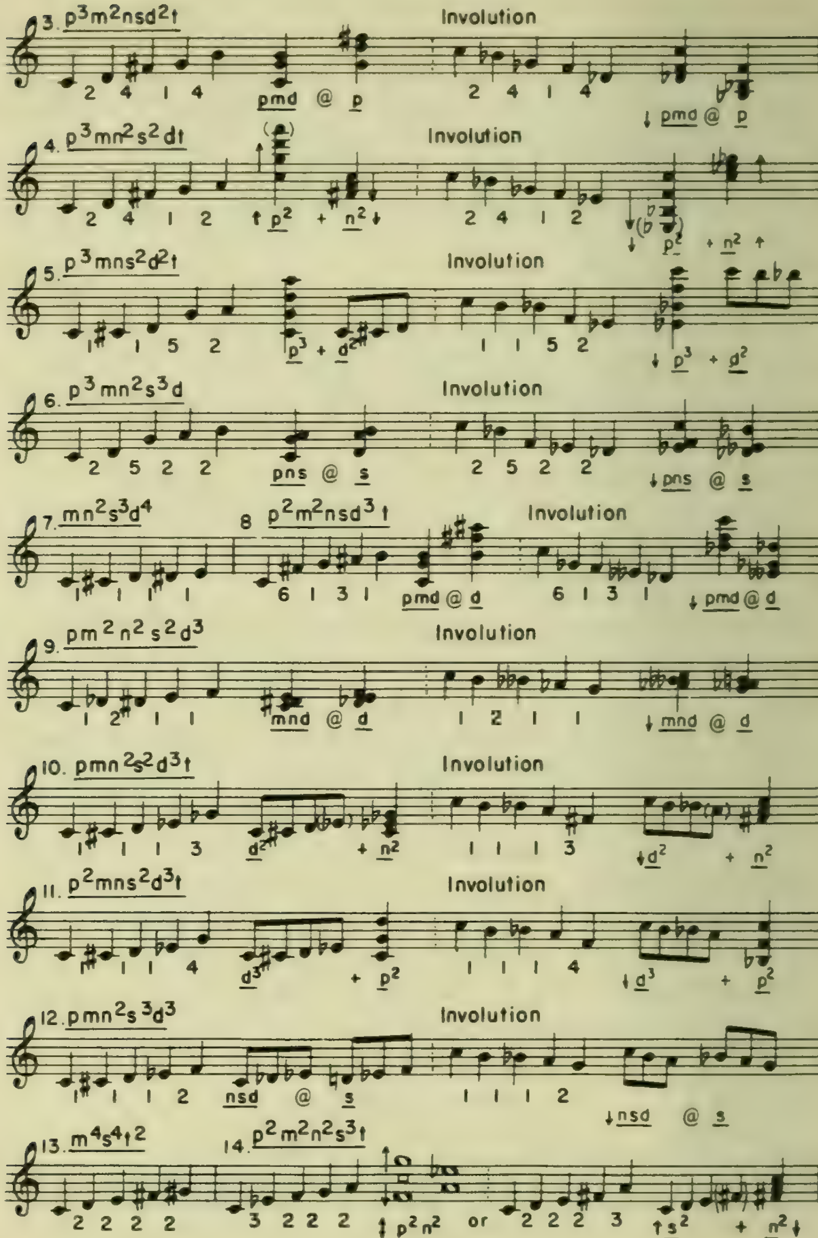
Recapitulation of Pentad Forms

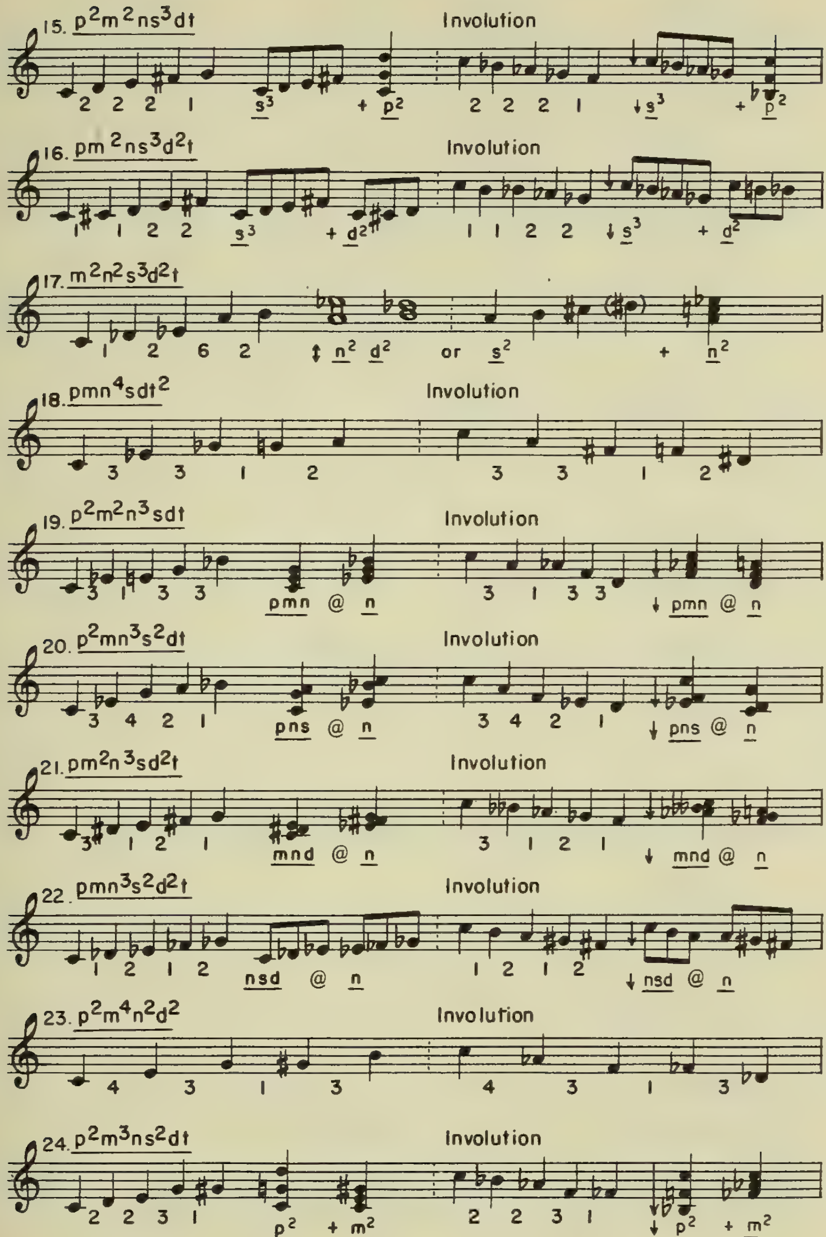
We have now encountered all the pentad forms which are found in the twelve-tone equally tempered scale. It is wise, therefore, to summarize them here. The student should review them carefully, play them and listen to them in all of their inversions and experiment with them, both melodically and harmonically. All of the pentads are projected above C for comparison and, where the pentad is not isometric, the involution is projected *downward* from C.

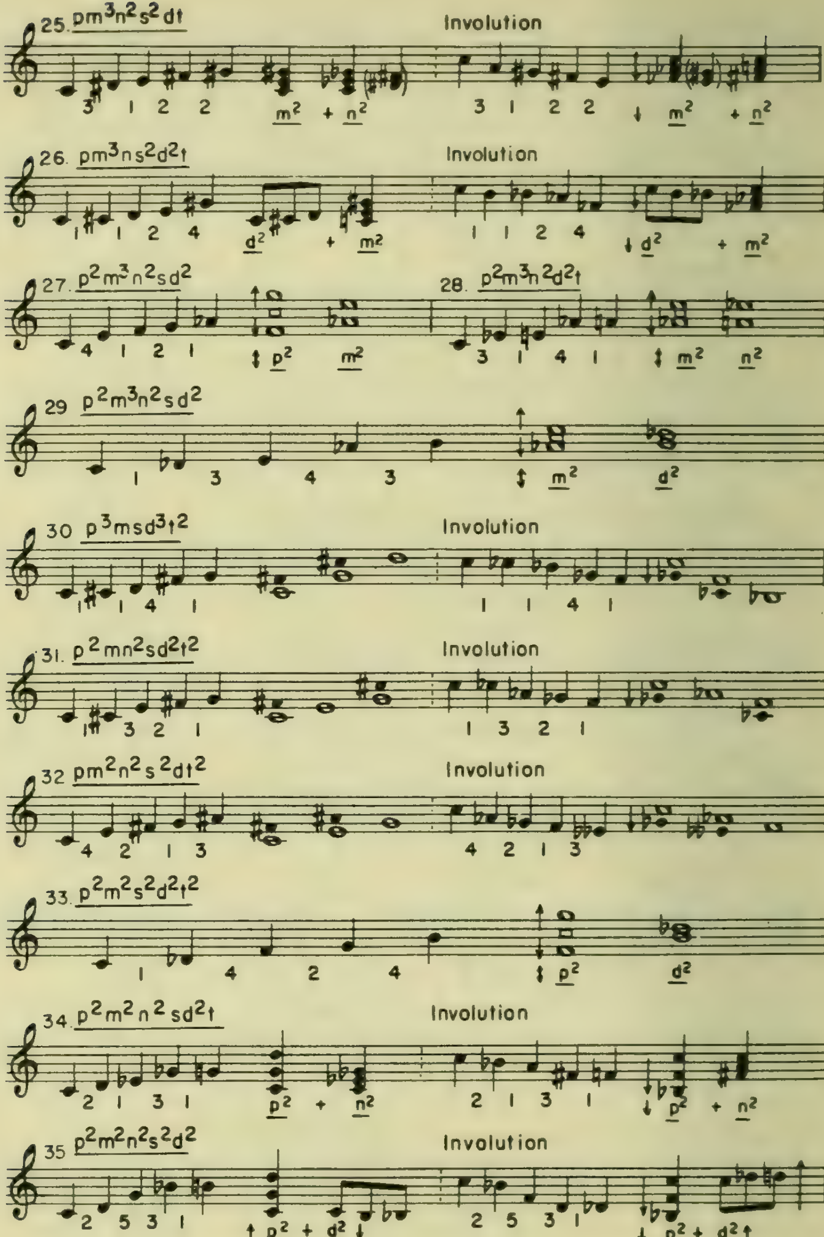
Pentads numbered 1 to 5 predominate in perfect fifths, while number 6 contains an equal number of perfect fifths and major seconds. Pentads numbered 7 to 11 predominate in minor seconds, with number 12 containing an equal number of minor seconds and major seconds. Pentad number 13 has major thirds, major seconds, and tritones in equal strength. Pentads numbered 14 to 17 predominate in major seconds. Pentad number 18 predominates in minor thirds and tritones; numbers 19 to 22 predominate in minor thirds. Pentads 23 to 29 predominate in major thirds. The tritone, considering its double valency, dominates pentads 30 to 33, and the remaining pentads, numbers 34 to 38, are neutral in character.

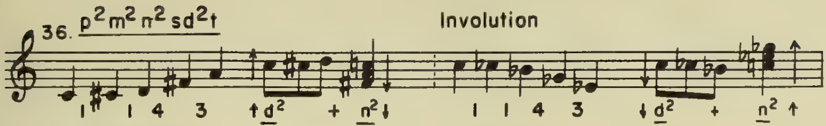
Example 37-1

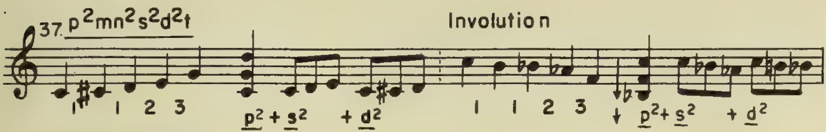



3. p3m2nsd2t
Involution
2 4 1 4
pmd @ p
2 4 1 4
pmd @ p
4. p3mn2s2dt
Involution
2 4 1 2
p2 + n2
2 4 1 2
p2 + n2
5. p3mns2d2t
Involution
1 1 5 2
p3 + d2
1 1 5 2
p3 + d2
6. p3mn2s3d
Involution
2 5 2 2
pns @ s
2 5 2 2
pns @ s
7. mn2s3d4
1 1 1 1
8 p2m2nsd3t
Involution
6 1 3 1
pmd @ d
6 1 3 1
pmd @ d
9. pm2n2s2d3
Involution
1 2 1 1
mnd @ d
1 2 1 1
mnd @ d
10. pmn2s2d3t
Involution
1 1 1 3
d2 + n2
1 1 1 3
d2 + n2
11. p2mns2d3t
Involution
1 1 1 4
d3 + p2
1 1 1 4
d3 + p2
12. pmn2s3d3
Involution
1 1 1 2
nsd @ s
1 1 1 2
nsd @ s
13. m4s4t2
2 2 2 2
14. p2m2n2s3t
3 2 2 2
p2n2 or
2 2 2 3
s2 + n2


15. p2m2ns3dt
Involution
2 2 2 1 s3 + p2
2 2 2 1 s3 + p2
16. pm2ns3d2t
Involution
1 1 2 2 s3 + d2
1 1 2 2 s3 + d2
17. m2n2s3d2t
1 2 6 2 n2 d2 or s2 + n2
18. pmn4sdt2
Involution
3 3 1 2
3 3 1 2
19. p2m2n3sdt
Involution
3 1 3 3 pmn @ n
3 1 3 3 pmn @ n
20. p2mn3s2dt
Involution
3 4 2 1 pns @ n
3 4 2 1 pns @ n
21. pm2n3sd2t
Involution
3 1 2 1 mnd @ n
3 1 2 1 mnd @ n
22. pmn3s2d2t
Involution
1 2 1 2 nsd @ n
1 2 1 2 nsd @ n
23. p2m4n2d2
Involution
4 3 1 3
4 3 1 3
24. p2m3ns2dt
Involution
2 2 3 1 p2 + m2
2 2 3 1 p2 + m2


25. $pm^3n^2s^2dt$
Involution
26. $pm^3ns^2d^2t$
Involution
27. $p^2m^3n^2sd^2$
28. $p^2m^3n^2d^2t$
29 $p^2m^3n^2sd^2$
30 $p^3msd^3t^2$
Involution
31. $p^2mn^2sd^2t^2$
Involution
32 $pm^2n^2s^2dt^2$
Involution
33. $p^2m^2s^2d^2t^2$
34. $p^2m^2n^2sd^2t$
Involution
35 $p^2m^2n^2s^2d^2$
Involution


36. p²m²n²sd²t
Involution
1 1 4 3 ↑d² + n²↓
1 1 4 3 ↓d² + n² ↑


37. p²mn²s²d²t
Involution
1 1 2 3 p² + s² + d²
1 1 2 3 ↓ p²+ s² + d²


38. p²mn²s²d²t
2 1 6 1 ↕ n² s²

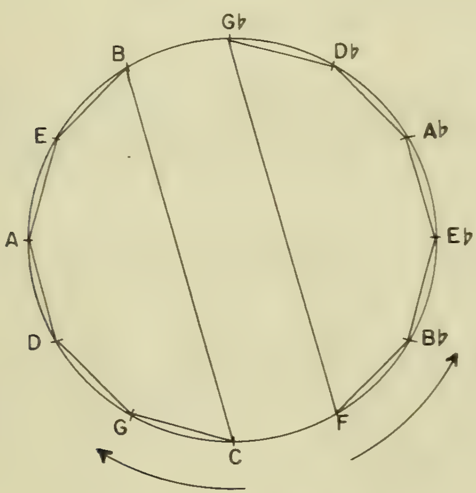
Part V

THE THEORY OF COMPLEMENTARY SONORITIES

The Complementary Hexad

WE COME NOW, logically, to the rather complicated but highly important theory of *complementary sonorities.* We have seen that the projection of five perfect fifths above the tone C produces the hexad C-G-D-A-E-B.

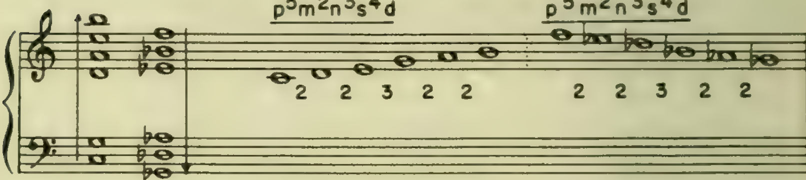
EXAMPLE 38-1



Referring to our twelve-tone circle, we note that these six tones form a figure having five equal sides and the baseline from C to B. We note, also, that the *remaining tones* form a *complementary pattern* beginning with F and proceeding counter-

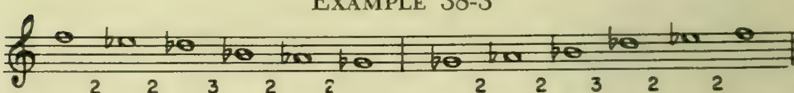
clockwise to G♭. This complementary hexad has the same formation as its counterpart and, of course, the same intervallic analysis.

EXAMPLE 38-2



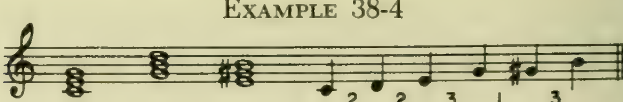
Since the hexad ↓F-E♭-D♭-B♭-A♭-G♭ is the isometric involution of the original, it will be clear that the formation is the same whether we proceed clockwise or counterclockwise. That is, if instead of beginning at F and proceeding counterclockwise, we begin at G♭ and proceed clockwise, the result is the same. We note, also, that the complementary hexad on G♭ is merely the transposition of the original hexad on C:

EXAMPLE 38-3



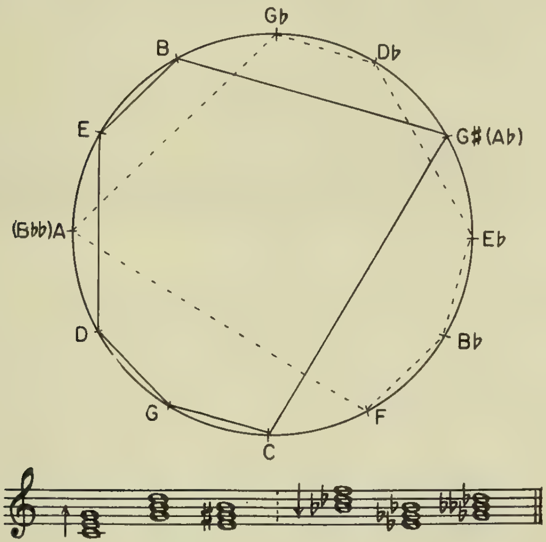
A more complicated example of complementary hexads occurs where the original hexad is not isometric. If we consider, for example, the hexad composed of major triads we find an important difference. Taking the major triad C-E-G, we form a second major triad on G—G-B-D, and a third major triad on E—E-G♯-B. Rearranging these tones melodically, we produce the hexad $C_2D_2E_3G_1G\sharp_3B$:

EXAMPLE 38-4



If we now diagram this hexad, we produce the pattern indicated in the following example, the major triad hexad being indicated by solid lines and the complementary hexad by dotted lines:

EXAMPLE 38-5

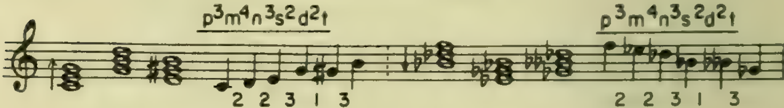


Here it will be observed that the complementary hexad F-B♭-E♭-D♭-G♭-A (B♭♭) is not the transposition but the *involution* of the original, and that the pattern of the first can be *duplicated only in reverse*, that is, by beginning at F and proceeding *counterclockwise*. The validity of this statement may be tested by *rotating* the pattern of the complementary hexad *within the circle* and attempting to find a position in which the second form exactly duplicates the original. It will then be discovered that the two patterns cannot be made to conform in this manner. They will conform only if the point F is placed upon C and the second pattern is *turned over*—similar to the turning over of a page. In this "mirrored" position, the two patterns will conform.

Transferring the above to musical notation, we observe again

that the complementary hexad to the hexad $C_2D_2E_3G_1G\sharp_3B$ is its involution, $\downarrow F_2E\flat_2D\flat_3B\flat_1B\flat\flat_3G\flat$. It will be noted further that as the first hexad was produced by the imposition of major triads upon the tones of a major triad, so the second hexad is a combination of three minor triads, the minor triad being the involution of the major triad:

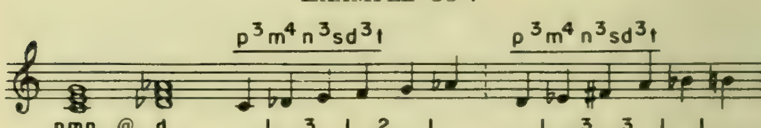
Example 38-6

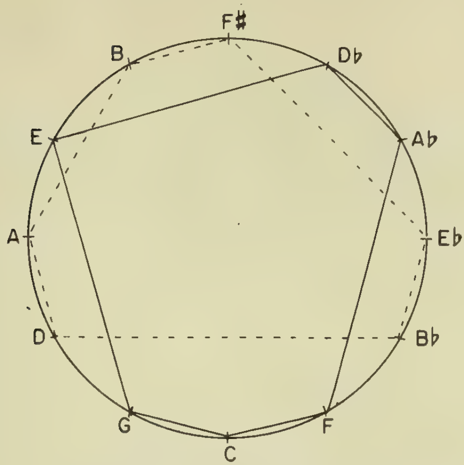


As might be expected, the intervallic analysis of the two sonorities is identical: three perfect fifths, four major thirds, three minor thirds, two major seconds, two minor seconds, and one tritone, $p^3m^4n^3s^2d^2t$.

The third, and most complicated, type of complementary hexad occurs when the remaining six tones form neither a transposition nor an involution of the original hexad but an entirely *new* hexad, yet having the same intervallic analysis. For example, the triad C-E-G at the interval of the minor second forms the hexad C-E-G + D♭-F-A♭, or $C_1D\flat_3E_1F_2G_1A\flat$. Its complementary hexad consists of the remaining tones, $D_1E\flat_3F\sharp_3A_1B\flat_1B\natural$. Both hexads have the same intervallic analysis, $p^3m^4n^3sd^3t$ but, as will be observed in Example 38-7, the two scales bear no other similarity one to the other.

Example 38-7

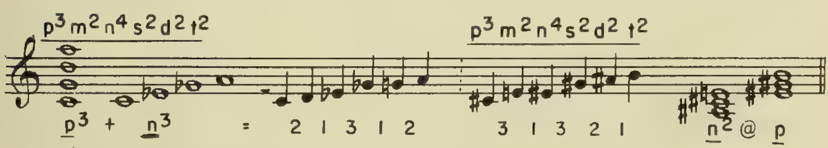




THE COMPLEMENTARY HEXAD

A fourth type includes the "isomeric twins" discussed in Part III, Chapters 27 to 32. If, for example, we superimpose three perfect fifths and three minor thirds above C we produce the hexad C-G-D-A plus C-E♭G♭A, or C_2-D_1-$\text{E}\flat_3$-$\text{G}\flat_1$-$\text{G}\natural_2$-A. The *remaining tones*, $\text{C}\sharp_3\text{E}\natural_1\text{E}\sharp_3\text{G}\sharp_2\text{A}\sharp_1$B, will be seen to consist of two minor thirds *at the interval of* the perfect fifth, A♯-C♯-E♮ plus E♯-G♯-B.

Example 38-8



39

The Hexad "Quartets"

WE ARE NOW READY to consider the more complex formations resulting from the projection of triads at intervals which are foreign to their own construction. We have already noted in the previous chapter that every six-tone scale has a *complementary* scale consisting in each case of the *remaining six tones* of the twelve-tone scale.

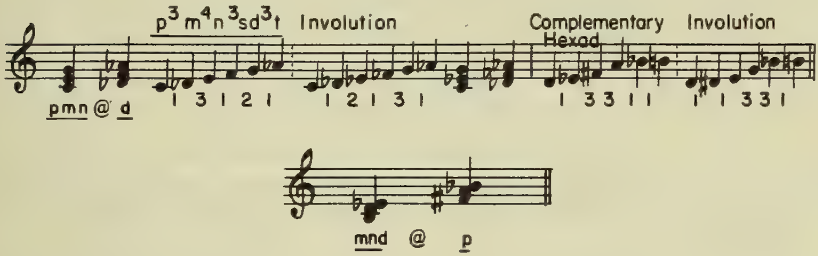
We have also noted that these complementary scales vary in their formation. In certain cases, as in the example of the six tone–perfect-fifth projection cited in Example 38-3, the complementary scale is simply a *transposition* of the original scale. In other cases, as in the major-triad projection referred to in Example 38-5, the complementary scale is the *involution* of the original scale. However, in fifteen cases the complementary scale has an entirely different order, although the same intervallic analysis.

We have already observed in Part III, Chapters 27 to 33, the formation of what we have called the *isomeric twins*—seven pairs of isometric hexads with identical intervallic analysis. A still more complex formation occurs where the original hexad is *not* isometric, for here the original scale and the complementary "twin" *will each have its own involution*. In other words, these formations result in eight *quartets of hexads*: the original scale, the involution of the original scale, the complementary scale, and the involution of the complementary scale.

The first of these is the scale formed by two major triads *pmn*

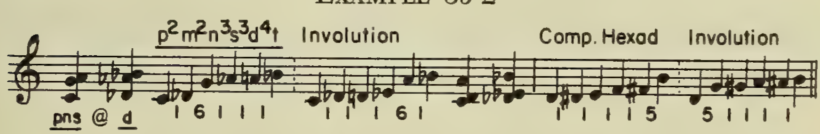
at the interval of the minor second, already referred to. Its involution will have the order, 12131, or $C_1D\flat_2E\flat_1F\flat_3G_1A\flat$, having the same analysis and consisting of two minor triads at the interval of the minor second. The complementary scale of the original will consist of the tones $D_1E\flat_3F\sharp_3A_1B\flat_1B\natural$, also with the analysis $p^3m^4n^3sd^3t$. Begun on B, it may be analyzed as $B_3D_1E\flat + F\sharp_3A_1B\flat$, or two triads *mnd* at the interval of the perfect fifth. This scale will in turn have its involution, having again the same analysis:

EXAMPLE 39-1



The triad *pns* at the interval of the minor second forms the six-tone scale C-G-A + D$\flat$-A$\flat$-B$\flat$, or $C_1D\flat_6G_1A\flat_1A\natural_1B\flat$, $p^2m^2n^3s^3d^4t$. Its involution becomes $C_1D\flat_1D\natural_1E\flat_6A_1B\flat_1$. The complementary scale of the original is $D_1D\sharp_1E_1F_1F\sharp_5B$, with its involution:

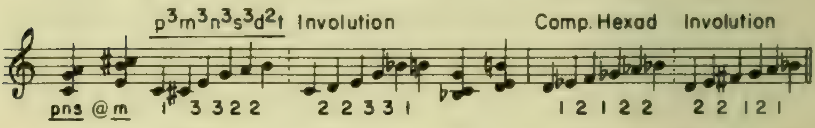
EXAMPLE 39-2



The triad *pns* at the interval of the major third forms the six-tone scale C-G-A + E-B-C$\sharp$, or $C_1C\sharp_3E_3G_2A_2B$, $p^3m^3n^3s^3d^2t$.

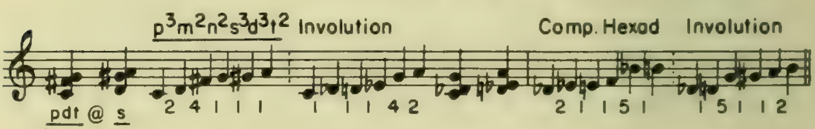
Its involution becomes $C_2D_2E_3G_3B\flat_1B\natural$. The complementary scale is $D_1E\flat_2F_1G\flat_2A\flat_2B\flat$, with its involution:

EXAMPLE 39-3



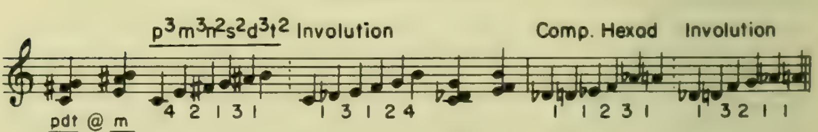
The triad *pdt* at the interval of the major second forms the six-tone scale C-F♯-G + D-G♯-A, or $C_2D_4F\sharp_1G_1G\sharp_1A$, $p^3m^2n^2s^3d^3t^2$. Its involution becomes $C_1C\sharp_1D\natural_1E\flat_4G_2A$. The complementary scale is $D\flat_2E\flat_1E\natural_1F_5B\flat_1B\natural$, with its involution:

EXAMPLE 39-4



The triad *pdt* at the interval of the major third forms the six-tone scale C-F♯-G + E-A♯-B, or $C_4E_2F\sharp_1G_3A\sharp_1B$, $p^3m^3n^2s^2d^3t^2$. Its involution is $C_1D\flat_3E_1F_2G_4B$. The complementary scale of $C_4E_2F\sharp_1G_3A\sharp_1B$ is $D\flat_1D\natural_1E\flat_2F_3A\flat_1A\natural$, with its involution:

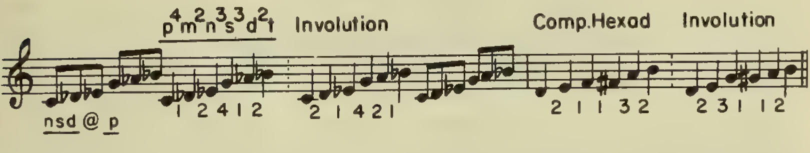
EXAMPLE 39-5



The triad *nsd* at the interval of the perfect fifth forms the six-tone scale C-D♭-E♭ + G-A♭-B♭, or $C_1D\flat_2E\flat_4G_1A\flat_2B\flat$, $p^4m^2n^3s^3d^2t$. Its involution becomes $C_2D_1E\flat_4G_2A_1B\flat$. The complementary hexad of C-D♭-E♭-G-A♭-B♭ is $D_2E_1F_1F\sharp_3A_2B$, with

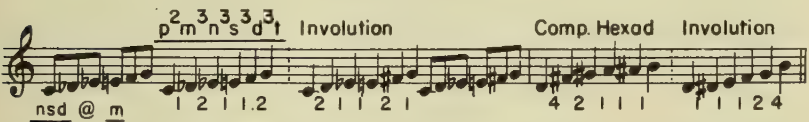
its involution. These hexads, with their preponderance of perfect fifths and secondary strength in major seconds and minor thirds, are most closely related to the perfect-fifth series:

EXAMPLE 39-6



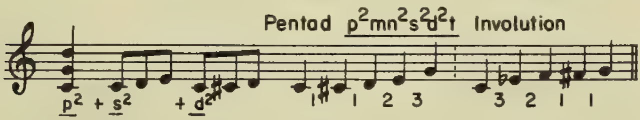
The triad *nsd* at the interval of the major third forms the six-tone scale C-D♭-E♭ + E♮-F-G, or $C_1D\flat_2E\flat_1E\natural_1F_2G$, $p^2m^3n^3s^3d^3t$. Its involution becomes $C_2D_1E\flat_1E\natural_2F\sharp_1G$. The complementary hexad of C-D♭-E♭-E♮-F-G is $D_4F\sharp_2G\sharp_1A_1A\sharp_1B$, with its involution. This quartet of hexads is neutral in character, with an equal strength of major thirds, minor thirds, major seconds, and minor seconds:

EXAMPLE 39-7



The last of these quartets of six-tone isomeric scales is somewhat of a maverick, formed from the combination of the intervals of the perfect fifth, the major second, and the minor second. If we begin with the tone C and project simultaneously two perfect fifths, two major seconds, and two minor seconds, we form the pentad C-G-D + C-D-E + C-C♯-D, or $C_1C\sharp_1D_2E_3G$, with its involution $C_3E\flat_2F_1F\sharp_1G$, $p^2mn^2s^2d^2t$:

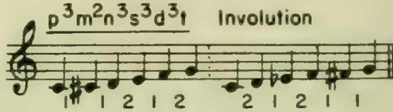
EXAMPLE 39-8



If we now form a six-tone scale by adding first a fifth *below* C,

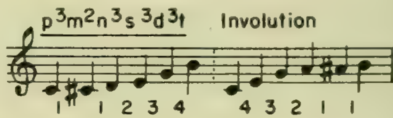
we form the scale $C_1C\sharp_1D_2E_1F_2G$, with its involution $C_2D_1E\flat_2F_1F\sharp_1G$:

EXAMPLE 39-9



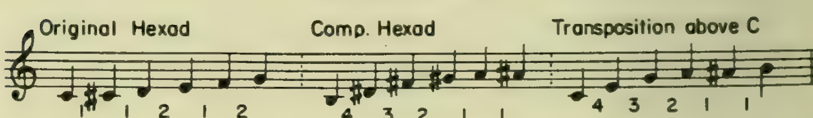
If we add the minor second below C, we form the six-tone scale $C_1C\sharp_1D_2E_3G_4B$, with its involution $C_4E_3G_2A_1A\sharp_1B$:

EXAMPLE 39-10



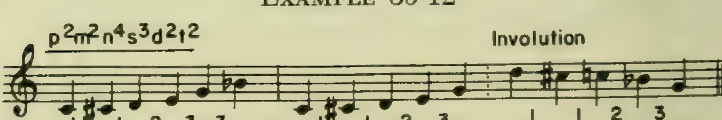
Upon examining these four scales, Examples 9 and 10, we find that they all have the same intervallic analysis, $p^3m^2n^3s^3d^3t$. We also discover in Example 11 that the complementary hexad of Example 9 is the same scale as the involution of the scale in Example 10:

EXAMPLE 39-11



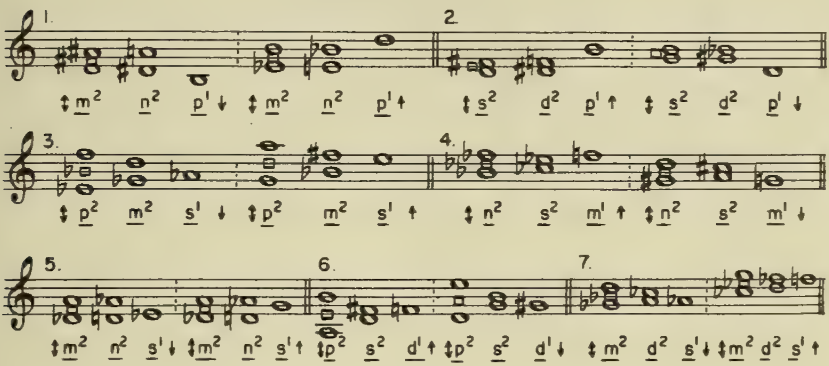
(If we take the third possibility and add a major second below C, we form the six-tone scale $C_1C\sharp_1D_2E_3G_3B\flat$, which is an *isometric* scale with the analysis $p^2m^2n^4s^3d^2t^2$, already discussed in Chapter 29. It will be noted that this scale contains both the pentad $C_1C\sharp_1D_2E_3G$ and its involution $\downarrow D_1C\sharp_1C\natural_2B\flat_3G$.

EXAMPLE 39-12



The complementary hexads of Examples 39-1 to 39-7, inclusive, may all be analyzed as projection by involution, as illustrated in Example 39-13:

EXAMPLE 39-13



Part VI

COMPLEMENTARY SCALES

40

Expansion of the Complementary-Scale Theory

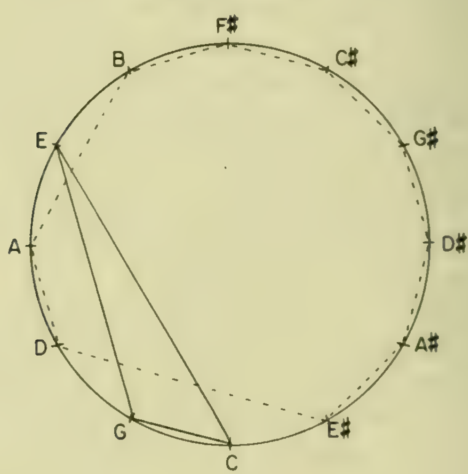
We have noted that every six-tone scale has a *complementary* six-tone scale consisting of all of the notes which are not present in the original scale, and that these scales have the *same* intervallic analysis. An analysis of all of the sonorities of the twelve-tone scale will reveal the fact that *every* sonority has a complementary sonority composed of the *remaining tones* of the twelve-tone scale and that the complementary scale will always have the same *type* of intervallic analysis, that is, the predominance of the same interval or intervals. In other words, every two-tone interval has a complementary ten-tone scale, every triad has a complementary nine-tone scale, every tetrad has a complementary eight-tone scale, every pentad has a complementary seven-tone scale, and every six-tone scale has another complementary six-tone scale.

For example, the major triad will be found to have a nine-tone scale as its counterpart, a scale which is saturated with major triads and whose intervallic analysis has a predominance of the intervals of the perfect fifth, major third, and minor third which make up the major triad. This nine-tone scale we shall call the *projection* of the major triad, since it is in fact the *expansion* or projection of the triad to the nine-tone order. The importance of this principle to the composer can hardly be overestimated, since it allows the composer to *expand any tonal relation with complete consistency.*

The process of arriving at such an expansion of tonal resources is not an entirely simple one, and we shall therefore examine it carefully, step by step, until the general principle is clear. The major triad C-E-G has a complementary nine-tone scale consisting of the *remaining nine tones of the chromatic scale,* the tones C♯-D-D♯-F-F♯-G♯-A-A♯ and B. We shall observe in analyzing this scale that it has seven perfect fifths, seven major thirds, and seven minor thirds, but only six major seconds, six minor seconds, and three tritones—*that it predominates in the same three intervals* which form the *major triad.*

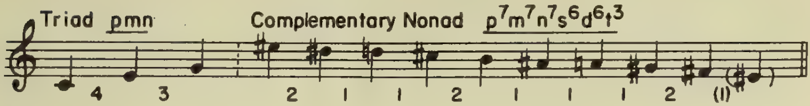
If we again revert to our circle and plot the major triad C-E-G, we find, proceeding counterclockwise, the complementary figure E♯-A♯-D♯-G♯-C♯-F♯-B-A and D:

Example 40-1



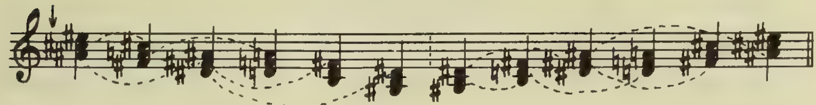
Since, as has already been noticed, clockwise rotation implies proceeding "upward" in perfect fifths and counterclockwise rotation implies proceeding "downward" in perfect fifths, we may transfer the above diagram to musical notation as follows:

EXAMPLE 40-2



If we analyze the complementary nine-tone scale, we find that it consists of a nine-tone projection downward from E♯, or upward from G♯, not of the major triad, but of its involution, the minor triad:

EXAMPLE 40-3



If we now form the involution of the nine-tone sonority by constructing a scale which has the same order of half and whole tones proceeding in the *opposite* direction, we construct the following scale:

EXAMPLE 40-4

Analyzing this scale, we find it to consist of the nine-tone projection of the *major* triad:

EXAMPLE 40-5

We may therefore state the general principle that the nine-tone projection of a triad is the *involution of its complementary scale.* We shall find, later, that this same principle applies also to the projection of tetrads and pentads.

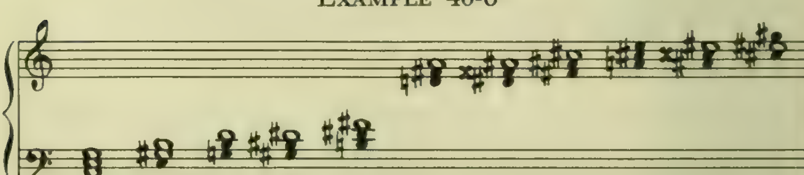
The tone which is used as the initial tone of the descending

complementary scale—in this case E♯ (or F♮)—we shall call the converting tone. Its choice in the case of the superposition of perfect fifths or minor seconds is simple. For example, if we superimpose twelve perfect fifths above C, the final tone reached is F, which becomes the initial tone of the descending complementary scale. The complementary heptad of the perfect-fifth pentad C-D-E-G-A becomes the scale $F_2E\flat_2D\flat_2C\flat_1B\flat_2A\flat_2G\flat_{(1)}$(F). The seven-tone projection of C-D-E-G-A becomes therefore the complementary heptad projected upward from C, or $C_2D_2E_2F\sharp_1G_2A_2B_{(1)}$(C) (See Ex. 41-1, lines 4 and 6.)

The converting tone of any triad is almost equally simple to determine being the final tone arrived at in the upward projection of the original triad. For example, if we superimpose major triads upon the tones of the major triad C-E-G and continue superimposing major triads upon each resultant new tone until all twelve tones have been employed, the final tone arrived at will be the converting tone for the complementary scales of that formation. Beginning with the major triad C-E-G, we form the triads (E)-G♯-B and (G)-B-D, giving the new tones G♯, B, and D. Superimposing major triads above G♯, B, and D, we form the triads (G♯)-(B♯)-D♯, (B)-D♯-F♯ and (D) F♯ A giving the new tones D♯, F♯, and A. Again superimposing major triads on D♯, F♯, and A, we form the triads (D♯)-(F𝄪)-A♯, (F♯)-A♯-C♯, and (A)-C♯-(E), giving the new tones A♯ and C♯.

Finally, superimposing major triads above A♯ and C♯, we form the triads (A♯)-(C𝄪)-E♯ and (C♯)-E♯-(G♯), giving the final twelfth tone E♯. This tone becomes the converting tone, that is, the initial tone of the descending complementary scale.

Example 40-6



The complementary heptad of the pentad composed of two major triads at the perfect fifth, C-E-G + G-B-D, or $C_2D_2E_3G_4B$, becomes, therefore, $\downarrow F_2E\flat_2D\flat_3B\flat_1A_1A\flat_2G\flat$. The projection of C-D-E-G-B is therefore $C_2D_2E_3G_1G\sharp_1A_2B$. (See Ex. 42-1, lines 2 and 5.)

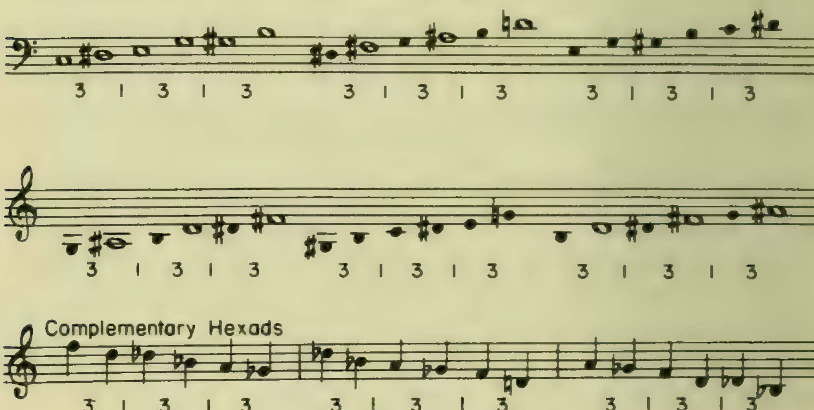
In many other cases, however, the choice of the converting tone must be quite arbitrary. For example, in the case of any sonority composed entirely of major seconds, the choice is entirely arbitrary. The whole-tone hexad above C, for example, is $C_2D_2E_2F\sharp_2G\sharp_2A\sharp$. Since this scale form superimposed on the original tones produces no new tones but merely octave duplications, it is obvious that the converting tone of the scale $C_2D_2E_2F\sharp_2G\sharp_2A\sharp$ will be B-A-G-F-E$\flat$ or D$\flat$, giving the complementary scales $\downarrow B_2A_2G_2F_2E\flat_2D\flat$, $\downarrow A_2G_2F_2E\flat_2D\flat_2B$, $\downarrow G_2F_2E\flat_2D\flat_2B_2A$, $\downarrow F_2E\flat_2D\flat_2B_2A_2G$, $\downarrow E\flat_2D\flat_2B_2A_2G_2F$, or $\downarrow D\flat_2B_2A_2G_2F_2E\flat$. The choice of F as the converting tone in Example 40-7 is therefore entirely arbitrary.

EXAMPLE 40-7

Take, again, the major-third hexad in Example 40-8, $C_3D\sharp_1E_3G_1G\sharp_3B$. If we superimpose this intervallic order, 31313, upon each of the tones of the hexad, we form the hexads $C_3D\sharp_1E_3G_1G\sharp_3B$; $(D\sharp)_3F\sharp_1(G)_3A\sharp_1(B)_3D\natural$; $(E)_3(G)_1(G\sharp)_3(B)_1(C)_3(D\sharp)$; $(G)_3A\sharp_1(B)_3D_1(D\sharp)_3F\sharp$; $(G\sharp)_3(B)_1(C)_3(D\sharp)_1(E)_3(G\natural)$; and $(B)_3D_1(D\sharp)_3F\sharp_1(G)_3A\sharp$, giving the new tones F$\sharp$, A$\sharp$, and D and producing the nine-tone scale

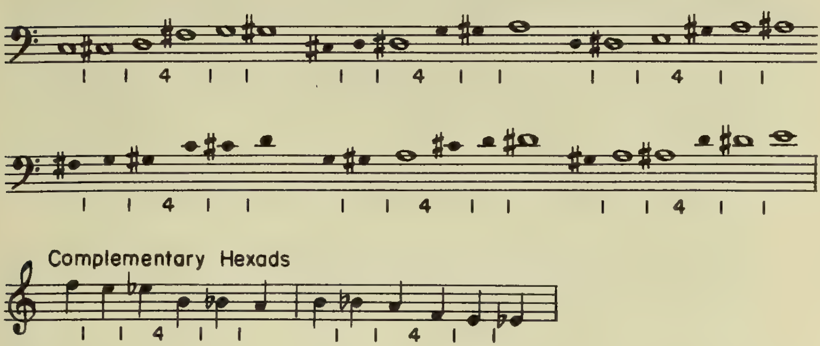
$C_2D_1D\sharp_1E_2F\sharp_1G_1G\sharp_2A\sharp_1B_{(1)}$(C.) The remaining tones, F, D$\flat$, and A, are all equally the result of further superposition and are therefore all possible converting tones, giving the descending complementary scales $\downarrow F_3D_1D\flat_3B\flat_1A_3G\flat$, $\downarrow D\flat_3B\flat_1A_3G\flat_1F_3D\natural$, and $\downarrow A_3G\flat_1F_3D_1D\flat_3B\flat$. Our choice of F is therefore an arbitrary choice from among three possibilities.

EXAMPLE 40-8



One final example may suffice. The tritone hexad of Example 40-9 contains the tones $C_1C\sharp_1D_4F\sharp_1G_1G\sharp$. This scale form superimposed upon the original tones gives the hexads $C_1C\sharp_1D_4F\sharp_1G_1G\sharp$; $(C\sharp)_1(D)_1D\sharp_4(G)_1(G\sharp)_1A$; $(D)_1D\sharp_1E_4(G\sharp)_1A_1A\sharp$; $(F\sharp)_1(G)_1(G\sharp)_4(C)_1(C\sharp)_1(D)$; $(G)_1(G\sharp)_1A_4(C\sharp)_1(D)_1D\sharp$; and $(G\sharp)_1A_1A\sharp_4(D)_1D\sharp_1E$, with the new tones D$\sharp$, E, A and A$\sharp$, producing the ten-tone scale $C_1C\sharp_1D_1D\sharp_1E_2F\sharp_1G_1G\sharp_1A_1A\sharp_{(2)}(C)$. The remaining tones, F and B, are therefore both possible converting tones giving the descending complementary scale of the hexad $C_1C\sharp_1D_4F\sharp_1G_1G\sharp$ as $\downarrow F_1E_1E\flat_4B_1B\flat_1A$ or $\downarrow B_1B\flat_1A_4F_1E_1E\flat$. Our choice of F is therefore an arbitrary choice between two possibilities. (See the Appendix.)

EXAMPLE 40-9

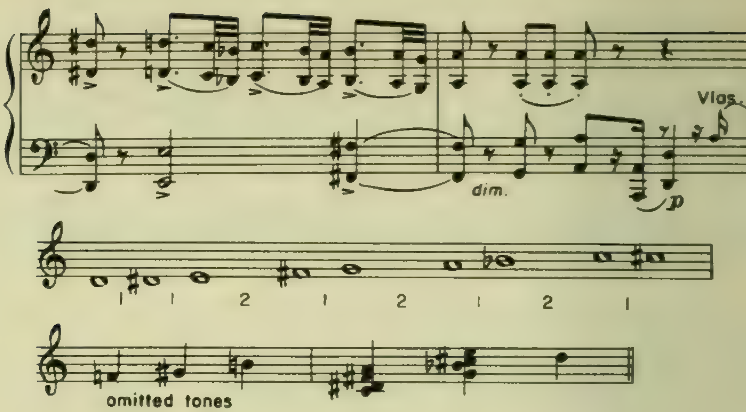


In certain cases where sonorities are built-up from tetrads or pentads through connecting hexads to the projection of the complementary octads or heptads respectively, the converting tone of the connecting hexad is used.*

An understanding of the theory of complementary scales is especially helpful in analyzing contemporary music, since it shows that complex passages may be analyzed accurately and effectively by an examination of the *tones which are not used* in the passage. Let us take, for example, the moderately complex tonal material of the opening of the Shostakovitch Fifth Symphony:

EXAMPLE 40-10

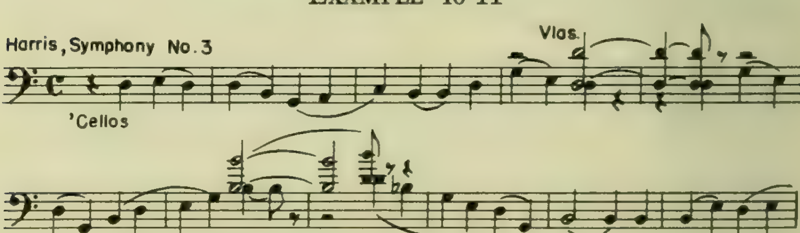
* A "connecting hexad" is defined as any hexad which *contains* a specific pentad and is also a *part* of that pentad's seven-tone projection.



An examination of the opening theme shows not only the presence of the tones D-D♯-E-F♯-G-A-B♭-C-C♯, but the *absence* of the tones F, G♯, and B. Since F-G♯-B is the *basic minor third triad,* it becomes immediately apparent that the complementary nine-tone theme must be the *basic nine-tone minor third scale.* A re-examination of the scale confirms the fact that it is composed of two diminished tetrads at the interval of the perfect fifth plus a second foreign tone a fifth above the first foreign tone—the formation of the minor-third nonad as described in Chapter 13.

This type of "analysis by omission" must, however, be used with caution, lest a degree of complexity be imputed which was never in the mind of the composer. The opening of the Third Symphony of Roy Harris offers a fascinating example of music which, at first glance, might seem much more complex than it actually is.

Example 40-11



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If we examine the first twenty-seven measures of this symphony, we shall find that the composer in one long melodic line makes use of the tones G-A♭-A♮-B♭-B♮-C-C♯-D-D♯-E-F-F♯; in other words, all of the tones of the chromatic scale. Upon closer examination, however, we find that this long line is organized into a number of expertly contrived sections, all linked together to form a homogeneous whole. The first seven measures consist of the perfect-fifth projection C-G-D-A-E-B, or melodically, G-A-B-C-D-E, a perfect-fifth hexad with the tonality apparently centering about G.

The next phrase, measures 8 to 12, drops the tone C and adds the tone B♭. This proves to be another essentially perfect-fifth projection: the perfect-fifth pentad G-D-A-E-B (G-A-B-D-E) with an added B♭, producing a hexad with both a major and minor third. (See Example 39-6, Chapter 39, complementary hexad.) Measure 15 adds a momentary A♭ which may be analyzed as a lowered passing tone or as a part of the minor-second tetrad G-A♭-A♮-B♭. Measures 16 to 18 establish a cadence consisting of two major triads at the relationship of the major third—B♭-D-F plus D-F♯-A (D-F-F♯-A-B♭—Example 22-2).

Measures 19 to 22 establish a new perfect-fifth hexad on D—D-A-E-B-F♯-C♯ (D-E-F♯-A-B-C♯), which will be seen to be a transposition of the original hexad of the first seven measures. In measure 23 the modulation to a B tonality is accomplished by the involution of the process used in measures 16 to 18, that is, two minor triads at the relationship of the major third:

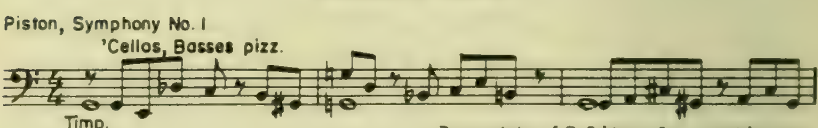
A♯	F♯
F♯	(D♮)
D♯	B

Measures 24 to 27 return to the pure-fifth hexad projection G-D-A-E-B-F♯, in the melodic form B-D-E-F♯-G-A, a transposition of the hexad which introduced the theme.

The student may well ask whether any such detailed analysis went on in the mind of the composer as he was writing the passage. The answer is probably, "consciously—no, subconsciously—yes." Even the composer himself could not answer the question with finality, for even he is not conscious of the workings of the subconscious during creation. What actually happens is that the composer uses both his intuition and his conscious knowledge in selecting material which is homogeneous in character and which accurately expresses his desires.

A somewhat more complicated example may be cited from the opening of the Walter Piston First Symphony:

EXAMPLE 40-12



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Here the first three measures, over a pedal tone, G, in the tympani, employ the tones G-G♯-A-B♭-B♮-C-C♯(D♭)-D-E, all of the tones except F, F♯, and D♯, in which case the nine-tone scale might be considered to be a projection of the triad *nsd*.

Such an analysis might, indeed, be justified. However, a simpler analysis would be that the first five beats are composed of two similar tetrads, $C_1D\flat_3E_3G$ and $G_1G\sharp_3B_3D$, at the interval of the perfect fifth; and that the remainder of the passage consists of two similar tetrads, $B\flat_1B\natural_1C_4E$ and $G_1G\sharp_1A_4C\sharp$, at the interval of the minor third. Both analyses are factually correct and supplement one another.

41

Projection of the Six Basic Series with Their Complementary Sonorities

We may now begin the study of the projection of all sonorities with the simplest and most easily understood of the projections, that of the perfect-fifth series. Here the relationship of the involution of complementary seven-, eight-, nine-, and ten-tone scales to their five-, four-, three-, and two-tone counterparts will be easily seen, since all perfect-fifth scales are isometric.

Referring to Chapter 5, we find that the ten-tone perfect-fifth scale contains the tones C-G-D-A-E-B-F♯-C♯-G♯-D♯ or, arranged melodically, C-C♯-D-D♯-E-F♯-G-G♯-A-B. We will observe that the *remaining* tones of the twelve-tone scale are the tones F and B♭. If we now examine the nine-tone–perfect-fifth scale, we find that it contains the tones C-G-D-A-E-B-F♯-C♯-G♯ or, arranged melodically, C-C♯-D-E-F♯-G-G♯-A-B. We observe that the remaining tones are the tones F, B♭, and E♭.

If we now build up the entire perfect-fifth projection above C, we find that the complementary interval to the ten-tone scale is the perfect fifth beginning on F and constructed downward; the complementary three-tone chord to the nine-tone scale consists of two perfect fifths beginning on F and formed downward, F-B♭-E♭; the complementary four-tone chord to the eight-tone scale consists of three perfect fifths below F, F-B♭-E♭-A♭; and the complementary five-tone scale to the seven-tone scale consists of four perfect fifths below F, F-B♭-E♭-A♭-D♭.

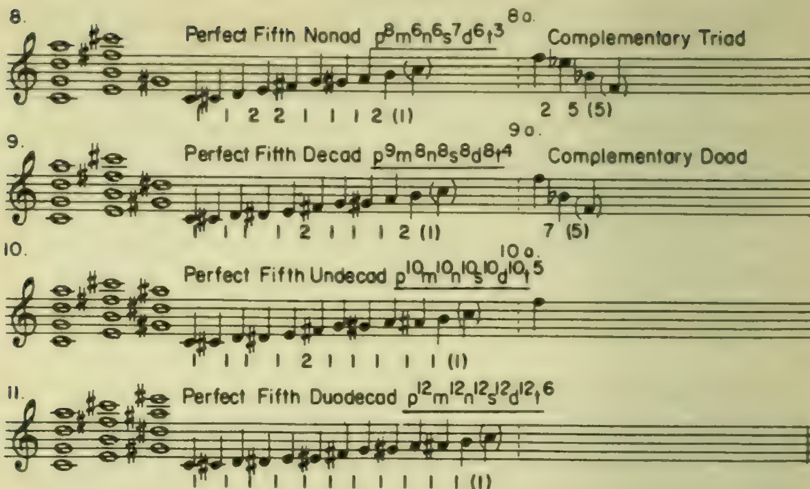
The first line of Example 41-1 gives the perfect fifth with its complementary decad. The projection of the doad of line 1 is

therefore the decad of line 9, which is the involution of the complementary decad of line 1.

Line 2 gives the perfect-fifth triad with its complementary nonad. The projection of the triad becomes the nonad, line 8, which is the involution of the complementary nonad of line 2.

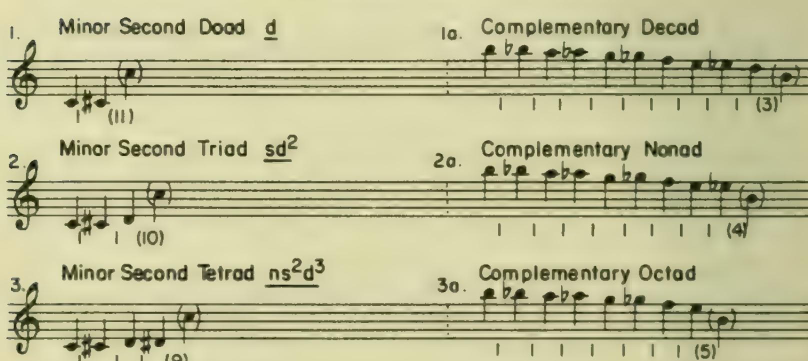
Compare, therefore, line 1*a* with line 9, 2*a* with 8, 3*a* with 7, 4*a* with 6, and 5*a* with 5. Note also that 9*a* is the involution of 1, 8*a* the involution of 2, 7*a* the involution of 3, 6*a* the involution of 4, and 5*a* the involution of 5.

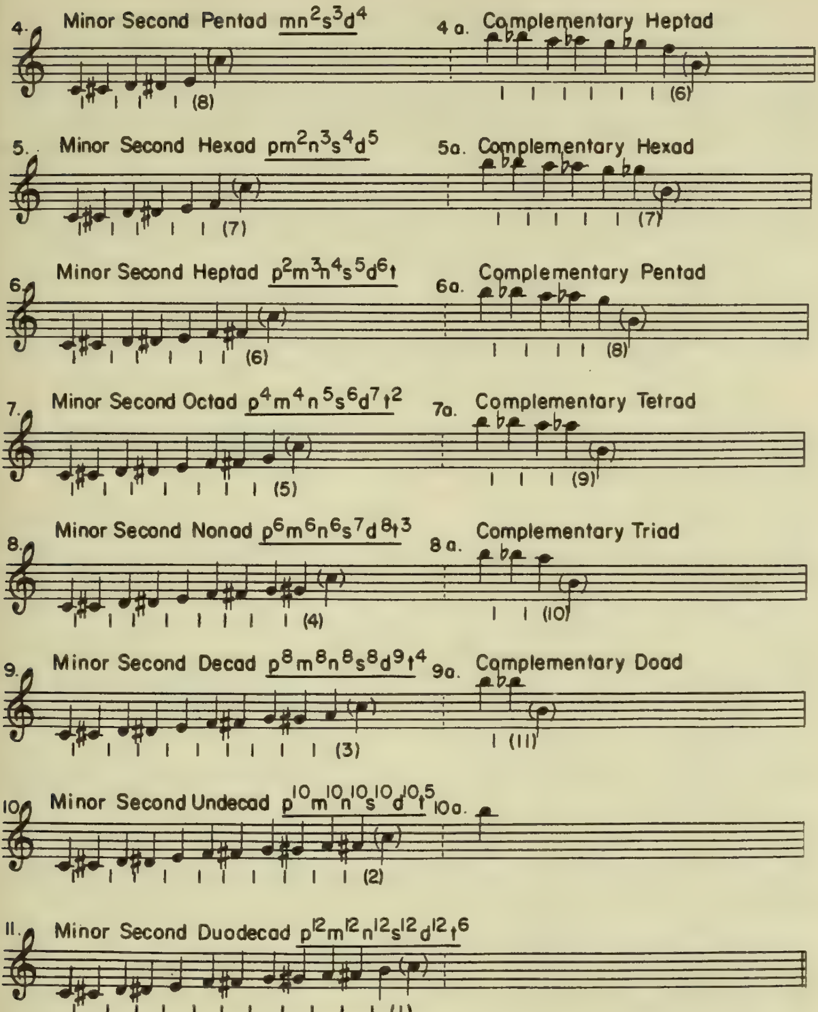
EXAMPLE 41-1



The minor-second series shows the same relationship between the two-tone interval and the ten-tone scale; between the triad and the nine-tone scale; the tetrad and the eight-tone scale, and the five-tone and the seven-tone scale. Line 9 is the involution of 1*a*; line 8 of 2*a*; line 7 of 3*a*, line 6 of 4*a*, and line 5 of 5*a*. Conversely, line 9*a* is the involution of 1, line 8*a* the involution of 2, line 7*a* of 3, line 6*a* of 4, and line 5*a* of 5.

Example 41-2





The major-second projection follows the same pattern, even though it is not a "pure" scale form. Note again that the decad in line 9 is the involution of the complementary decad, 1*a*; the nonad 8 is the involution of the complementary nonad 2*a*; and so forth. Note also that 9*a* is the involution of 1, 8*a* the involution of 2, and so forth.

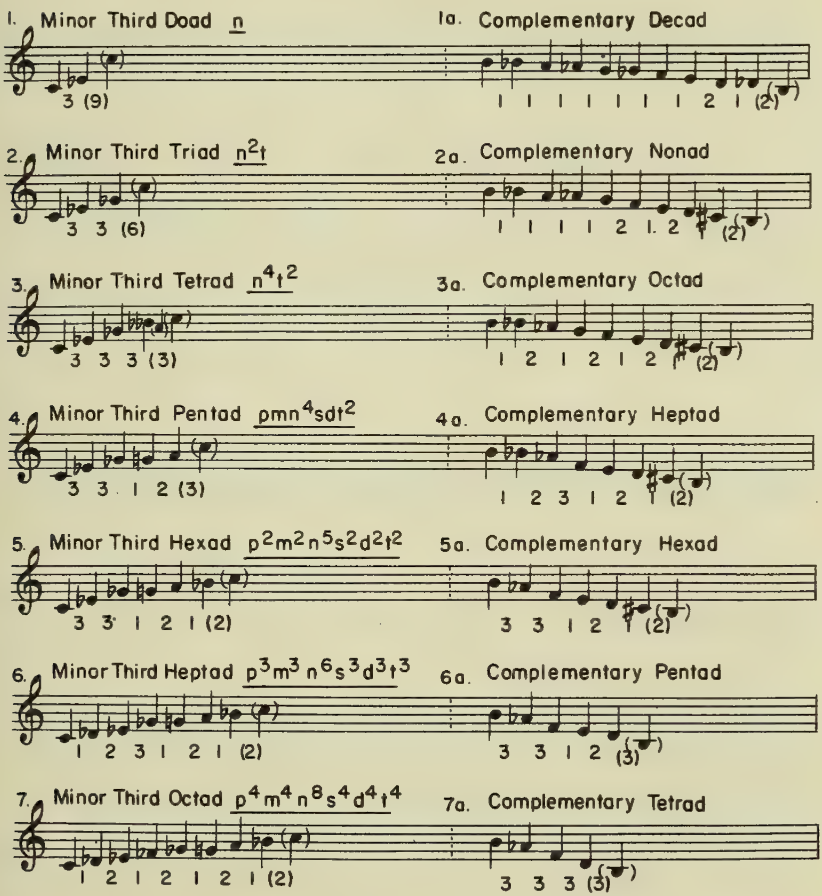
EXAMPLE 41-3

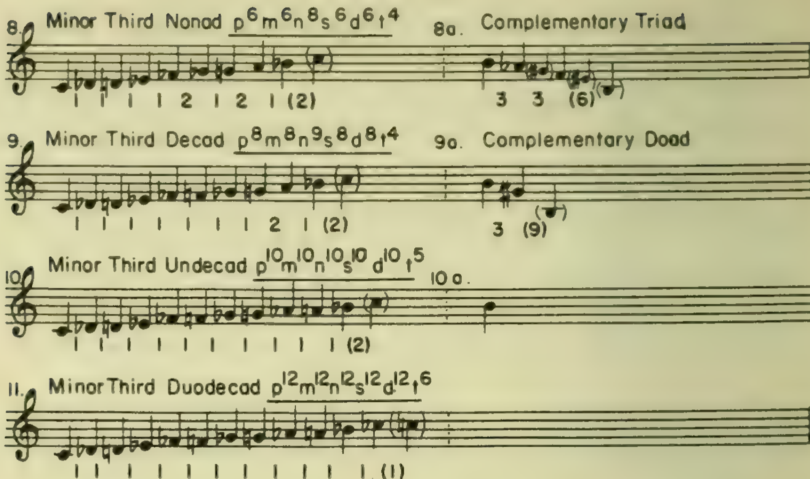

1. Major Second Doad s
2 (10)
1a. Complementary Decad
1 1 2 2 1 1 1 1 1 (1)
2. Major Second Triad ms2
2 2 (8)
2a. Complementary Nonad
2 2 2 1 1 1 1 1 (1)
3. Major Second Tetrad m2s3t
2 2 2 (6)
3a. Complementary Octad
2 2 2 1 1 1 1 (2)
4. Major Second Pentad m4s4t2
2 2 2 2 (4)
4a. Complementary Heptad
2 2 2 1 1 2 (2)
5. Major Second Hexad m6s6t3
2 2 2 2 2 (2)
5a. Complementary Hexad
2 2 2 2 2 (2)
6. Major Second Heptad p2m6n2s6d2t3
2 2 2 1 1 2 (2)
6a. Complementary Pentad
2 2 2 2 (4)
7. Major Second Octad p4m6n4s7d4t3
2 2 2 1 1 1 1 (2)
7a. Complementary Tetrad
2 2 2 (6)
8. Major Second Nonad p6m7n6s8d6t3
2 2 2 1 1 1 1 1 (1)
8a. Complementary Triad
2 2 (8)
9. Major Second Decad p8m8n8s9d8t4
1 1 2 2 1 1 1 1 1 (1)
9a. Complementary Doad
2 (10)
10. Major Second Undecad p10m10n10s10d10t5
1 1 1 1 2 1 1 1 1 1 (1)
10a.
11. Major Second Duodecad p12m12n12s12d12t6
1 1 1 1 1 1 1 1 1 1 1 (1)

The minor-third projection follows the same pattern, with the exception that the minor-third scale forms are not all isometric. It should be noted that while the three-, four-, eight-, and nine-tone formations are isometric, the five-, six-, and seven-tone scale each has its involution. (See Chapters 11 through 13.)

The student should examine with particular care the eight-tone minor-third scale, noting the characteristic alternation of a half-step and whole step associated with so much of contemporary music.

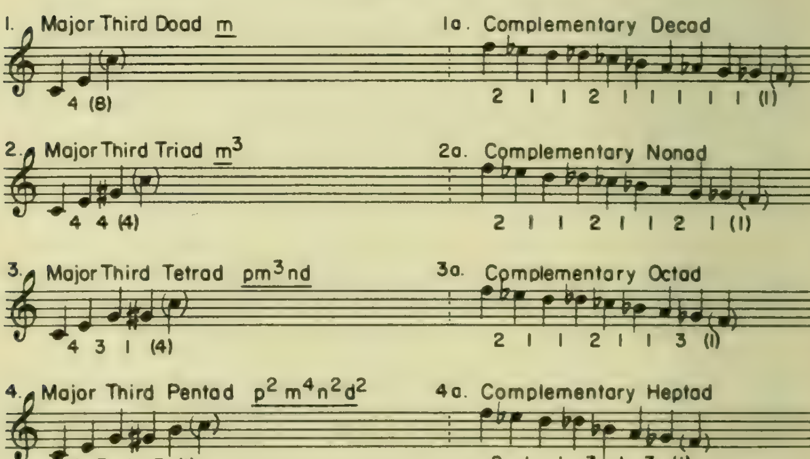
EXAMPLE 41-4

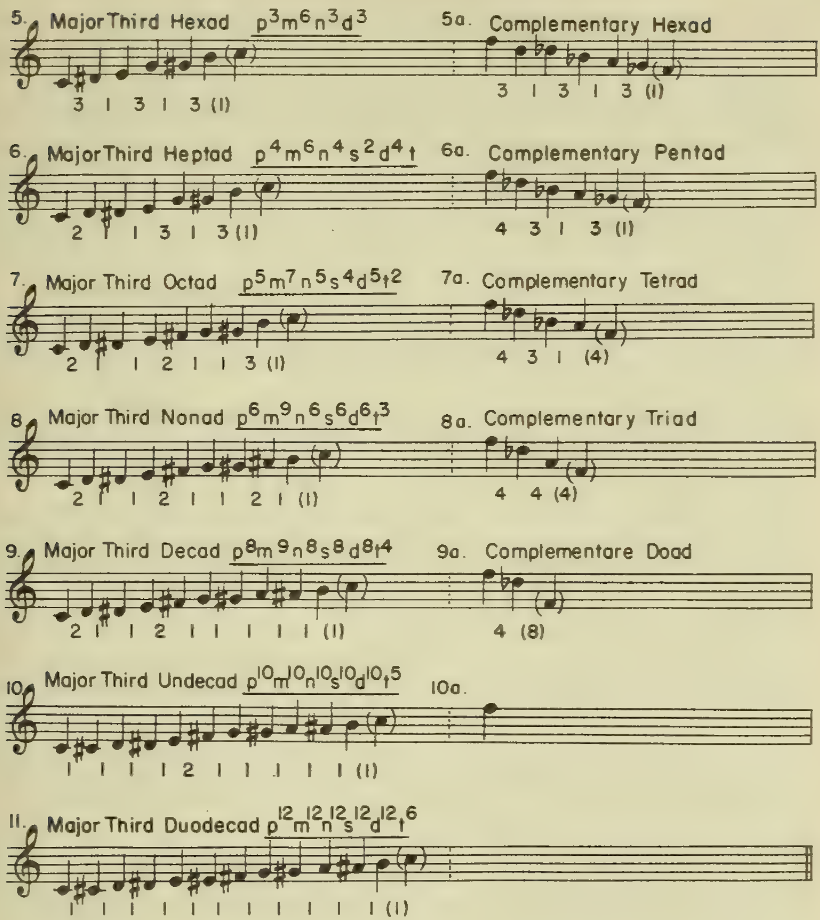




The major-third projection forms isometric types at the three-, six-, and nine-tone projections; the four-, five-, seven-, and eight-tone projections all having involutions. (See Chapters 14 and 15.) The student should examine especially the nine-tone major-third scale with its characteristic progression of a whole step followed by two half-steps, or vice-versa.

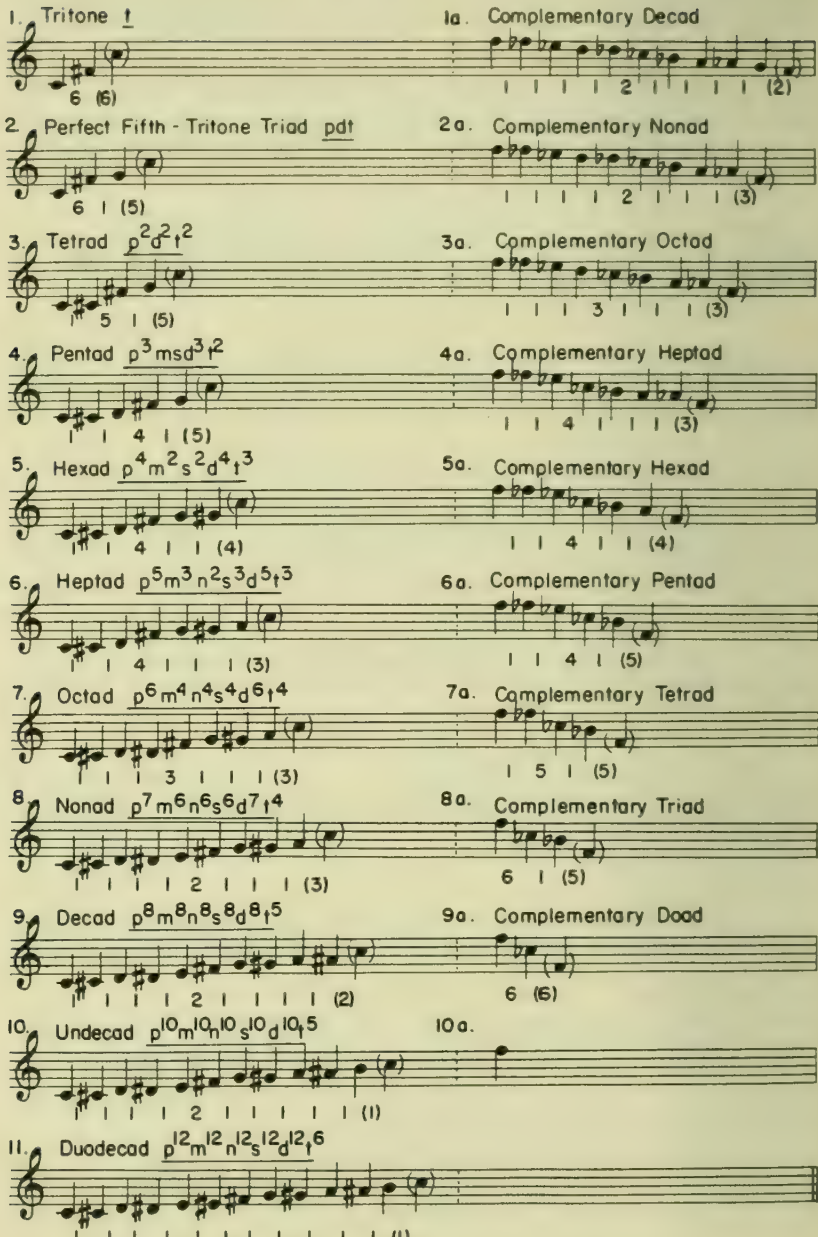
Example 41-5





The projection of the tritone upon the perfect-fifth series produces a series of scales which predominate in tritones—remembering the double valency of the tritone discussed in previous chapters. All of the scales follow the general pattern of the triad *pdt,* with a preponderance of tritones and secondary importance of the perfect fifth and minor second. The four-, six-, and eight-tone forms are isometric, whereas the three-, five-, seven-, and nine-tone forms have involutions.

EXAMPLE 41-6


1. Tritone $\underline{t}$
6 (6)
1a. Complementary Decad
1 1 1 1 2 1 1 1 1 (2)
2. Perfect Fifth - Tritone Triad $\underline{pdt}$
6 1 (5)
2a. Complementary Nonad
1 1 1 1 2 1 1 1 (3)
3. Tetrad $\underline{p^2d^2t^2}$
1 5 1 (5)
3a. Complementary Octad
1 1 1 3 1 1 1 (3)
4. Pentad $\underline{p^3msd^3t^2}$
1 1 4 1 (5)
4a. Complementary Heptad
1 1 4 1 1 1 (3)
5. Hexad $\underline{p^4m^2s^2d^4t^3}$
1 1 4 1 1 (4)
5a. Complementary Hexad
1 1 4 1 1 (4)
6. Heptad $\underline{p^5m^3n^2s^3d^5t^3}$
1 1 4 1 1 1 (3)
6a. Complementary Pentad
1 1 4 1 (5)
7. Octad $\underline{p^6m^4n^4s^4d^6t^4}$
1 1 1 3 1 1 1 (3)
7a. Complementary Tetrad
1 5 1 (5)
8. Nonad $\underline{p^7m^6n^6s^6d^7t^4}$
1 1 1 1 2 1 1 1 (3)
8a. Complementary Triad
6 1 (5)
9. Decad $\underline{p^8m^8n^8s^8d^8t^5}$
1 1 1 1 2 1 1 1 1 (2)
9a. Complementary Dyad
6 (6)
10. Undecad $\underline{p^{10}m^{10}n^{10}s^{10}d^{10}t^5}$
1 1 1 1 2 1 1 1 1 1 (1)
10a.
11. Duodecad $\underline{p^{12}m^{12}n^{12}s^{12}d^{12}t^6}$
1 1 1 1 1 1 1 1 1 1 1 (1)

An excellent example of the gradual expansion of the projection of perfect fifths will be found in Bernard Rogers' "Portrait" for Violin and Orchestra (Theodore Presser Company). The first two and a half measures consist of the tones D-E-F (triad *nsd*). The third, fourth, and fifth measures add, successively, the tones G, A, and C, forming the perfect-fifth hexad, D-E-F-G-A-C (F-C-G-D-A-E).

This material suffices until the fifteenth measure which adds the next perfect fifth, B. The seventeenth measure adds C♯, the nineteenth measure adds F♯, and the twenty-first measure adds G♯, forming the perfect-fifth decad, F♮-C-G-D-A-E-B-F♯-C♯-G♯.

In the twenty-third measure this material is exchanged in favor of a completely consistent modulation to another perfect-fifth projection, the nonad composed of the tones A♭-E♭-B♭-F-C-G-D-A♮-E♮. This material is then used consistently for the next twenty-four measures.

In the forty-seventh measure, however, the perfect-fifth projection is suddenly abandoned for the harmonic basis F♯-G-A-C♯; the sombre, mysterious *pmnsdt* tetrad, rapidly expanding to a similar *pmnsdt* tetrad on A (A-B♭-C-E), and again to a similar tetrad on C♯ (C♯-D♮-E-G♯), as harmonic background.

The opening of the following *Allegro di molto* makes a similarly logical projection, beginning again with the triad *nsd* (F-G♭-A♭) and expanding to the nine-tone projection of the triad *nsd*, E♭-E♮-F-G♭-G♮-A♭-A♮-B♭-C, in the first four measures.

The projection of the most complex of the basic series, the tritone, is beautifully illustrated by a passage which has been the subject of countless analyses by theorists, the phrase at the beginning of Wagner's *Tristan and Isolde*. If we analyze the opening passage as one unified harmonic-melodic conception, it proves to be an eight-tone projection of the tritone–perfect-fifth relationship, that is, $A_1 A\sharp_1 B_3 D_1 D\sharp_1 E_1 F_3 G\sharp_{(1)}$ (A). Sensitive listening to this passage, even without analysis, should convince the student of the complete dominance of this music by the tritone relationship. (See Example 41-6, line 7.)

This consistency of expression is, I believe, generally characteristic of master craftsmen, and an examination of the works of Stravinsky, Bartok, Debussy, Sibelius, and Vaughn-Williams—to name but a few—will reveal countless examples of a similar expansion of melodic-harmonic material.

The keenly analytical student will also find that although no composer confines himself to only one *type* of material, many composers show a strong predilection for certain kinds of tonal material—a predilection which may change during his lifetime. It might in many cases be more analytically descriptive to refer to a composer as essentially a "perfect-fifth composer," a "major-third composer," a "minor-second-tritone composer," and the like—although no composer limits himself exclusively to one vocabulary—rather than as an "impressionist," "neoclassicist," or other similar classifications.

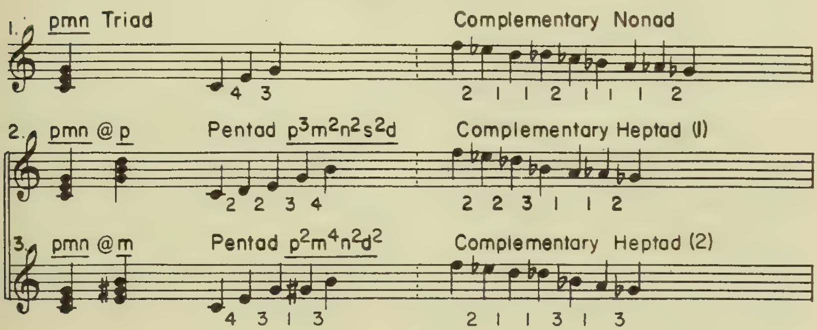
42

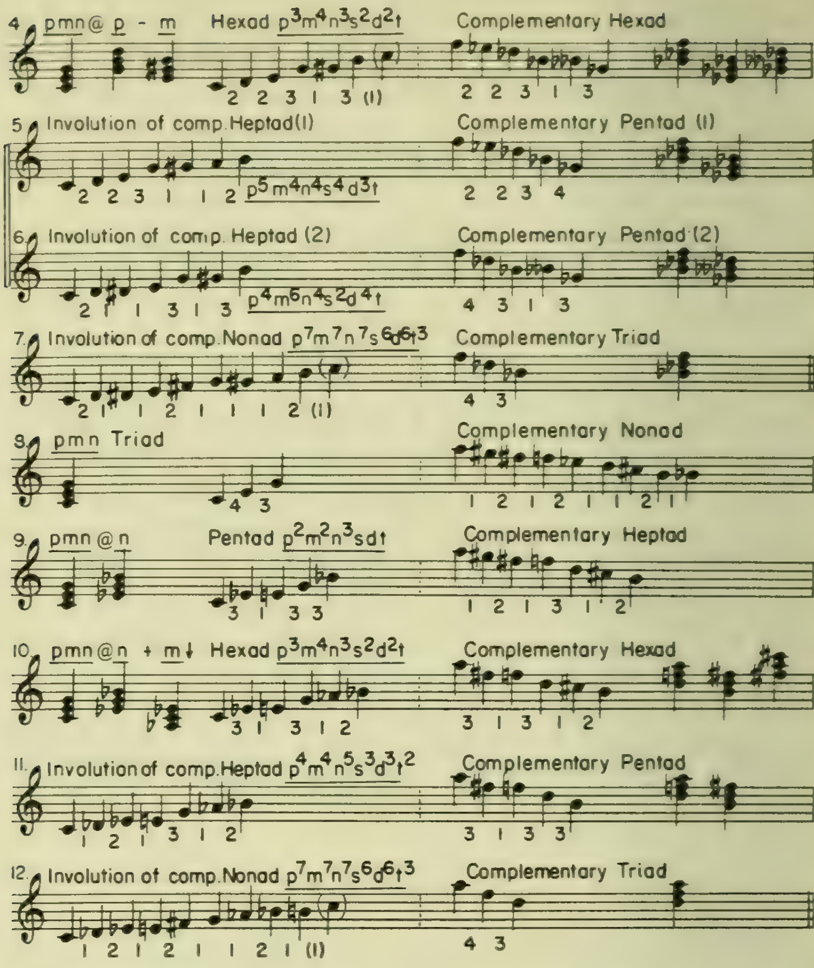
Projection of the Triad Forms with Their Complementary Sonorities

BEFORE BEGINNING THE STUDY of the complementary sonorities or scales of the triad projections, the student should review Part II, Chapters 22 to 26 inclusive. We have seen that any of the triads *pmn, pns, pmd, mnd,* and *nsd,* projected upon one of its own tones or intervals, produces a pentad. The triad projected upon all three of its tones produces a hexad which is "saturated" with the original triad form. The seven-tone scales have the same characteristics as their five-tone counterparts, and the nine-tone scale follows the pattern of the original triad.

Let us now examine Example 42-1, which presents the projection of the major triad *pmn.* Since the projection of the triads *pns, pmd, mnd,* and *nsd* follow the same principle, the careful study of one should serve them all.

EXAMPLE 42-1





The first line of Example 42-1 shows the major triad C-E-G and, separated by a dotted line, its complementary nonad—the remaining tones of the chromatic scale begun on F and projected downward. The second line shows the pentad formed by the superposition of a second major triad, on G, again with its complementary scale. The third line shows the second pentad formed by the superposition of a major triad upon the tone E with its complementary scale.

The fourth line shows the hexad formed by the combination of the three major triads, on C, on G, and on E, with its complementary hexad. It will be noted that the complementary scale has the same relationship in involution—in other words, the similar projection of three *minor* triads.

The fifth line shows the projection of the first pentad, line 2, by taking the order of intervals in the complementary heptad (second part of line 2) and projecting them *upward*. Its complementary pentad (second part of line 5) in turn becomes the involution of the pentad of line 2, having the same order of half-steps—2234—but projected *downward* and therefore representing the relationship of two *minor* triads at the perfect fifth.

The sixth line shows the projection of the second heptad (line 3) by taking the order of half-steps in the complementary heptad in the second part of line 3 and projecting it *upward*. Its complementary pentad (second part of line 6) becomes in turn the involution of the pentad of line 3 and presents, therefore, the relationship of two *minor* triads at the interval of the major third.

Line seven is formed by the projection *upward* of the order of half-steps in the complementary scale of the original triad (second part of line 1). Its complementary triad in turn is the involution of the original triad of line 1, that is, the *minor* triad.

Note the consistency of interval analysis as the projection progresses from the three-tone to the six-tone to the nine-tone formation: three tone—pmn, six-tone—$p^3m^4n^3s^2d^2t$; nine-tone—$p^7m^7n^7s^6d^6t^3$. In all of them we see the characteristic domination of the intervals p, m, and n.

In examining the hexad we discover the presence of one additional relationship, that of two major triads at the concomitant interval of the minor third—E-G♯-B and G♮-B-D. Lines 8 to 12 explore this relationship by transposing it down a major third so that the basic triad is again C major. Line 8 gives the major triad C-E-G with its complementary nonad begun on A

and projected downward (A being the converting tone of the connecting hexad of line 10).

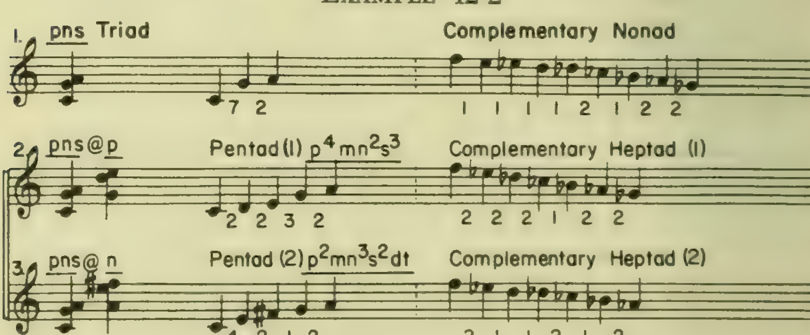
Line 9 gives the pentad formed by the relationship of two major triads at the interval of the minor third, with its complementary heptad. Line 10 is the transposition of line 4, beginning the original hexad of line 4 on E and transposing it down a major third to C, the order of half-steps becoming 313 (1)22; with its accompanying complementary hexad which is also its involution.

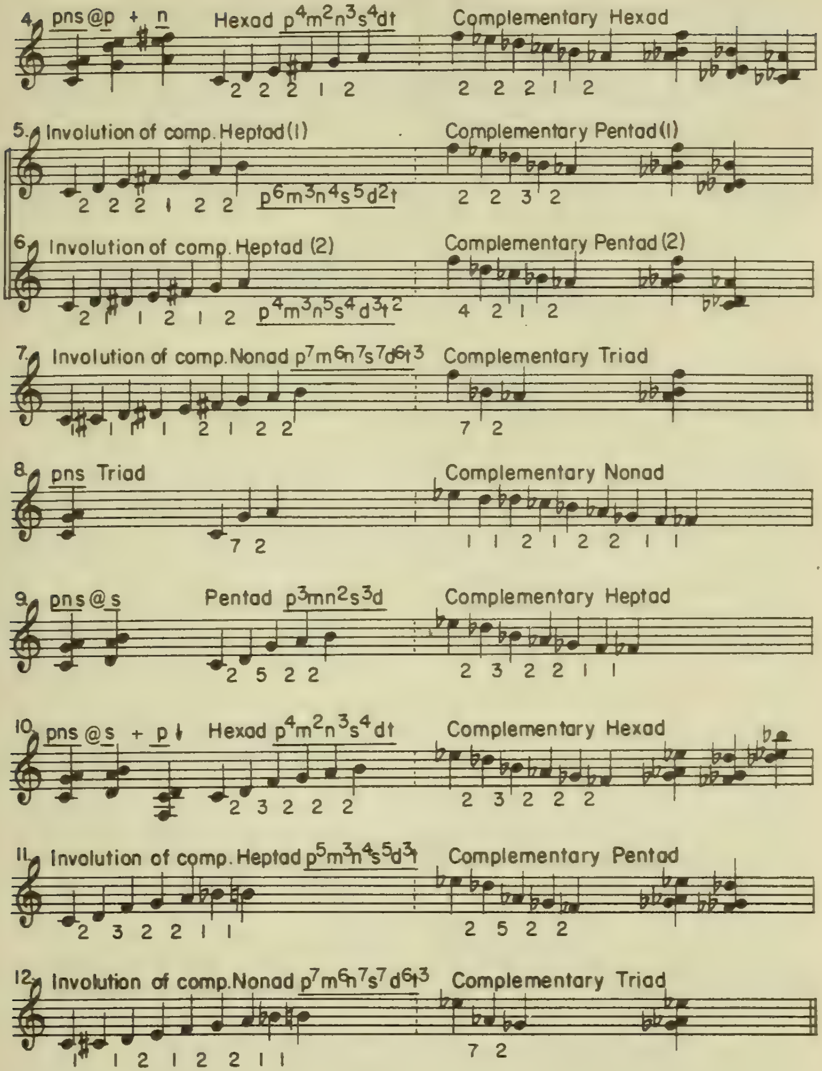
Line 11 is the projection of the order of half-steps of the complementary heptad (second part of line 9) *upward*. Its complementary pentad will be seen to be the involution of line 9, or the relationship of two *minor* triads at the interval of the minor third.

Line 12 gives the projection upward of the order of half-steps of the complementary nonad (second part of line 8), its complementary sonority being the *minor* triad D-F-A, which is the involution of the major triad of line 8. It should be observed that the nonads of lines 7 and 12 are the *same scale*, line 12 having the same order of half-steps as line 7, if we begin the nonad of line 12 on E, a major third above C.

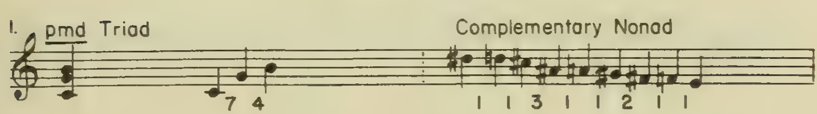
Study the relationships within the *pmn* projection carefully and then proceed to the study of the projection of the triad *pns* (Example 42-2), the triad *pmd* (Example 42-3), the triad *mnd* (Example 42-4), and the triad *nsd* (Example 42-5).

EXAMPLE 42-2



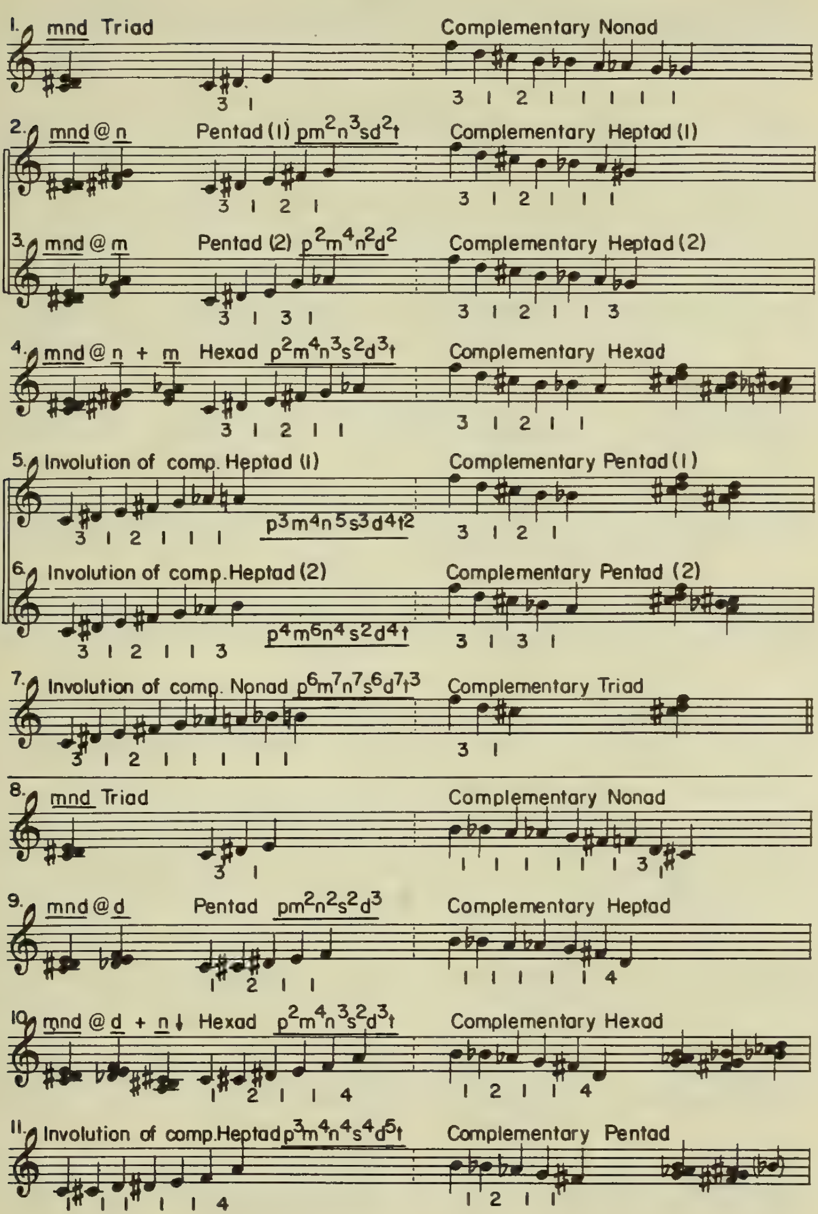

4. pns@p + n
Hexad $p^4m^2n^3s^4dt$
Complementary Hexad
2 2 2 1 2
2 2 2 1 2
5. Involution of comp. Heptad (1)
$p^6m^3n^4s^5d^2t$
Complementary Pentad (1)
2 2 2 1 2 2
2 2 3 2
6. Involution of comp. Heptad (2)
$p^4m^3n^5s^4d^3t^2$
Complementary Pentad (2)
2 1 1 2 1 2
4 2 1 2
7. Involution of comp. Nonad $p^7m^6n^7s^7d^6t^3$
Complementary Triad
1 1 1 1 2 1 2 2
7 2
8. pns Triad
Complementary Nonad
7 2
1 1 2 1 2 2 1 1
9. pns@s
Pentad $p^3mn^2s^3d$
Complementary Heptad
2 5 2 2
2 3 2 2 1 1
10. pns @s + p↓
Hexad $p^4m^2n^3s^4dt$
Complementary Hexad
2 3 2 2 2
2 3 2 2 2
11. Involution of comp. Heptad $p^5m^3n^4s^5d^3t$
Complementary Pentad
2 3 2 2 1 1
2 5 2 2
12. Involution of comp. Nonad $p^7m^6n^7s^7d^6t^3$
Complementary Triad
1 1 2 1 2 2 1 1
7 2

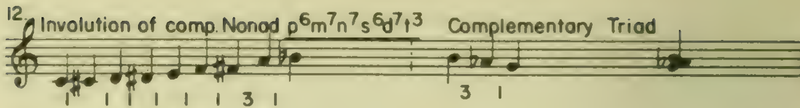
Example 42-3


1. pmd Triad
Complementary Nonad
7 4
1 1 3 1 1 2 1 1

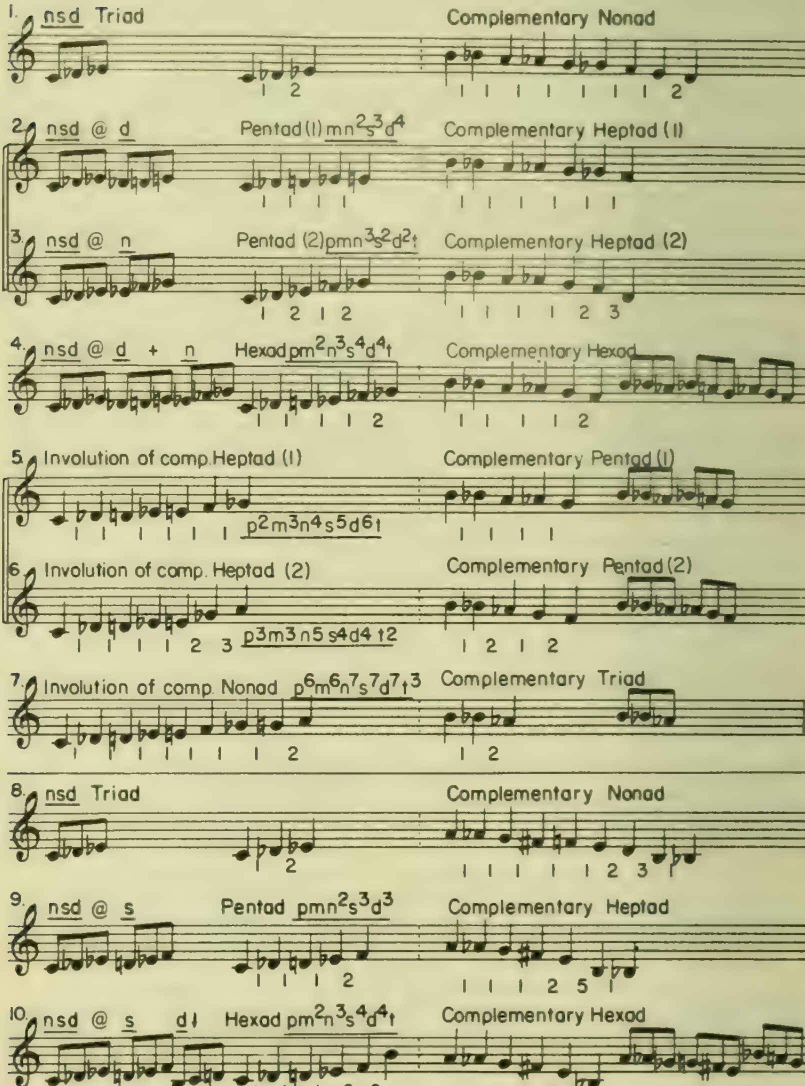

2. pmd @ p
Pentad (1) p3m2nsd2t
Complementary Heptad (1)
2 4 1 4
2 3 1 1 3 1
3. pmd @ d
Pentad (2) p2m2nsd3t
Complementary Heptad (2)
6 1 3 1
1 1 4 1 3 1
4. pmd @ p + d
Hexad p3m4n2s2d3t
Complementary Hexad
2 4 1 3 1
2 4 1 3 1
5. Involution of comp. Heptad (1)
Complementary Pentad (1)
2 3 1 1 3 1
p5m4n3s3d4t2
2 4 1 4
6. Involution of comp. Heptad (2)
Complementary Pentad (2)
1 1 4 1 3 1
p4m4n3s3d5t2
6 1 3 1
7. Involution of comp. Nonad p7m7n6s6d7t3
Complementary Triad
1 1 3 1 1 2 1 1
7 4
8. pmd Triad
Complementary Nonad
7 4
3 1 1 2 1 1 1 1
9. pmd @ m
Pentad p2m4n2d2
Complementary Heptad
3 1 3 4
3 1 1 2 1 3
10. pmd @ m + p↓
Hexad p3m4n2s2d3t
Complementary Hexad
3 1 1 2 4
3 1 1 2 4
11. Involution of comp. Heptad p4m6n4s2d4t
Complementary Pentad
3 1 1 2 1 3
3 1 3 4
12. Involution of comp. Nonad p7m7n6s6d7t3
Complementary Triad
3 1 1 2 1 1 1 1
7 4

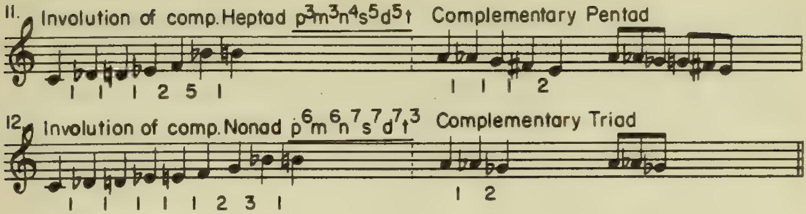
Example 42-4


1. mnd Triad
Complementary Nonad
2. mnd @ n
Pentad (1) pm2n3sd2t
Complementary Heptad (1)
3. mnd @ m
Pentad (2) p2m4n2d2
Complementary Heptad (2)
4. mnd @ n + m
Hexad p2m4n3s2d3t
Complementary Hexad
5. Involution of comp. Heptad (1)
p3m4n5s3d4t2
Complementary Pentad (1)
6. Involution of comp. Heptad (2)
p4m6n4s2d4t
Complementary Pentad (2)
7. Involution of comp. Nonad p6m7n7s6d7t3
Complementary Triad
8. mnd Triad
Complementary Nonad
9. mnd @ d
Pentad pm2n2s2d3
Complementary Heptad
10. mnd @ d + n
Hexad p2m4n3s2d3t
Complementary Hexad
11. Involution of comp. Heptad p3m4n4s4d5t
Complementary Pentad


12. Involution of comp. Nonad p6m7n7s6d7t3
Complementary Triad

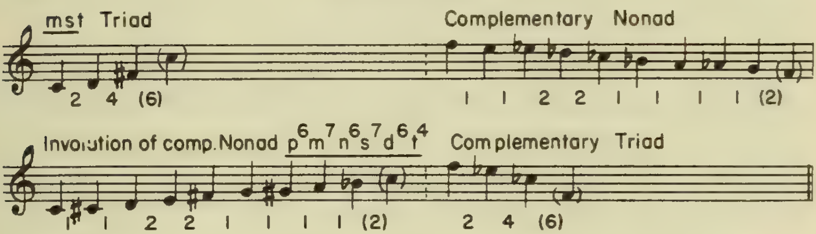
EXAMPLE 42-5


1. nsd Triad
Complementary Nonad
2. nsd @ d
Pentad (1) mn2s3d4
Complementary Heptad (1)
3. nsd @ n
Pentad (2) pmn3s2d2t
Complementary Heptad (2)
4. nsd @ d + n
Hexad pm2n3s4d4t
Complementary Hexad
5. Involution of comp. Heptad (1)
p2m3n4s5d6t
Complementary Pentad (1)
6. Involution of comp. Heptad (2)
p3m3n5s4d4t2
Complementary Pentad (2)
7. Involution of comp. Nonad p6m6n7s7d7t3
Complementary Triad
8. nsd Triad
Complementary Nonad
9. nsd @ s
Pentad pmn2s3d3
Complementary Heptad
10. nsd @ s d
Hexad pm2n3s4d4t
Complementary Hexad



Since the triad *mst* cannot be projected to the hexad by superposition, the simplest method of forming its nine-tone counterpart is to consider it as a part of the major-second hexad, and proceed as in Example 42-6:

EXAMPLE 42-6



The projection of the triad forms of the six basic series—p^2s, sd^2, ms^2, n^2t, m^3, and *pdt*—were shown in Chapter 41.

The opening of the author's *Elegy in Memory of Serge Koussevitzky* illustrates the projection of the minor triad *pmn*. The first six notes outline the minor triad at the interval of the major third, C-E♭-G + E♮-G-B. The addition of D and A in the second and fourth measures forms the seven-tone scale C-D-E♭-E♮-G-A-B, the projection of the pentad *pmn* @ *p*. The later addition of A♭ and F♯ produces the scale C-D-E♭-E♮-F♯-G-A♭-A♮-B, which proves to be the projection of the major triad *pmn*. (See Ex. 42-1, line 7.)

43

The pmn-Tritone Projection with Its Complementary Sonorities

We may combine the study of the projection of the triad *mst* with the study of the *pmn*-tritone projection, since the triad *mst* is the most characteristic triad of this projection. Line 1 in Example 43-1 gives the *pmn*-tritone hexad with its complementary hexad. Line 2 gives the triad *mst* with its complementary nonad, begun on A and projected downward.

Lines 3 and 4 give the two characteristic tetrads *pmnsdt*, with their respective complementary octads. Lines 5 and 6 give the two characteristic pentads with their complementary heptads, and line 7 gives the hexad with its complementary involution, two minor triads at the interval of the tritone.

Line 8 forms the heptad which is the projection of the pentad in line 5 by the usual process of taking the order of half-steps of the complementary heptad (second part of line 5) and projecting that order *upward*. Its complementary pentad (second part of line 8) will be seen to be the involution of the pentad in line 5.

Line 9 forms the second heptad by taking the complementary heptad of line 6 and projecting the same order of half-steps upward. Its complementary pentad becomes the involution of the pentad in line 6.

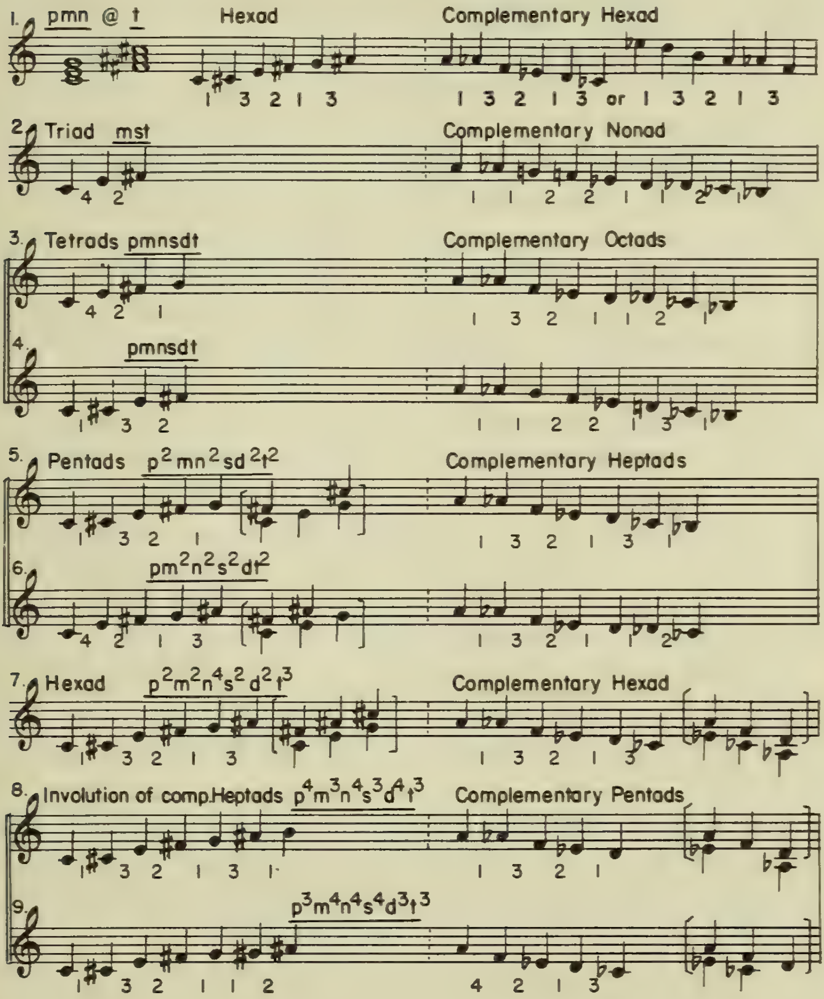
Line 10 forms the first eight-tone projection by taking the first complementary octad (second part of line 3) and projecting the same order of half-steps upward. Its complementary octad is the involution of the tetrad of line 3.

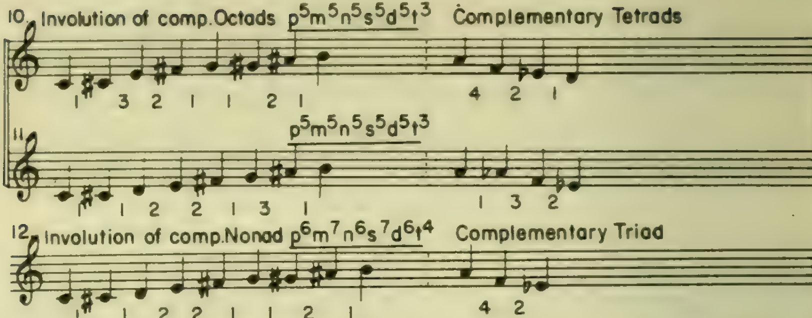
Line 11 forms the second eight-tone projection in the same manner, by taking the complementary octad of line 4 and

projecting the same order of half-steps upward. Its complementary tetrad becomes the involution of the tetrad of line 4.

Finally, line 12 is derived from the complementary nonad of line 2 projected upward, its complementary triad being the involution of the triad *mst* of line 2.

EXAMPLE 43-1





This projection offers possibilities of great tonal beauty to composers who are intrigued with the sound of the tritone. It is clearly allied to the minor-third projection but is actually *saturated* with tritones, the minor thirds being, in this case, incidental to the tritone formation. Notice the consistency of the projection, particularly the fact that the triad and the nonad, the two tetrads and the two octads, and the two pentads and the two heptads keep the same pattern of interval dominance.

The opening of the Sibelius Fourth Symphony—after the first sixteen measures (discussed in Chapter 45)—shows many aspects of the *pmn*-tritone relationship. The twentieth measure contains a clear juxtaposition of the C major and G♭ major triads, and the climax comes in the twenty-fifth measure in the tetrad C-E-F♯-G, *pmnsdt*, which with the addition of C♯ in measures twenty-seven and twenty-eight becomes C-C♯-E-F♯-G, the C major triad with a tritone added below the root and fifth.

The student will profit from a detailed analysis of this entire symphony, since it exhibits a fascinating variation between earlier nineteenth-century melodic-harmonic relationships and contemporary material.

The opening of the author's Symphony No. 2, *Romantic*, illustrates many aspects of this projection. The opening chord is a D♭ major triad with a tritone below the root and third, alternating with a G major triad with a tritone below its third and fifth. Later the principal theme employs the complete material of the projection of the pentad D♭-F-G-A♭-B, that is, D♭-D♮-F-G-A♭-A♮-B.

However, it is not necessary to examine only contemporary music or music of the late nineteenth century for examples of exotic scale forms. The strange and beautiful transition from the scherzo to the finale of the Beethoven Fifth Symphony is a magnificent example of the same projection. Beginning with the tones A♭ and C, the melody first outlines the configuration A♭-C-E♭-D-F♯, a major triad, A♭-C-E♭, with tritones above the root and third—D and F♯. It then rapidly expands, by the addition of G, A, and then E, to the scale A♭-A♮-C-D-E♭-E♮-F♯-G which is the eight-tone counterpart of A♭-C-D-E♭, *pmnsdt*, a characteristic tetrad of the *pmn*-tritone projection.

This projection is essentially melodic rather than harmonic, but the relationship is as readily apparent as if the tones were sounded simultaneously.

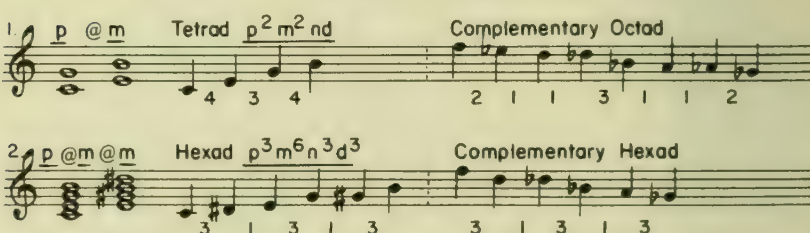
Projection of Two Similar Intervals at a Foreign Interval with Complementary Sonorities

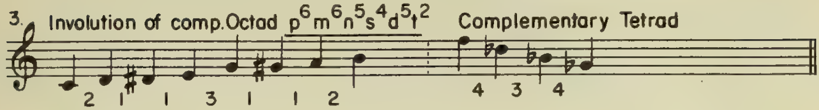
THE NEXT PROJECTION to be considered is the projection of those tetrads which are composed of two similar intervals at the relationship of a foreign interval. We shall begin with the examination of the tetrad C-E-G-B, formed of two perfect fifths at the interval of the major third, or of two major thirds at the interval of the perfect fifth. (See Examples 5-15 and 16.)

Line 1, Example 44-1, gives the tetrad *p* @ *m* with its complementary octad. Line 2 gives the hexad formed by the projection of this tetrad at the major third—(*p* @ *m*) @ *m*, with its complementary hexad. Line 3 forms the eight-tone projection of the original tetrad by the now familiar process of projecting *upward* the order of the complementary octad (second part of line 1).

Since all of these sonorities are isometric in character, there are no involutions to be considered.

EXAMPLE 44-1





The remaining tetrads are projected in similar manner: Example 44-2 presents the interval of the minor third at the relationship of the perfect fifth:

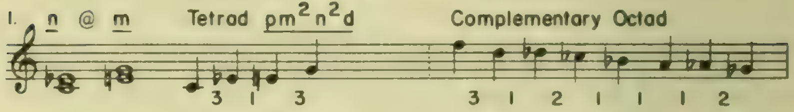
EXAMPLE 44-2

There follows the major third at the tritone;

EXAMPLE 44-3

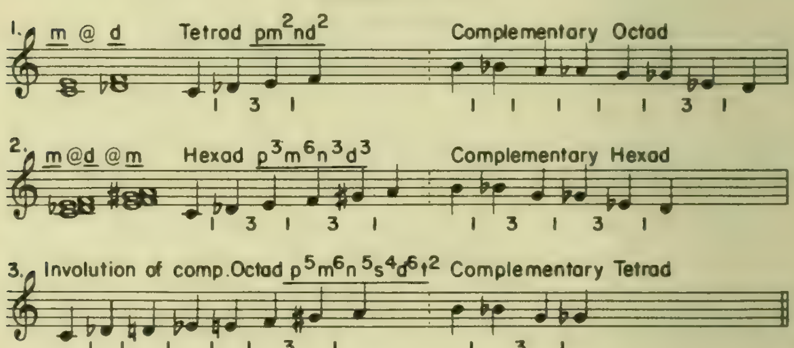
the minor third at the interval of the major third;

EXAMPLE 44-4



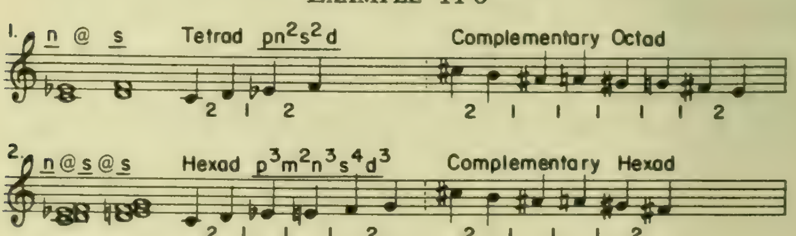
the major third at the interval of the minor second;

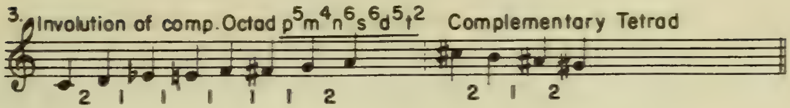
EXAMPLE 44-5



the minor third at the interval of the major second;

EXAMPLE 44-6





the minor third at the interval of the minor second;

EXAMPLE 44-7

and the perfect fifth at the interval of the minor second.

EXAMPLE 44-8

The reverse relationship of $(p @ m) @ p$; $(n @ p) @ n$; $(n @ m) @ n$; $(m @ d) @ d$; $(n @ s) @ n$; and $(n @ d) @ n$ are not used as connecting hexads in Examples 44-1, 2, 4, 5, 6, and 7 respectively because they all belong to the family of "twins" or

"quartets" discussed in Chapters 27-33, 39. The relationships of (*p* @ *d*) @ *p;* and (*p* @ *d*) @ *d*; are not used as connecting hexads for the same reason. The reverse relationship of Example 44-3, (*m* @ *t*) @ *t*, is not used because it reproduces itself enharmonically.

In the second movement of the Sibelius Fourth Symphony, the first nineteen measures are a straightforward presentation of the perfect-fifth heptad on F, expanded to an eight-tone perfect-fifth scale by the addition of a B♭ in measure twenty. (Compare the Beethoven example, Chapter 4, Example 15).

Measures twenty-five to twenty-eight present the heptad counterpart of the *pmn* @ *n* pentad. Measures twenty-nine to thirty-six, however, depart from the more conservative material of the opening being built on the expansion of the tetrad C-E-G♭-B♭ to its eight-tone counterpart C-D-E-F-G♭-A♭-B♭-B♮. (See Example 44-3.)

45

Simultaneous Projection of Intervals with Their Complementary Sonorities

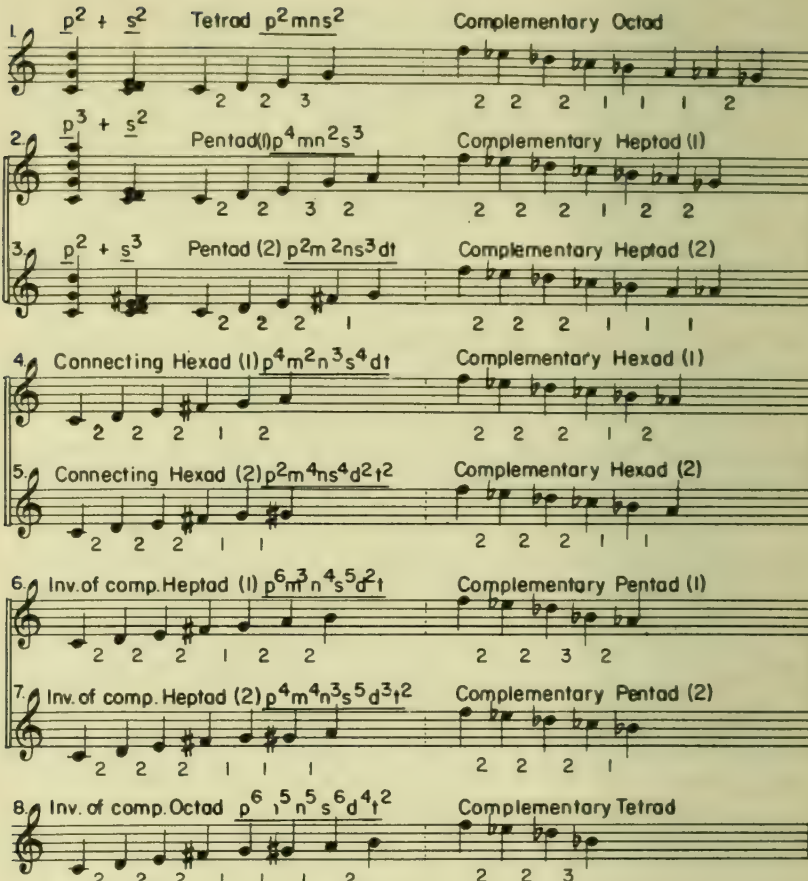
We come now to the projection of those sonorities formed by the simultaneous projection of different intervals. As we shall see, some of these projections result in tetrads which may be projected to their eight-tone counterparts, whereas others form pentads which may be projected to their seven-tone counterparts.

In Example 45-1 we begin with the simultaneous projection of the perfect fifth and the major second. Line 1 gives the projection of two perfect fifths and two major seconds above C, resulting in the tetrad C-D-E-G with its complementary octad. Line 2 increases the projection to three perfect fifths and two major seconds, producing the familiar perfect-fifth pentad, with its complementary heptad; while line 3 gives the pentad formed by the projection of two perfect fifths and three major seconds, with its complementary heptad.

Line 6 gives the heptad formed by projecting upward the order of the complementary heptad in line 2, with *its* own complementary pentad—which will be seen to be the isometric involution of the pentad of line 2. Line seven, in similar manner, gives the heptad which is the upward projection of the complementary heptad of line 3. Line 8 becomes the octad projection of the original tetrad.

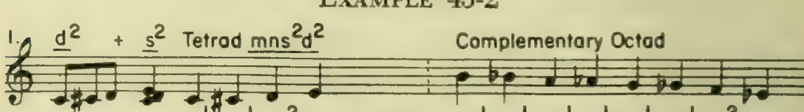
Lines 4 and 5 are the hexads which connect the pentads of lines 3 and 4 with the heptads of lines 6 and 7 respectively. There is a third connecting hexad, C-D-E-G-A-B, which is not included because it duplicates the perfect-fifth hexad projection.

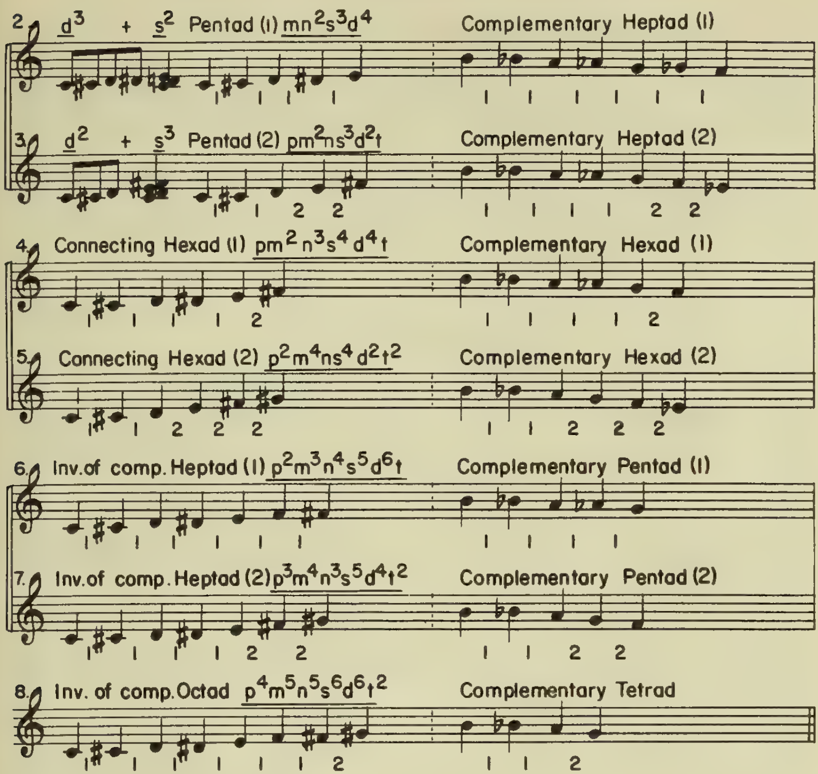
EXAMPLE 45-1



Example 45-2 gives the projection of the minor second and the major second which parallels in every respect the projection just discussed:

EXAMPLE 45-2



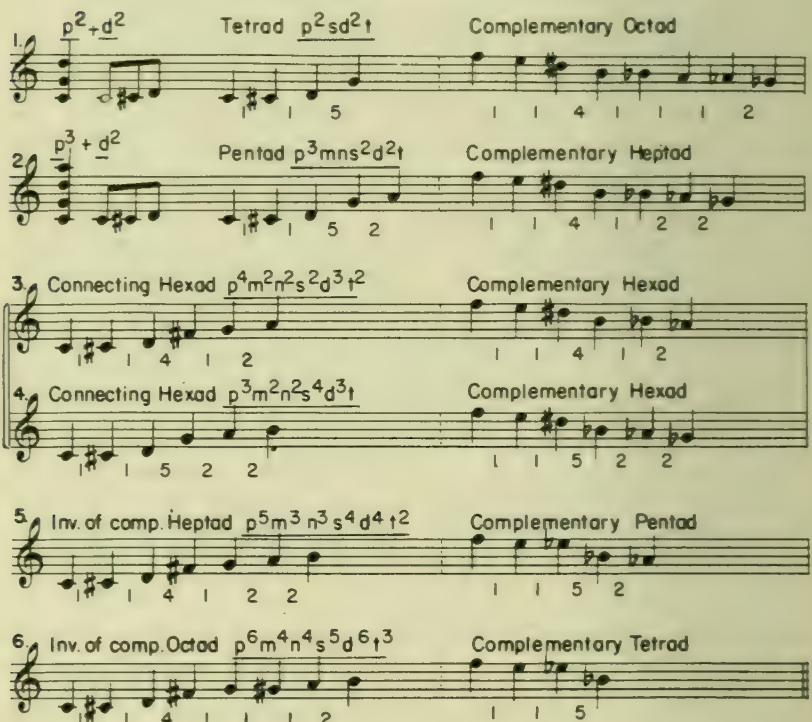


The third illustration is arranged somewhat differently, as it concerns a phenomenon which we encounter for the first time. In referring back to the simultaneous projection of the perfect fifth and the major second, we shall see that if we combine the two pentads of Example 45-1, line 2, formed of three perfect fifths plus two major seconds, and line 3, formed of two perfect fifths and three major seconds, we produce the hexad of line 4 which is a part of both of the heptads of lines 6 and 7.

Line 1 of Example 45-3 gives the tetrad formed by the simultaneous projection of two perfect fifths and two minor seconds, together with its complementary octad. Line 2 gives the pentad formed by the addition of a third perfect fifth—three

perfect fifths and two minor seconds—with its complementary heptad. Line 5 forms the heptad by projecting upward the complementary heptad of line 2. Its complementary pentad is the involution of the pentad of line 2. Line 6 forms the octad by projecting upward the complementary octad of line 1. The complementary tetrad of line 6 will be seen to be the involution of the original tetrad of line 1.

EXAMPLE 45-3



Example 45-4 is the same as 45-3, except that the pentad of line 2 is formed by the addition of a minor second—that is, two perfect fifths and three minor seconds—with its projected heptad in line 5, and the two connecting hexads of lines 3 and 4.

EXAMPLE 45-4

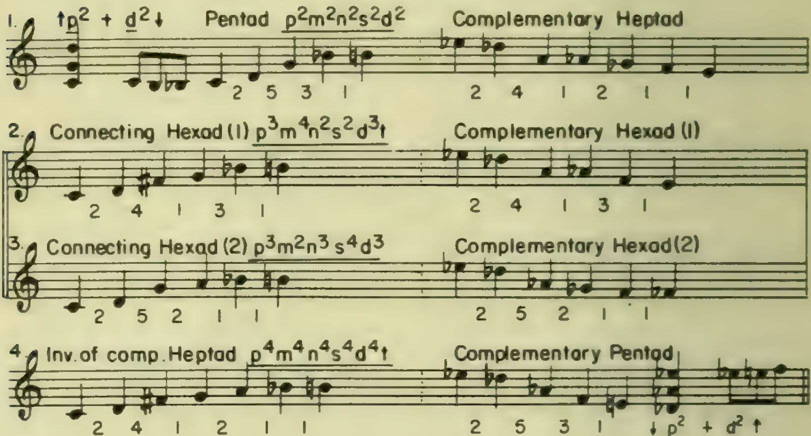
If we compare Examples 45-3 and 4 with Example 45-1, we shall observe an interesting difference. If we combine the two pentads in 45-1 formed by the projection of $p^3 + s^2$ and $p^2 + s^3$, we form the connecting hexad of line 4, C-D-E-F♯-G-A, which consists of three perfect fifths, C-D-G-A, plus three major seconds, C-D-E-F♯. However, if we combine the pentads of Examples 45-3 and 45-4, formed by the projection of $p^3 + d^2$ and $p^2 + d^3$, we form the hexad C-C♯-D-G-A + C-C♯-D-E♭-G, or C-C♯-D-E♭-G-A, which is *not* a connecting hexad for either projection.

The reason for this is that the hexad C-C♯-D-E♭-G-A is one of the isomeric "quartets" discussed in Chapter 39. It is the curious property both of the "twins" and the "quartets" of hexads, as we have already observed, that their complementary hexads are *not* their own involutions as is the case with all other

hexad forms. This type of hexad, therefore, does not serve as a connecting scale between a pentad and its heptad projection.

Example 45-5 gives the pentad formed by the projection of two perfect fifths upward and two minor seconds downward, with its projected heptad and connecting hexads:

EXAMPLE 45-5

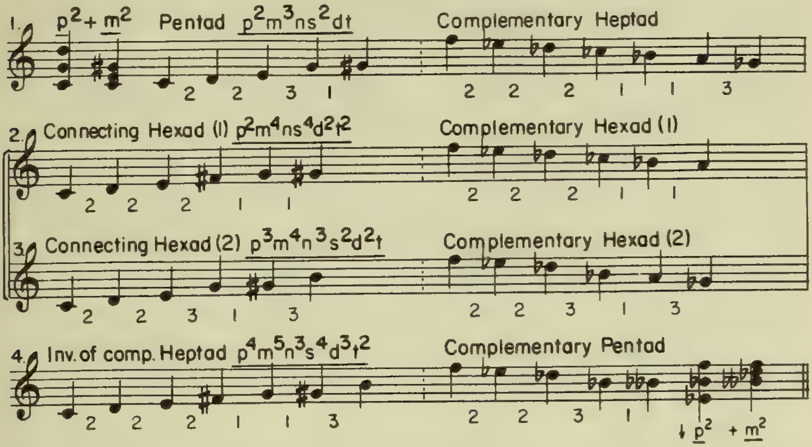


Example 45-6 gives the projection of two major seconds and two major thirds from the tetrad to the octad which is its counterpart, using the whole-tone scale as the connecting hexad:

EXAMPLE 45-6

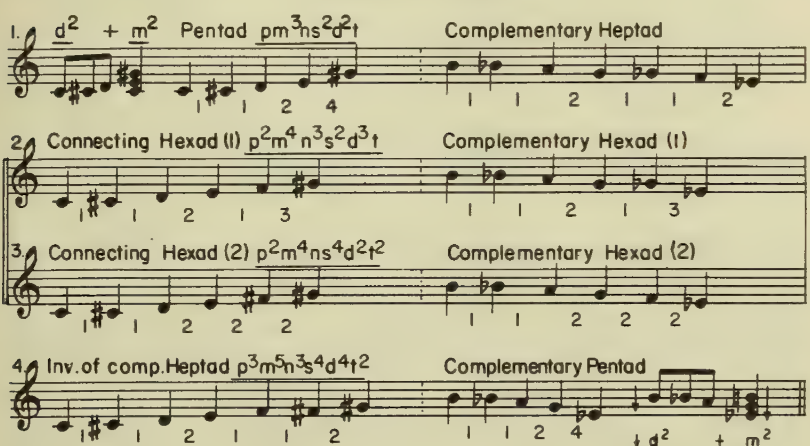
Example 45-7 gives the projection of the perfect fifth and major third:

EXAMPLE 45-7



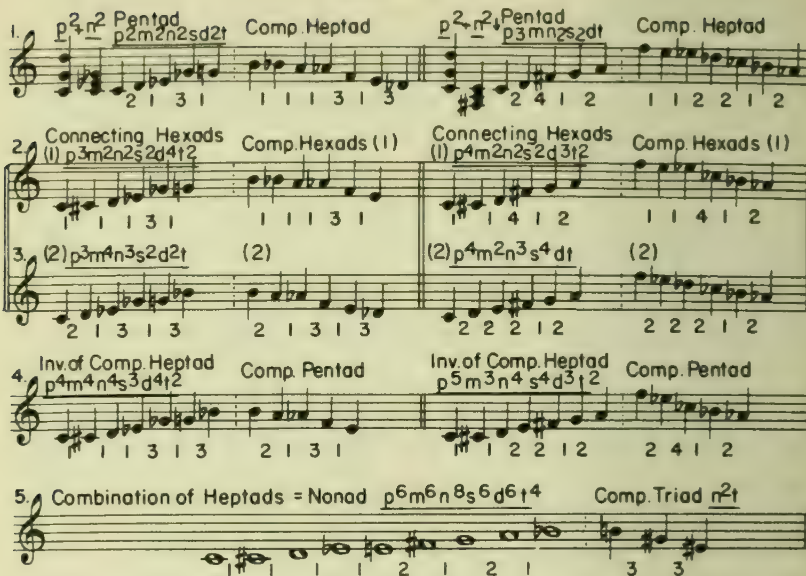
Example 45-8 gives the projection of the minor second and major third:

EXAMPLE 45-8



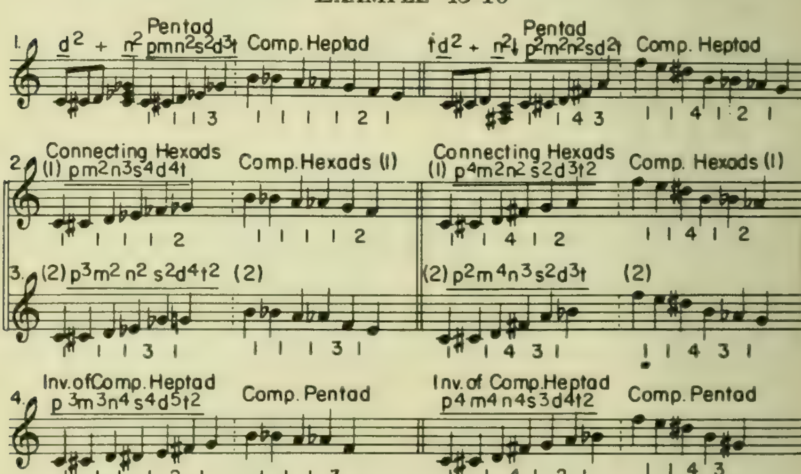
Example 45-9 gives the projection of the perfect fifth and minor third; with the second interval in both its upward and downward projection:

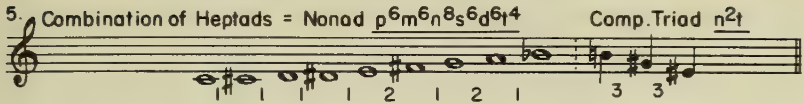
EXAMPLE 45-9



Example 45-10 gives the projection of the minor second and minor third:

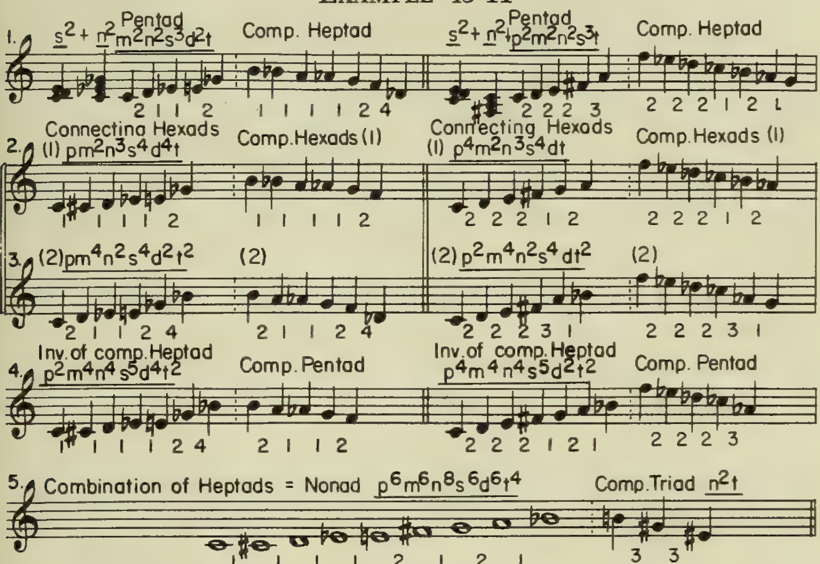
EXAMPLE 45-10





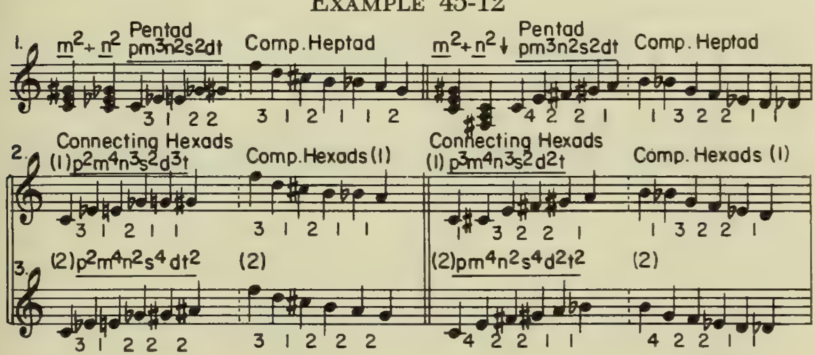
Example 45-11 gives the projection of the majoı second and minor third:

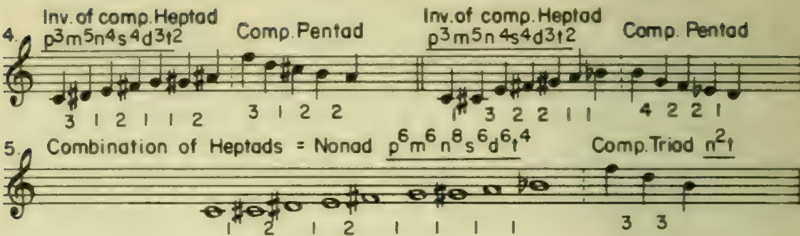
EXAMPLE 45-11



Example 45-12 gives the projection of the major third and minor third:

EXAMPLE 45-12

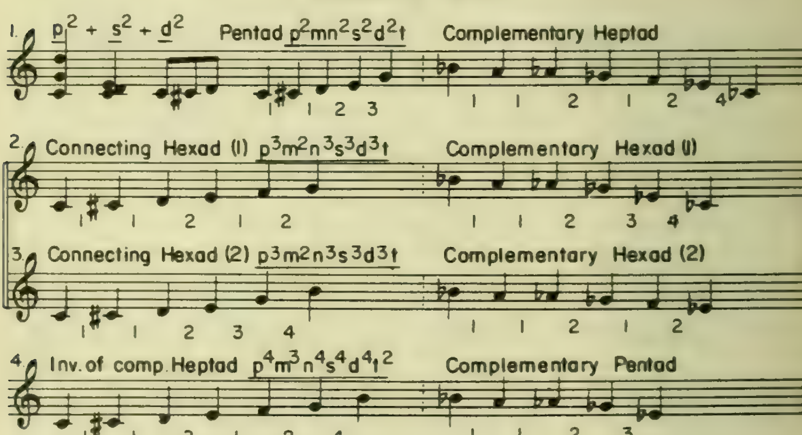




It will be noted that in Examples 9, 10, and 11, the minor third is projected both *up* and *down*, since in each case a new pentad results. It will also be observed that in all of these examples the combination of the heptads produces a minor-third nonad. In Example 12, however, only the involution of the first heptad results since the augmented triad is the same whether constructed up or down.

Finally, Example 45-13 shows the pentad formed by the simultaneous projection of two perfect fifths, two major seconds, and two minor seconds, with its seven-tone projection and connecting hexads.

Example 45-13



The hexads of Example 45-13 have already been discussed in Chapter 39, Examples 39-8, 9, 10, and 11. It will be noted again

that the complementary hexad of hexad (1) is the involution of hexad (2), and vice-versa.

Note: The projections $p^2 + s^2\downarrow$ and $d^2 + s^2\downarrow$ are not used since the former is the involution of $p^2 + s^3$ (Ex. 45-1, line 3), and the latter is the involution of d^2+s^3 (Ex. 45-2, line 3). Projections at m^2 are obviously the same whether projected up or down.

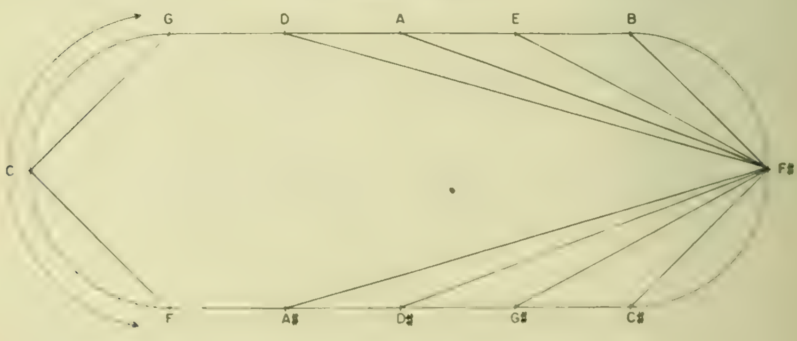
The opening of the first movement of the Sibelius Fourth Symphony, already referred to, furnishes a fine example of the projection illustrated in Example 45-1. The first six measures utilize the major-second pentad C-D-E-F♯-G♯. The seventh to the eleventh measures add the tones A, G, and B, forming the scale C-D-E-F♯-G-G♯-A-B, the projection of the tetrad C-D-E-G.

Projection by Involution with Complementary Sonorities

In chapter 34 we observed how isometric triads and pentads could be formed by simultaneous projection of intervals *above and below a given axis*. From this observation it becomes equally apparent that an isometric series, such as the projection of the perfect fifth, can be analyzed as a bidirectional projection as well as a superposition of intervals.

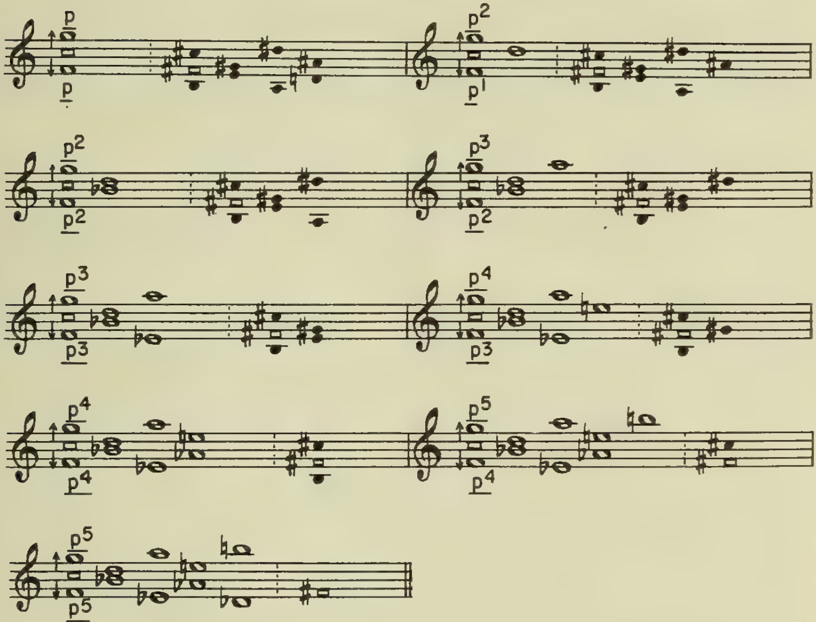
Example 46-1 illustrates this observation graphically. In order to make the illustration as clear as possible we have "stretched out" the circle to make an ellipse, placing C at one extreme and F♯ at the other. Now if we form a triad of perfect fifths by proceeding one perfect fifth *above* C and one perfect fifth *below* C, its complementary scale will be the nine-tone scale formed by the projection of the remaining tones above and below F♯ at the other extreme of the ellipse.

Example 46-1



Example 46-2 proceeds to illustrate the principle further by forming the entire perfect-fifth series above and below the axis C, the complementary scale in each case being the remaining tones above and below the axis of F♯.

EXAMPLE 46-2



It will be obvious that this principle may also be illustrated equally well by the projection of the minor-second scale above and below the starting tone.

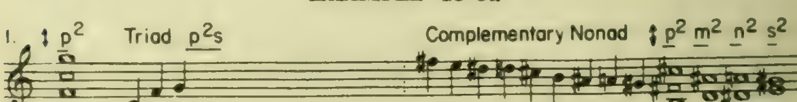
The projection of the basic series of the perfect fifth or the minor second by involution rather than by superposition does not, of course, add any new tonal material, but merely gives another explanation of the same material. However, if the projection is based upon the simultaneous involution of two *different* intervals, new and interesting sonorities and scales

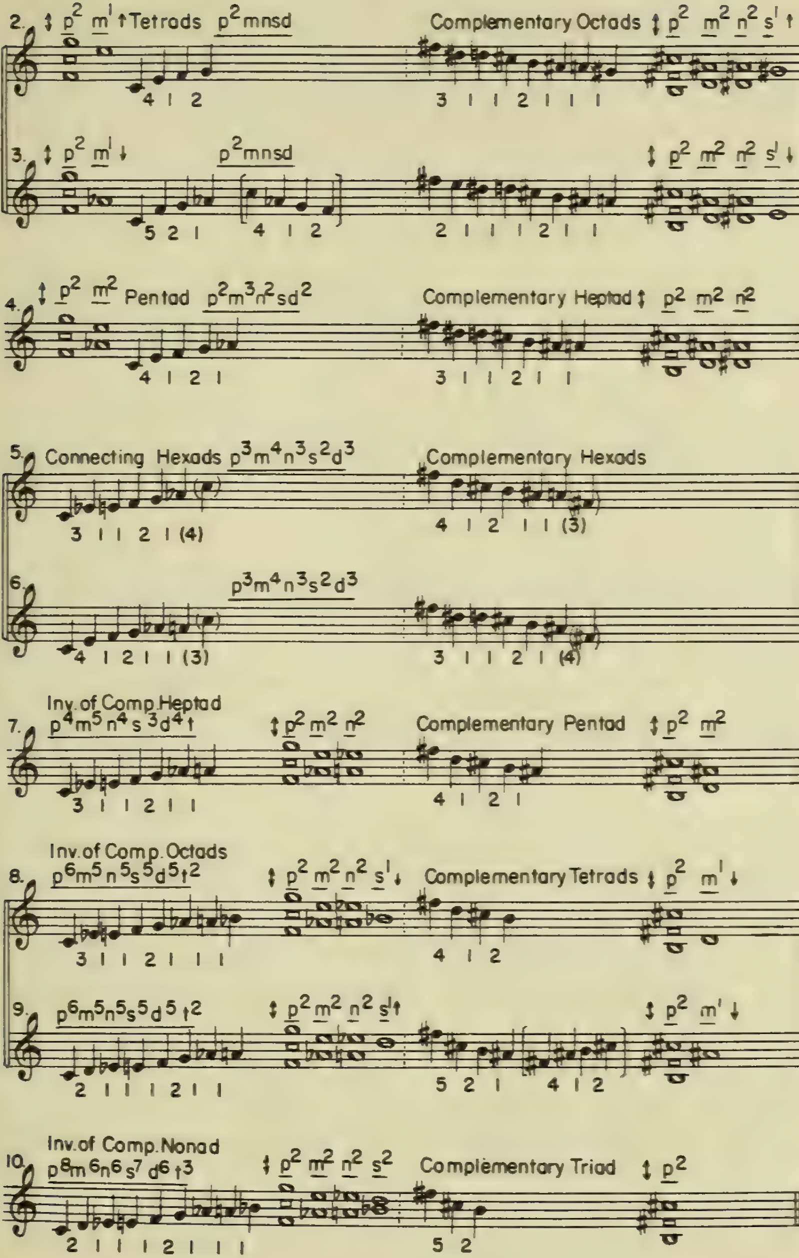
result. Example 46-3*a* shows the simultaneous projection by involution of the intervals of the perfect fifth and the major third above and below C.

The first line gives the perfect-fifth triad formed of a perfect fifth above and below C, with its complementary nine-tone scale arranged in the form of two perfect fifths, two major thirds, two minor thirds, and two major seconds above and below F♯. The second line adds the major third *above* C, with its complementary octad arranged in a similar manner, and the third line shows a perfect fifth above and below C, with a major third *below* C—the two tetrads being, of course, involutions of each other.

The fourth line gives the pentad formed of two perfect fifths and two major thirds above and below C, with its complementary heptad. Line 7 forms the projection of line 4 by the usual process of projecting upward the order of the complementary heptad of line 4, the tones of this scale being arranged as two perfect fifths, two major thirds, and two minor thirds above and below C. The right half of line 7 presents *its* complementary pentad arranged as two perfect fifths and two major thirds above and below F♯. Lines 5 and 6 give the connecting hexads between lines 4 and 7. Lines 8 and 9 form the octad projection by projecting upward the order of the complementary octads of lines 2 and 3, *their* complementary tetrads being the involutions of the original tetrads of lines 2 and 3. Line 10 forms the nonad which is the prototype of the original triad by projecting upward the complementary nonad of line 1. The complementary triad of this nonad is, of course, the same formation as the original triad of line 1.

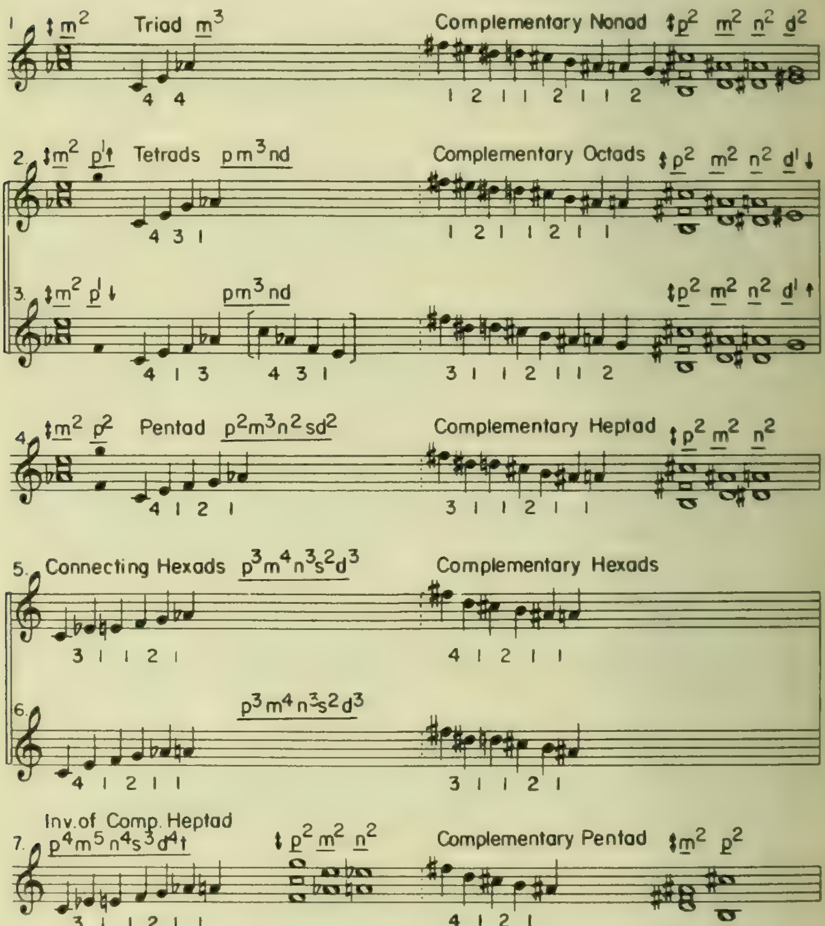
Example 46-3*a*

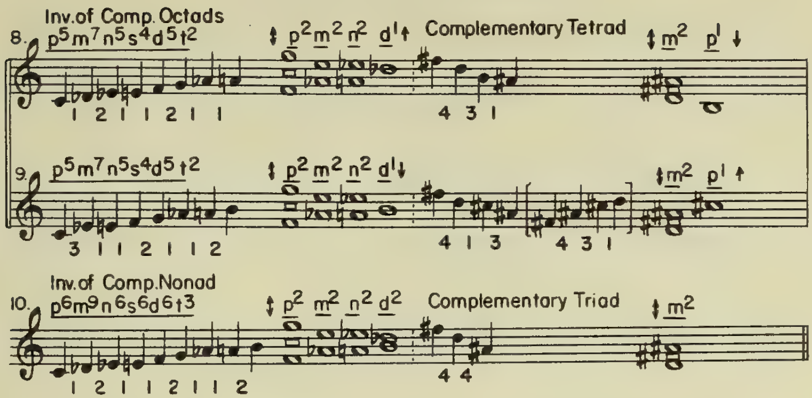



2. p2 m1 Tetrads p2mnsd
Complementary Octads p2 m2 n2 s1
4 1 2
3 1 1 2 1 1 1
3. p2 m1 p2mnsd p2 m2 n2 s1
5 2 1 4 1 2
2 1 1 1 2 1 1
4. p2 m2 Pentad p2m3n2sd2
Complementary Heptad p2 m2 n2
4 1 2 1
3 1 1 2 1 1
5. Connecting Hexads p3m4n3s2d3
Complementary Hexads
3 1 1 2 1 (4)
4 1 2 1 1 (3)
6. p3m4n3s2d3
4 1 2 1 1 (3)
3 1 1 2 1 (4)
7. Inv. of Comp. Heptad p4m5n4s3d4t p2 m2 n2
Complementary Pentad p2 m2
3 1 1 2 1 1
4 1 2 1
8. Inv. of Comp. Octads p6m5n5s5d5t2 p2 m2 n2 s1
Complementary Tetrads p2 m1
3 1 1 2 1 1 1
4 1 2
9. p6m5n5s5d5t2 p2 m2 n2 s1 p2 m1
2 1 1 1 2 1 1
5 2 1 4 1 2
10. Inv. of Comp. Nonad p8m6n6s7d6t3 p2 m2 n2 s2
Complementary Triad p2
2 1 1 1 2 1 1 1
5 2

Example 46-3*b* forms the projection of the same two intervals of the previous example in reverse, that is, two major thirds plus the perfect fifth rather than two perfect fifths plus the major third. The pentad, heptad and connecting hexads are, of course, the same, but the tetrads and octads are different.

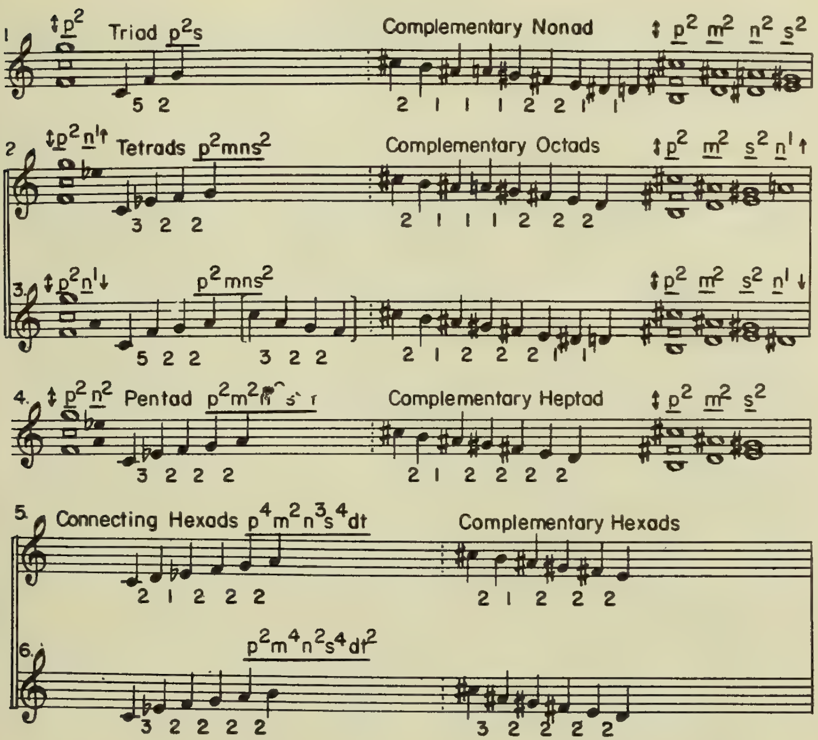
EXAMPLE 46-3*b*

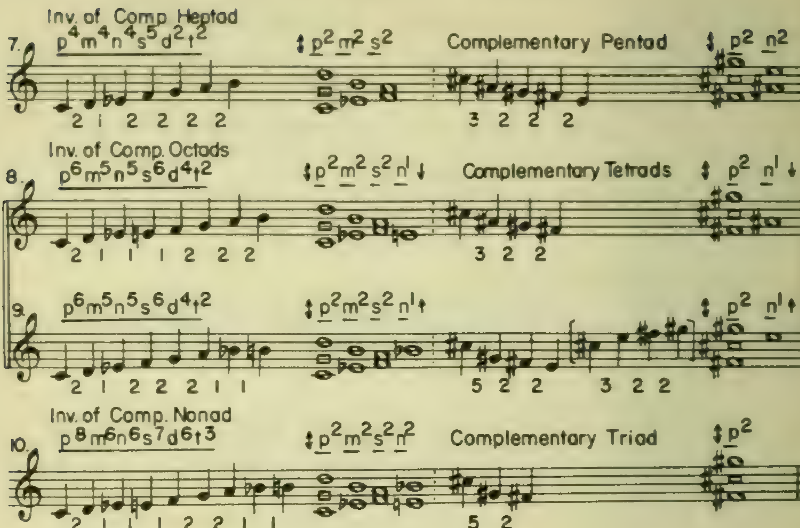




Example 46-4*a* continues the same process for the relationship of the perfect fifth and the minor third;

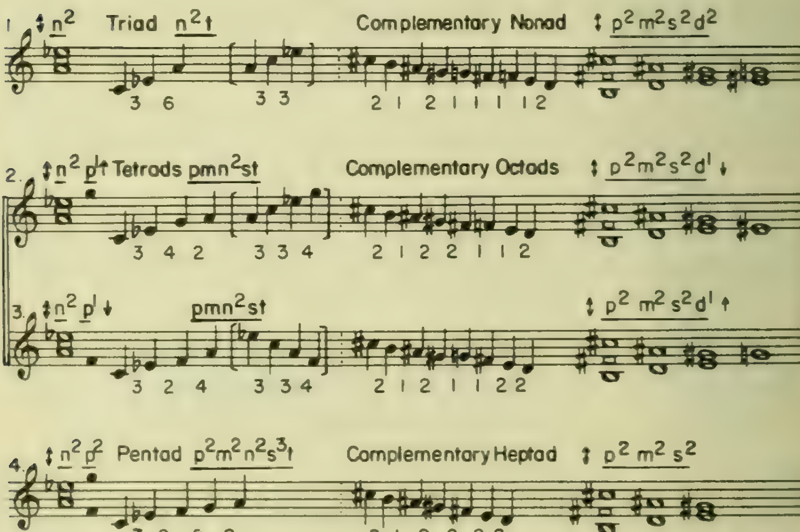
EXAMPLE 46-4*a*

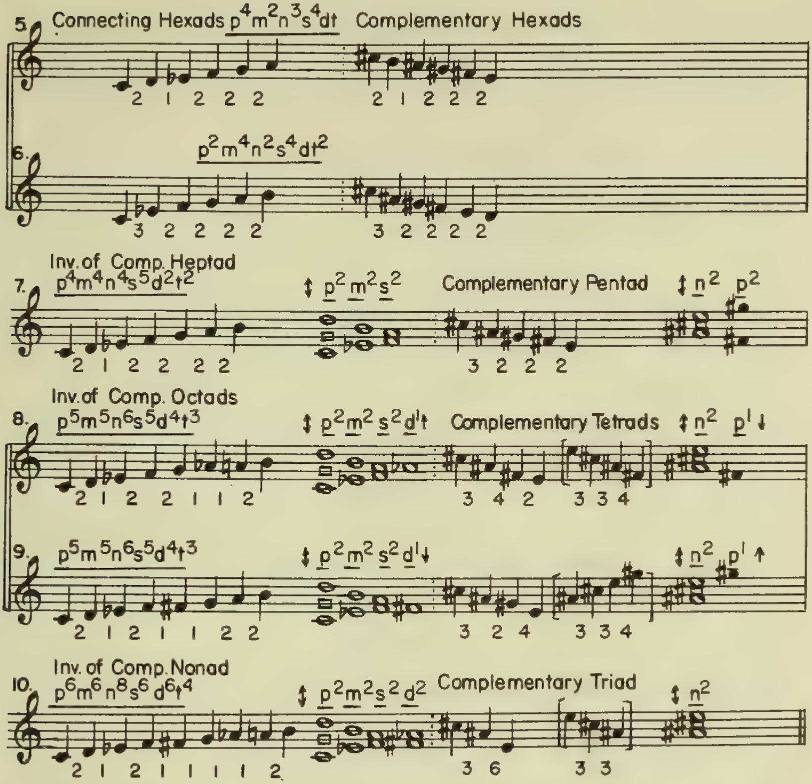




and Example 46-4*b* gives the reverse relationship—the minor third plus the perfect fifth:

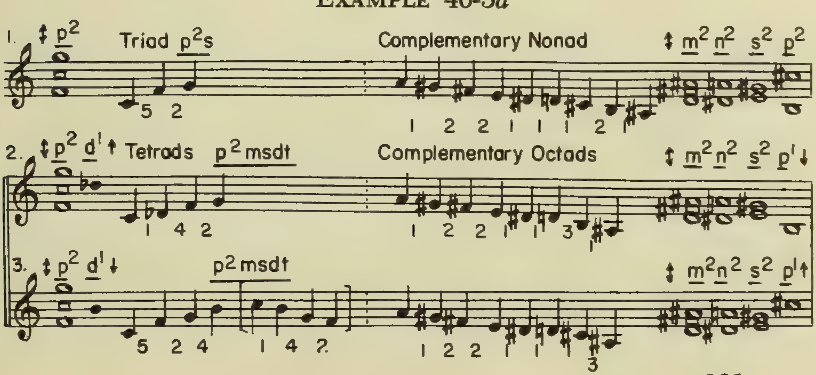
EXAMPLE 46-4*b*

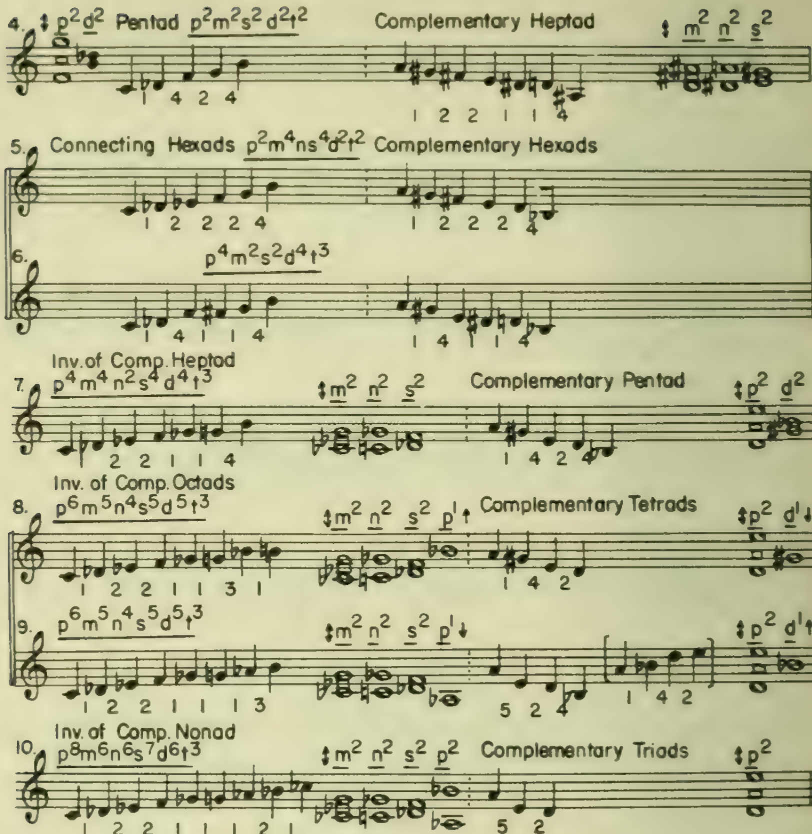




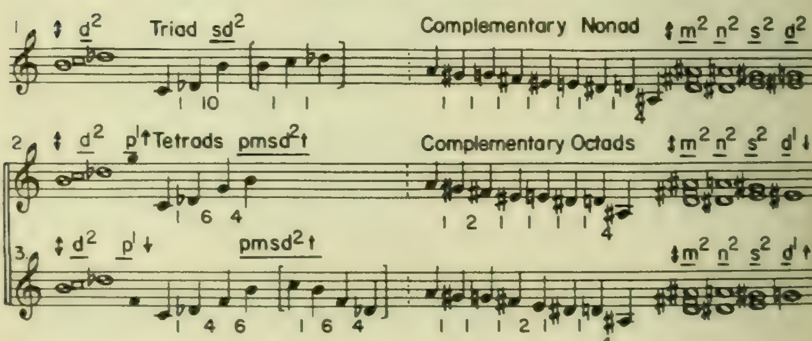
Example 46-5*a* gives the vertical projection of the perfect fifth and the minor second, and Example 46-5*b* the reverse relationship:

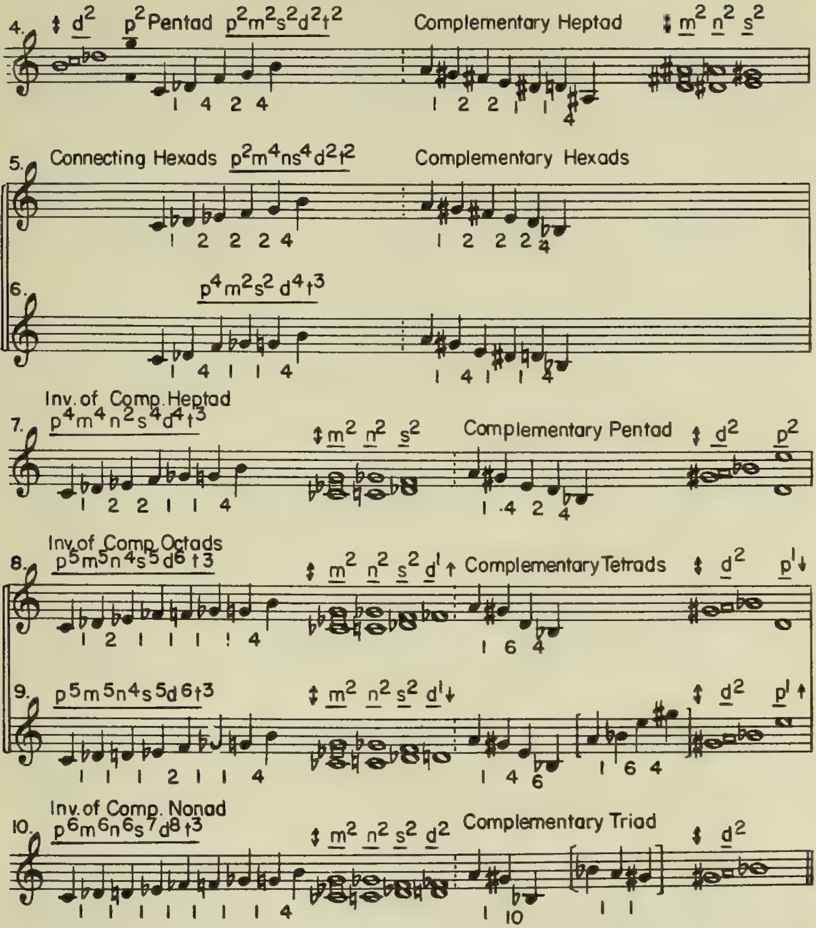
EXAMPLE 46-5*a*





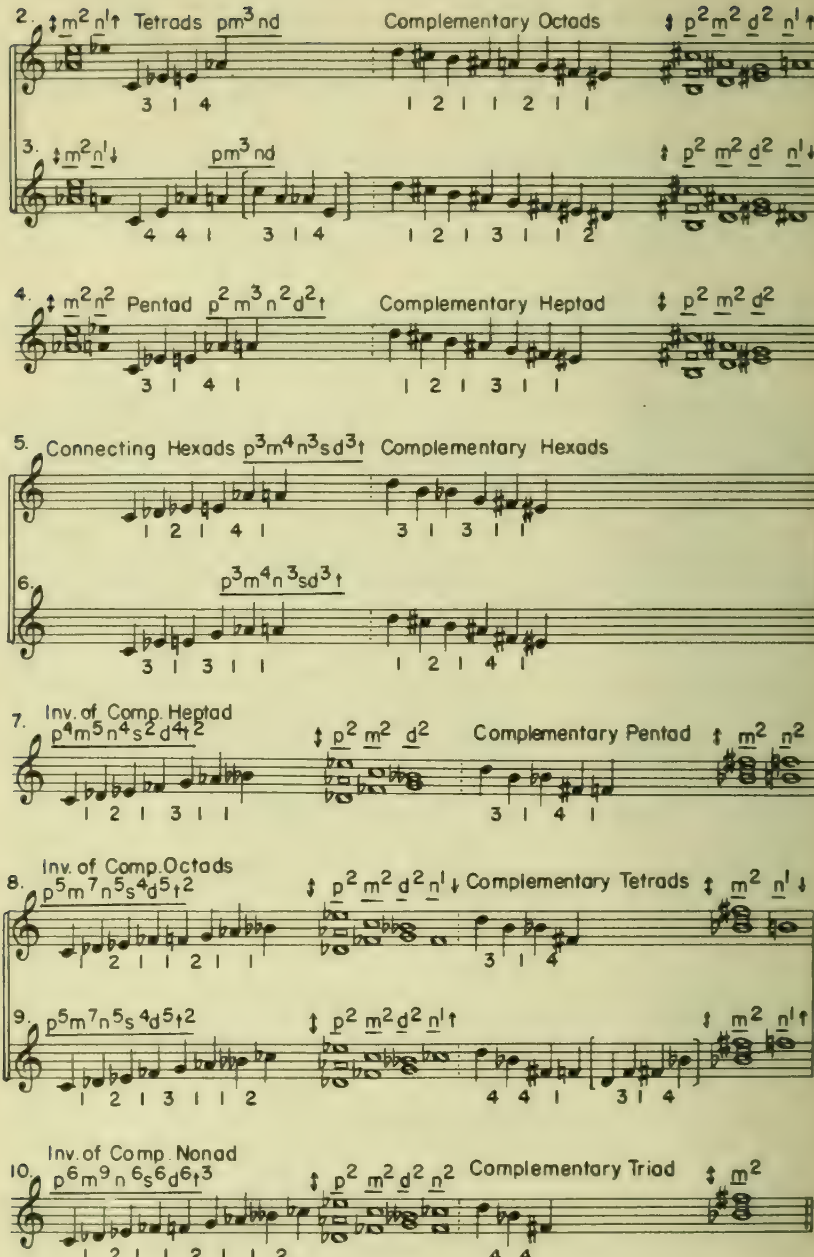
Example 46-5b



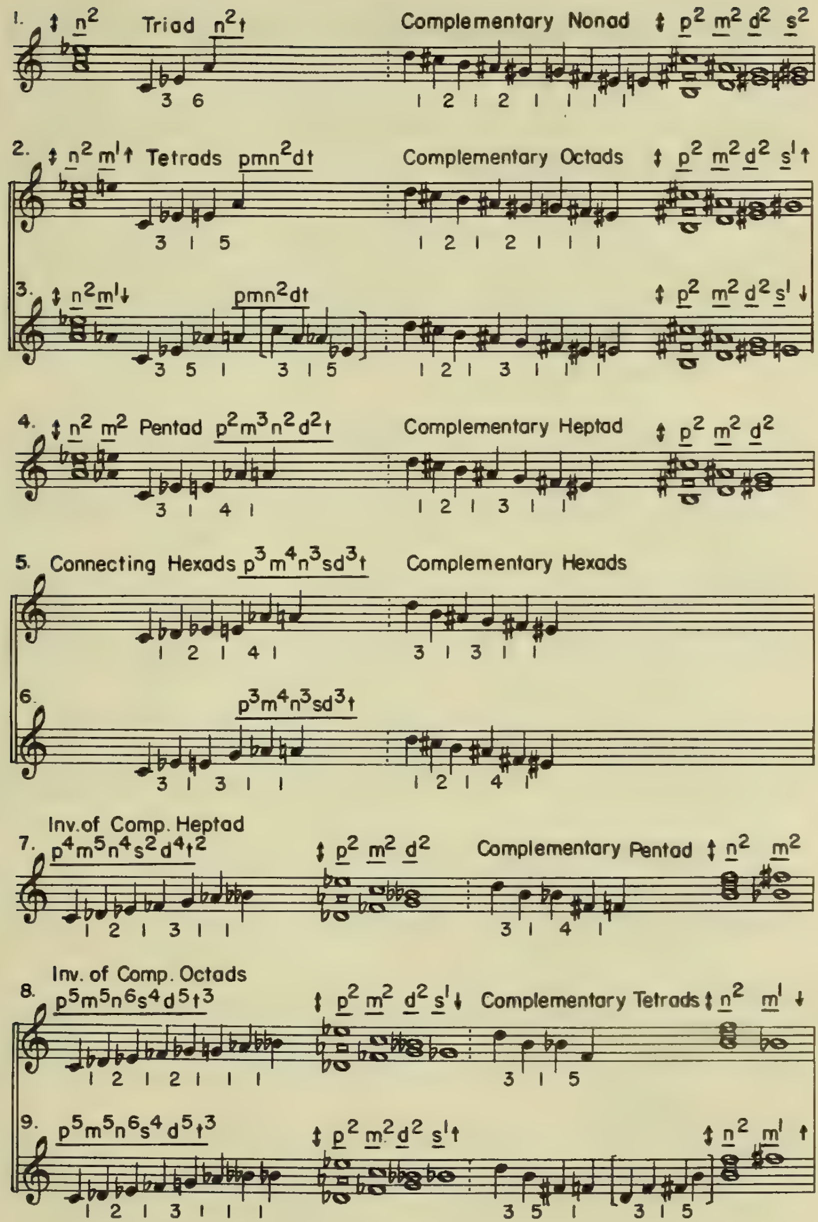


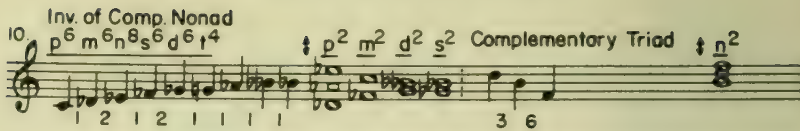
Example 46-6*a* presents the relationship of the major third and minor third, and 46-6*b* presents the reverse relationship:

Example 46-6*a*


2. m²n¹ Tetrads pm³nd
Complementary Octads
p² m² d² n¹
3 1 4
1 2 1 1 2 1 1
3. m²n¹
pm³nd
p² m² d² n¹
4 4 1 3 1 4
1 2 1 3 1 1 2
4. m² n² Pentad p² m³ n² d² t
Complementary Heptad
p² m² d²
3 1 4 1
1 2 1 3 1 1
5. Connecting Hexads p³m⁴n³sd³t Complementary Hexads
1 2 1 4 1
3 1 3 1 1
6. p³m⁴n³sd³t
3 1 3 1 1
1 2 1 4 1
7. Inv. of Comp. Heptad
p⁴m⁵n⁴s²d⁴t²
p² m² d²
Complementary Pentad
m² n²
1 2 1 3 1 1
3 1 4 1
8. Inv. of Comp. Octads
p⁵m⁷n⁵s⁴d⁵t²
p² m² d² n¹ Complementary Tetrads
m² n¹
1 2 1 1 2 1 1
3 1 4
9. p⁵m⁷n⁵s⁴d⁵t²
p² m² d² n¹
m² n¹
1 2 1 3 1 1 2
4 4 1 3 1 4
10. Inv. of Comp. Nonad
p⁶m⁹n⁶s⁶d⁶t³
p² m² d² n²
Complementary Triad
m²
1 2 1 1 2 1 1 2
4 4

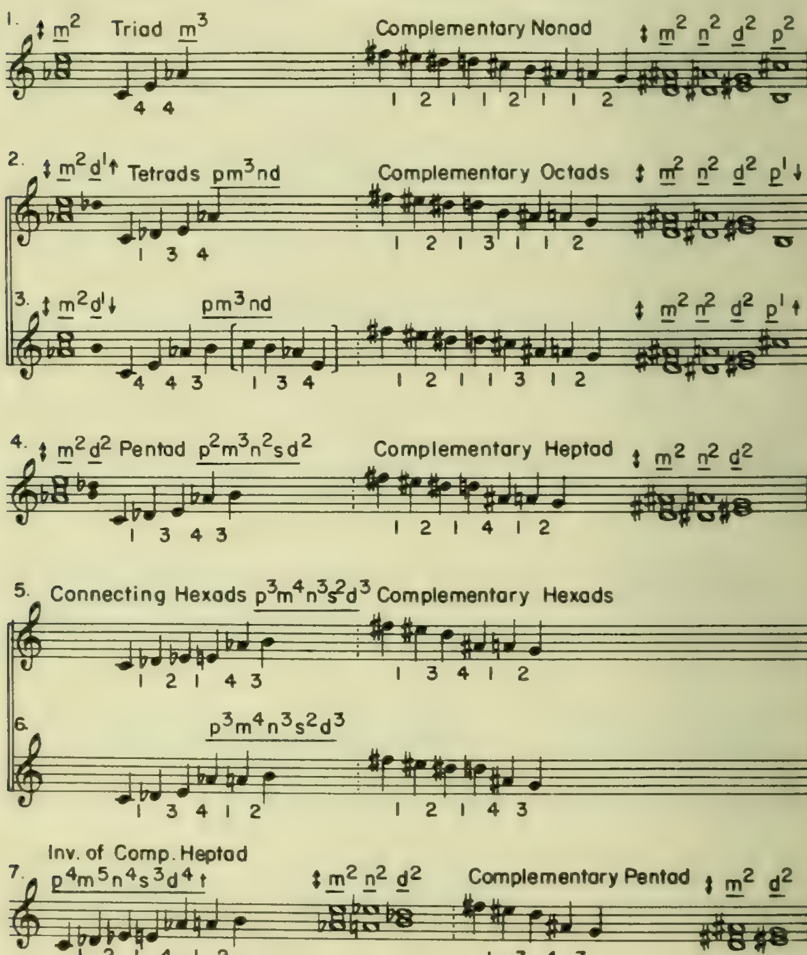
EXAMPLE 46-6*b*

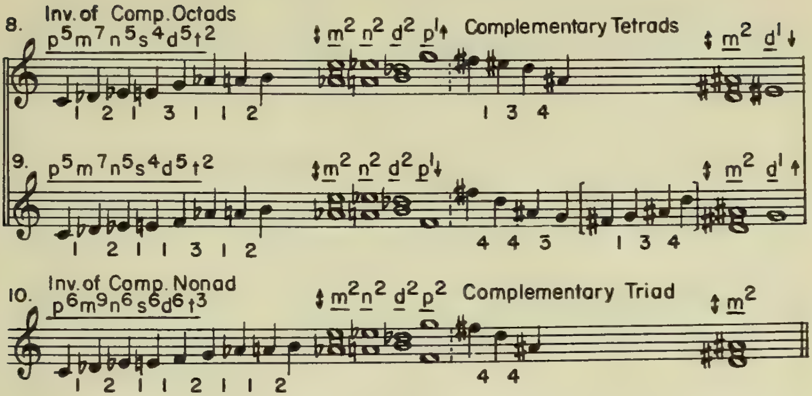




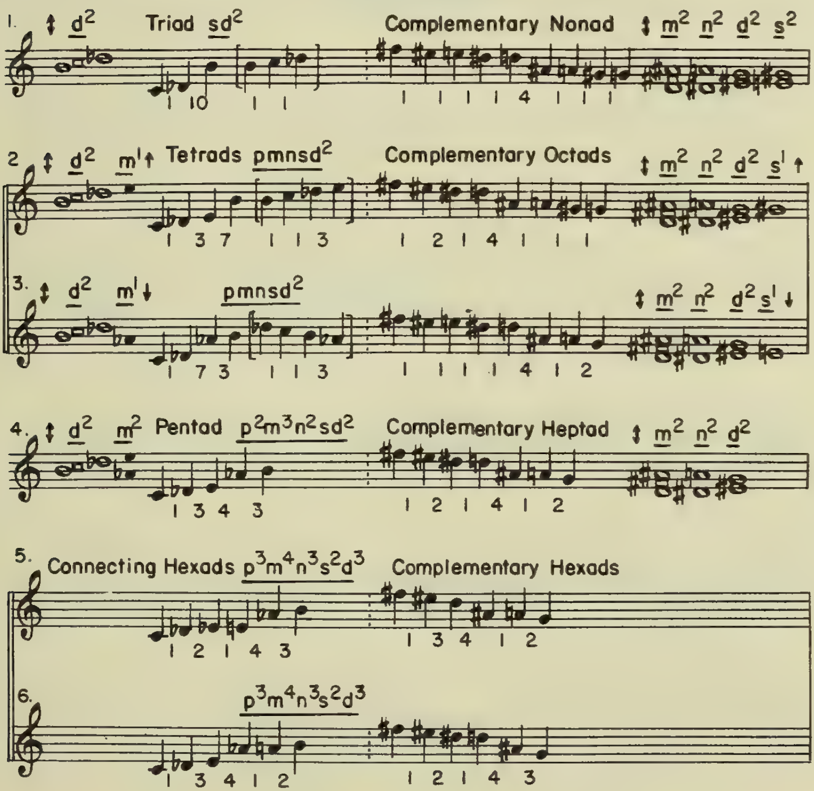
Examples 46-7*a* and 46-7*b* show the vertical projection of the major third and minor second:

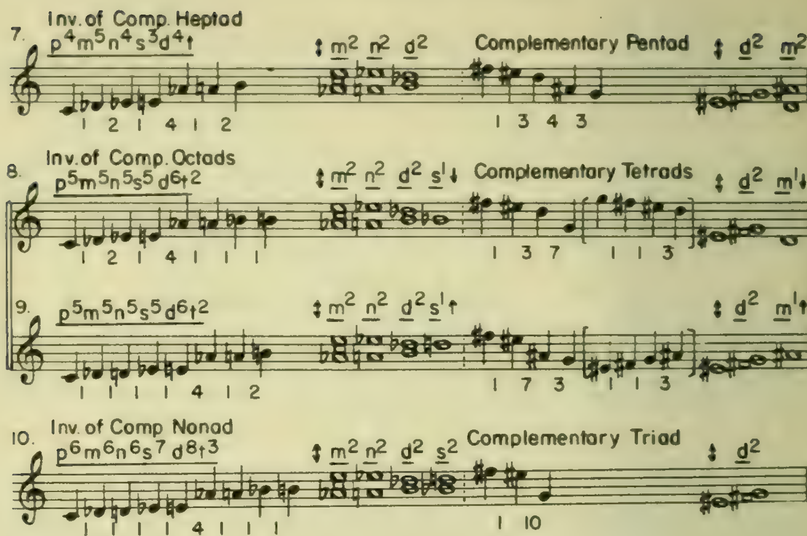
Example 46-7*a*





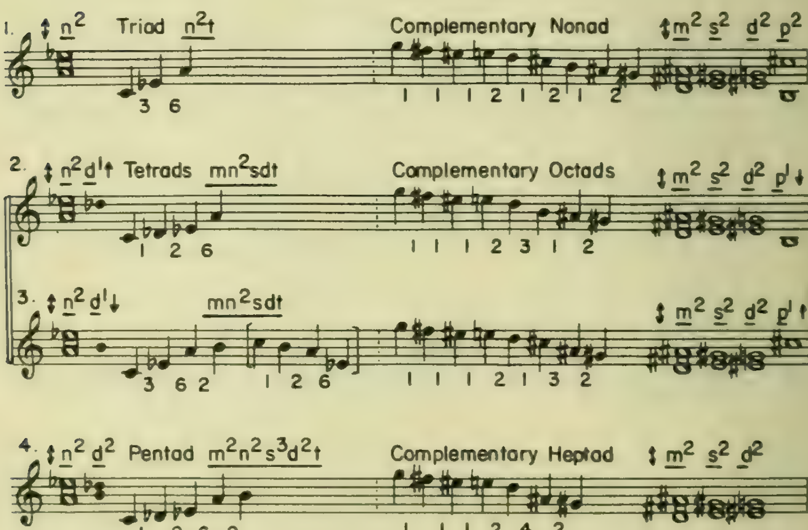
EXAMPLE 46-7*b*

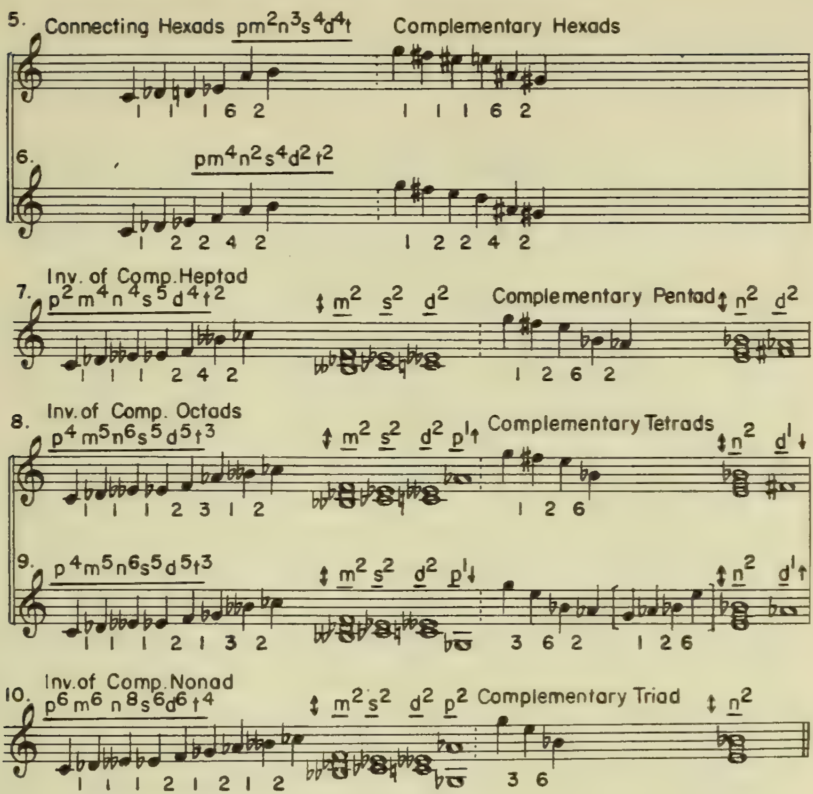




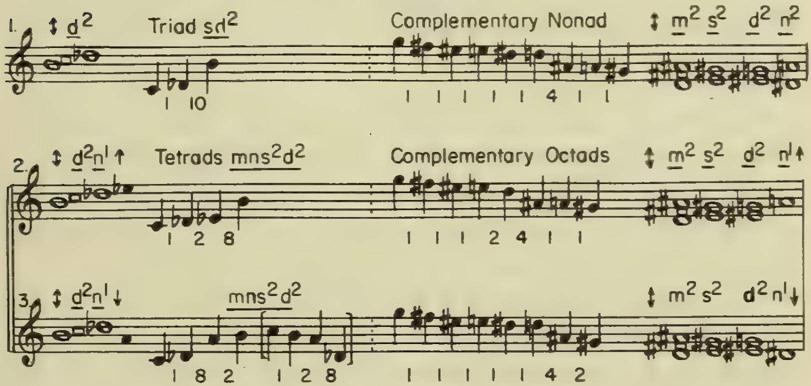
Examples 46-8*a* and 46-8*b* give the vertical projection of the minor third and the minor second:

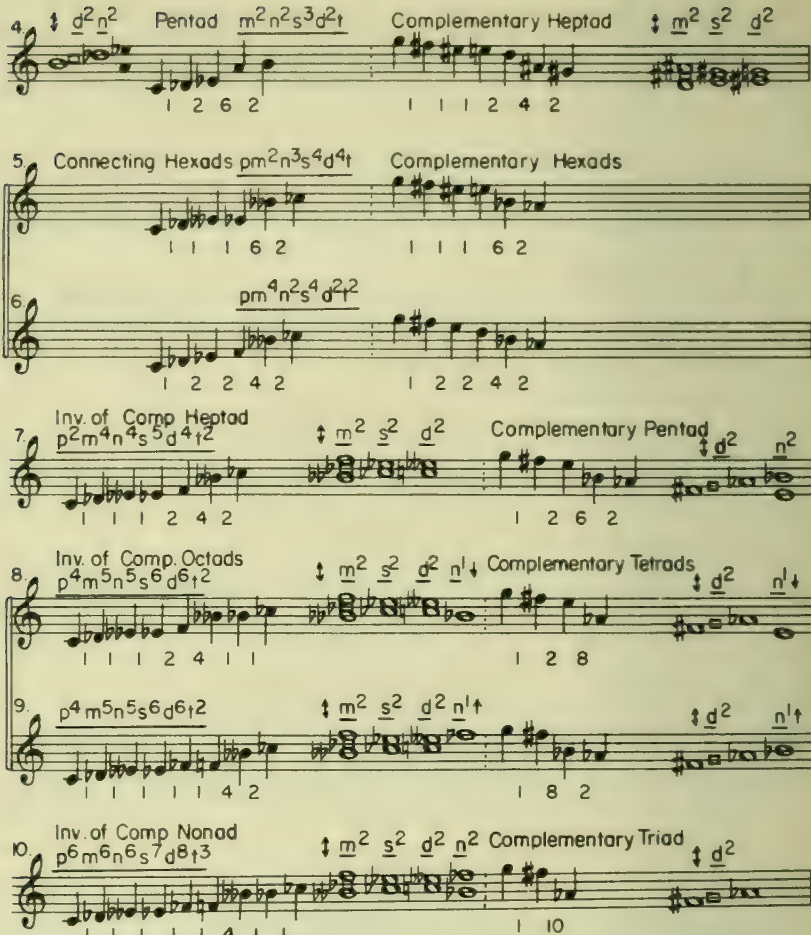
Example 46-8*a*





Example 46-8b





The vertical projection of the perfect fifth and major second duplicates the perfect-fifth series; the combination of the major second and the major third duplicates the major-second series; and the vertical projection of the minor second and major second duplicates the minor-second series.

The vertical projection of the minor third and major second results in a curious phenomenon which will be discussed in the following chapter.

47

The "Maverick" Sonority

THE VERTICAL PROJECTION of the minor third and major second forms a sonority which can be described only as a "maverick," because it is the only sonority in all of the tonal material of the twelve-tone scale which is *not itself a part of its own complementary scale.* It is, instead, a part of the "twin" of its own complementary scale. Because of its unique formation, we should examine it carefully.

In Example 47-1, line 1 gives the tone C with the minor third and major second above and below it. The second half of line 1 forms the descending complementary scale, beginning on G♯ and containing the remaining seven tones which are not a part of the original pentad, arranged both as a melodic scale and as two perfect fifths, two major seconds, and two minor seconds—one above and one below the tone F♯.

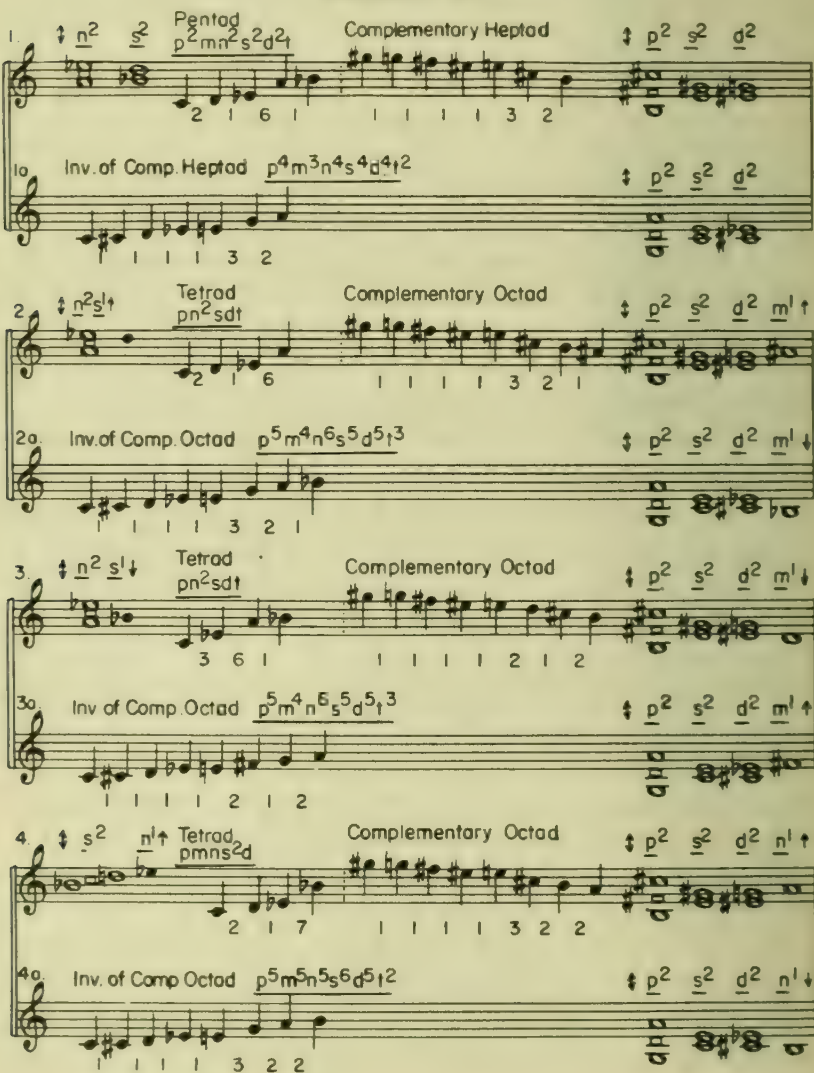
In line 1*a* we follow the usual process of projecting upward from C the order of the complementary heptad, producing the scale $C_1C\sharp_1D_1E\flat_1E\natural_3G_2A$—also arranged as two perfect fifths, two major seconds, and two minor seconds, one above and one below the tone D. We find, however, that the original pentad of line 1 is *not a part of its corresponding heptad* (line 1*a*). There can therefore be no connecting hexads.

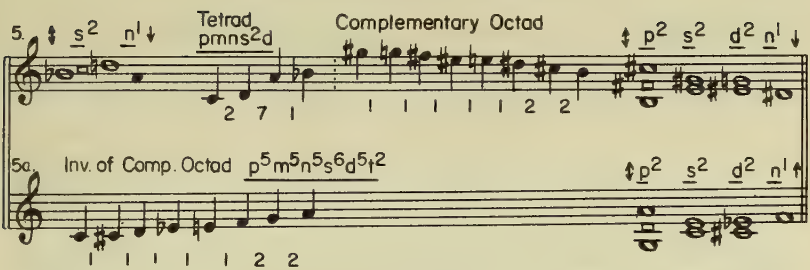
Line 2 gives the tetrad $C_2D_1E\flat_6A$ with its complementary octad, while line 2*a* forms the octad projection. Lines 3 and 3*a* give the tetrad $C_3E\flat_6A_1B\flat$ with its octad projection. Lines 4 and 4*a* form the projected octad of the tetrad $C_2D_1E\flat_7B\flat$, and lines

5 and 5*a* form the projected octad of the tetrad $C_2D_7A_1B\flat$.

The tetrads in lines 2 and 3 will be seen to be involutions, one of the other. In the same way, the tetrads of lines 4 and 5 form involutions of each other.

EXAMPLE 47-1

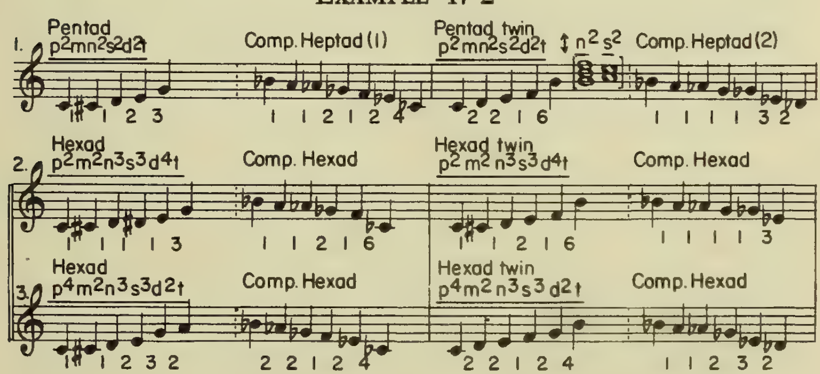


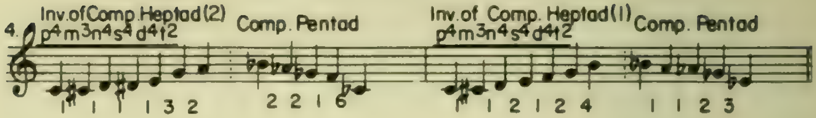


Example 47-2 shows the relationship of the pentad of the previous illustration to its twin, the pentad C-C♯-D-E-G, which has the same intervallic analysis, $p^2mn^2s^2d^2t$. The first line gives the two pentads, each with its complementary heptad. Line 4 gives the involution of the two complementary heptads but with the order *interchanged*, the *first* heptad of line 4 being the involution of the *second* complementary heptad of line 1, and vice versa. The "maverick" pentad C-D-E-F-B will be seen to be a part of the complementary scale of its "twin"—second part of line 4. The first pentad, C-C♯-D-E-G, will be seen to be a part both of its own related heptad and the related heptad of its maverick twin.

The connecting hexads also show an interesting relationship, the first connecting hexad of line 2 being the "twin" of the second connecting hexad of line 2; and the first connecting hexad of line 3 being the twin of the second connecting hexad of line 3.

EXAMPLE 47-2



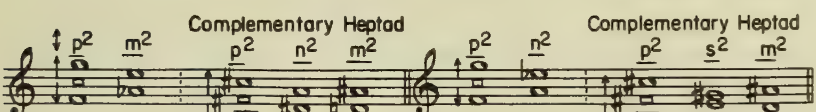

4.
Inv. of Comp. Heptad (2)
p4 m3 n4 s4 d4 t2
Comp. Pentad
Inv. of Comp. Heptad (1)
p4 m3 n4 s4 d4 t2
Comp. Pentad

48

Vertical Projection by Involution and Complementary Relationship

There is a type of relationship which occurs when intervals are projected by involution, as described in the previous two chapters, which explains the formation of the hexad "quartets" described in Chapter 39. If we compare in Example 48-1 the projection of two perfect fifths and two *major* thirds, one below and one above the tone C, together with its complementary heptad, with a similar projection of perfect fifths and *minor* thirds, together with its complementary heptad, we shall notice a very interesting difference.

Example 48-1



The complementary heptad of

$$\begin{matrix} \uparrow G & E \\ C & \\ \downarrow F & A\flat \end{matrix} \quad ,$$

that is, a perfect fifth and major third above and below C, is

$$\begin{matrix} \uparrow C\sharp & A & A\sharp \\ F\sharp & & \\ \downarrow B & D\sharp & D\natural \end{matrix},$$

which forms a perfect fifth, a minor third, and a major third above and below F♯. The complementary heptad of

$$\begin{matrix} \uparrow G & E\flat \\ C & \\ \downarrow F & A \end{matrix},$$

a perfect fifth and a minor third above and below C, is

$$\begin{matrix} \uparrow C\sharp & G\sharp & A\sharp \\ F\sharp & & \\ \downarrow B & E & D \end{matrix},$$

which forms a perfect fifth, major second, and major third above and below F♯.

In other words, the projection of $\updownarrow p^2m^2$ is $\updownarrow p^2m^2n^2$, whereas the projection of $\updownarrow p^2n^2$ is $\updownarrow p^2m^2s^2$. In the first pentad, the vertical projection of p and m is a part both of its own complementary heptad and of the complementary heptad of the vertical projection of p and n. In the case of the second pentad, however, the vertical projection of p and n is *not a part* of the vertical projection of its own complementary heptad, *but is a part* of the vertical projection of the complementary heptad of the pentad $\updownarrow p^2m^2$, that is, $\updownarrow p^2m^2n^2$.

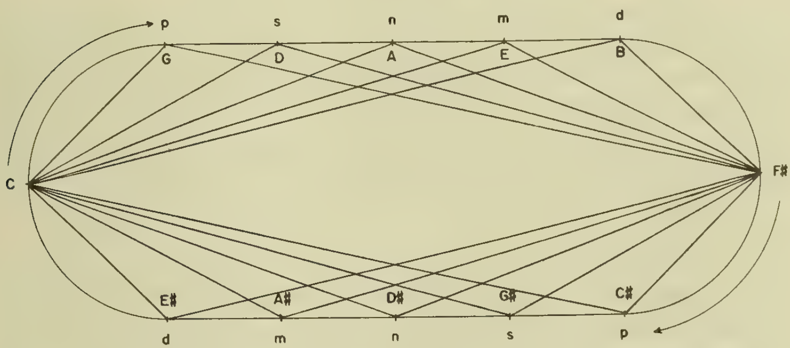
This phenomenon makes possible a fascinating "diagonal" relation between pentads and heptads formed by vertical projection, resulting in quartets of connecting hexads all of the members of which have the same intervallic analysis. In each case the "quartet" consists of two hexads having differing formations but with the same intervallic analysis, each with its own involution. (See Chapter 39.)

If the student will re-examine the material contained in Chapter 46, he will observe that the same phenomenon which

we have just observed in the vertical projection of the projection p^2n^2 also occurs in the vertical projections of p^2d^2, m^2n^2, m^2s^2, and n^2d^2. We have already discussed in detail in Chapter 47 the peculiarities of the vertical projection of n^2s^2.

The reason for this phenomenon becomes clear if we examine Example 48-2. Here again we have the circle of perfect fifths "stretched out" with C at one extreme of the ellipse and F♯ at the other. The letters *p, s, n, m,* and *d* at the top of the figure represent the intervals which the tones G, D, A, E, and B, and the tones F, B♭, E♭, A♭ and D♭, form above and below the tone C; while the letters *d, m, n, s,* and *p* below the figure represent the relationshhip of the tones E♯, A♯, D♯, G♯ and C♯, and the tones G, D, A, E, and B, below and above the tone, F♯.

EXAMPLE 48-2



Now if we project the intervals of the perfect fifth and the major third above and below the tone C, the remaining tones, which constitute the complementary heptad, become the perfect fifth, major third, and minor third above and below F♯. However, if we project the perfect fifth and the minor third above and below C, the complementary projection above and below F♯ becomes the perfect fifth, major second, and major third. Hence it becomes obvious that the projection of the minor third above and below the axis, C, cannot be found in the complementary scale above and below the axis, F♯, since the minor

third above and below C are the same tones as the minor third below and above F♯.

There follows the list of pentads formed by the projection of two intervals above and below the axis C, with their complementary heptads arranged above and below the axis F♯:

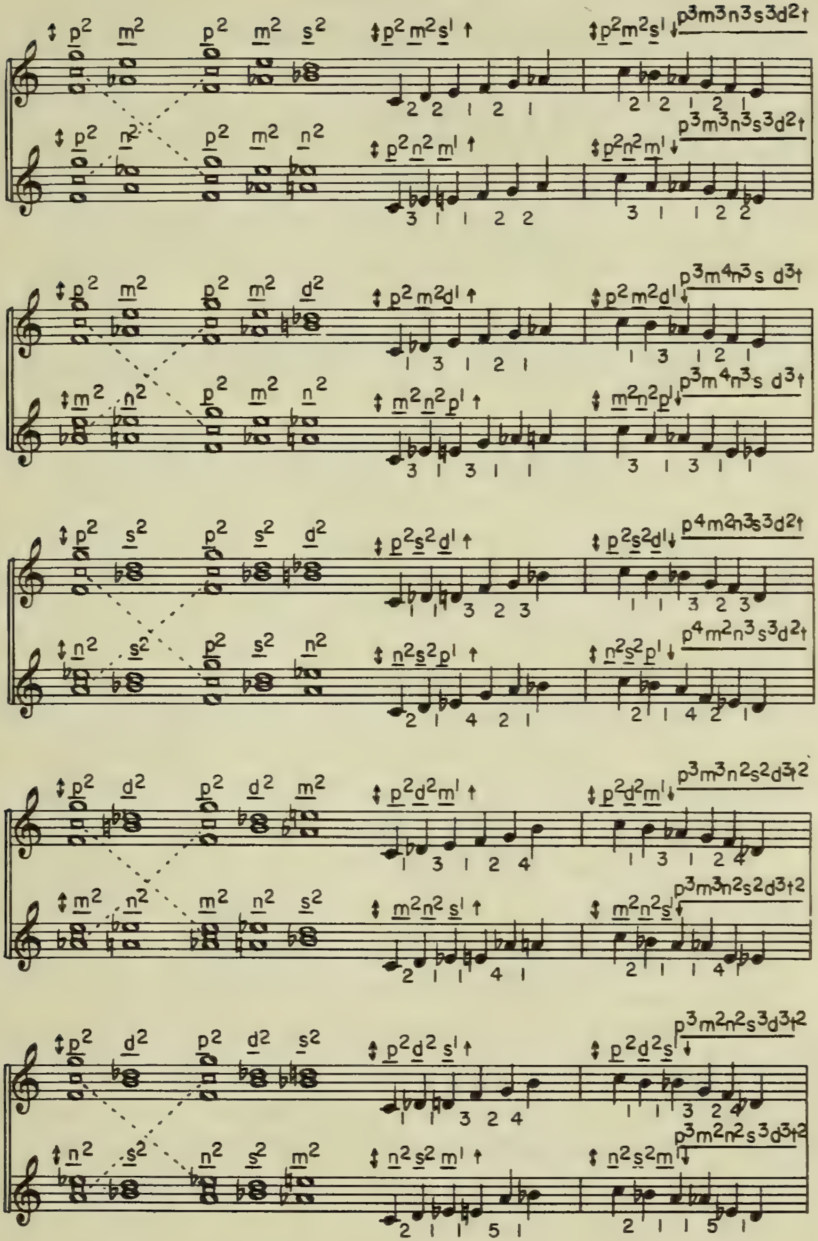
↕ p^2s^2	....	↕ $p^2s^2n^2$
p^2n^2	....	$p^2s^2m^2$
p^2m^2	....	$p^2n^2m^2$
p^2d^2	....	$s^2n^2m^2$
s^2n^2	....	$p^2s^2d^2$
s^2m^2	....	$p^2n^2d^2$
s^2d^2	....	$s^2n^2d^2$
n^2m^2	....	$p^2m^2d^2$
n^2d^2	....	$s^2m^2d^2$
m^2d^2	....	$n^2m^2d^2$

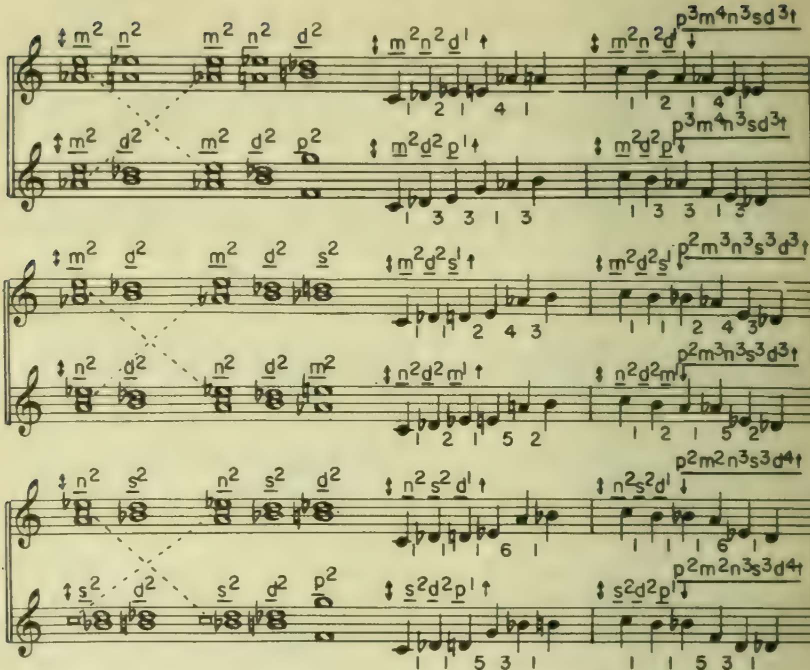
It will be noted that in four of the ten possible projections, the pentad contains the *same* vertical projection as its complementary heptad. In six of the projections, the heptad does *not* contain the vertical projection of the same intervals as its pentad prototype.

Example 48-3 works out all of the relationships based on this principle which result in the formation of the hexad "quartests." Lines 1 and 2 give the two pentads formed by the vertical projections p^2m^2 and p^2n^2. The heptad of line 1 is the projection of the pentad of line 2, while the heptad of line 2 is the projection of the pentad of line 1, as indicated by the dotted lines. The four connecting hexads, upon examination, prove to have the same intervallic analysis, the second hexad of line 1 being the involution of the first hexad of line 1; and the second hexad of line 2 being the involution of the first hexad of line two; the four together constituting a quartet having the same intervallic analysis.

All of the other hexads in this illustration are formed on the same principle and each quartet of scales has the same analysis.

EXAMPLE 48-3





There remains only one other group of hexads to be considered, the isometric twins discussed in Part III. Example 48-4 indicates that these sonorities may be considered as part of a projection from a tetrad to its related octad. Line 1*a* gives the tetrad formed by the projection of two minor thirds and a perfect fifth above C. Line 1*b* gives the isometric twins, the first formed by the simultaneous projection of three minor thirds and three perfect fifths, and the second formed by the relation of two minor thirds *at the interval of* the perfect fifth. The combination of these two hexads forms the octad of line 1*c*, which is the projection of the tetrad of line 1*a*.

Line 2*a* is similar in construction to line 1*a* except that the perfect fifth is projected *below* C. Line 2*b* is similar to line 1*b* except that in the first isometric twin the perfect fifths are

projected *below* C, and the second twin is formed of two minor thirds at the interval of the perfect fifth *below* C. (It will be observed that the twins of line 2*b* are merely different *versions* of those of line 1*b* since, if the order of the first twin in line 1*b* is begun on A, it will be seen to contain the same intervals as the first twin of line 2*b*: $A_3C_2D_1E\flat_3G\flat_1G\natural$. In the same way, if the order of the second twin of line 1*b* is begun on G, it will duplicate the intervals of the second twin of line 2*b*: $G_3B\flat_2C_1D\flat_2E\flat_3G\flat$.)

Line 2*c* is the octad formed by the combination of the hexads of line 2*b* and proves to be the projection of the tetrad of line 2*a*.

In similar manner, lines 3*a*, 3*b*, and 3*c* show the projection of the tetrad formed of two minor thirds and a major third above C, while 4*a*, 4*b*, and 4*c* show the projection of the tetrad formed of two minor thirds above C and a major third *below* C.

Lines 5*a*, 5*b*, and 5*c* explore the projection of two minor thirds and a major second above C, while lines 6*a*, 6*b*, and 6*c* show the projection of two minor thirds above and a major second *below* C.

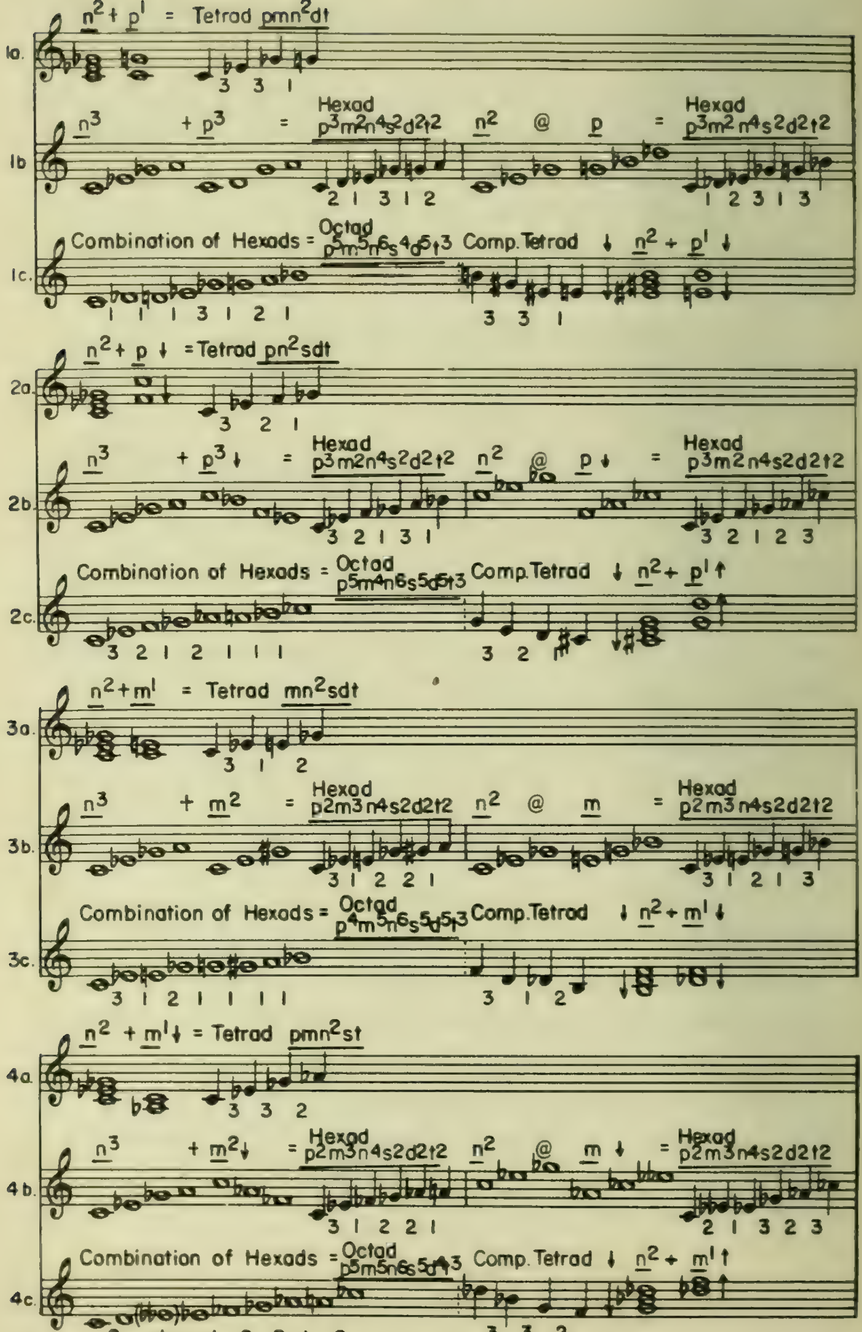
Lines 7*a*, 7*b*, and 7*c* and lines 8*a*, 8*b*, and 8*c* are concerned with the projection of two minor thirds and a minor second.

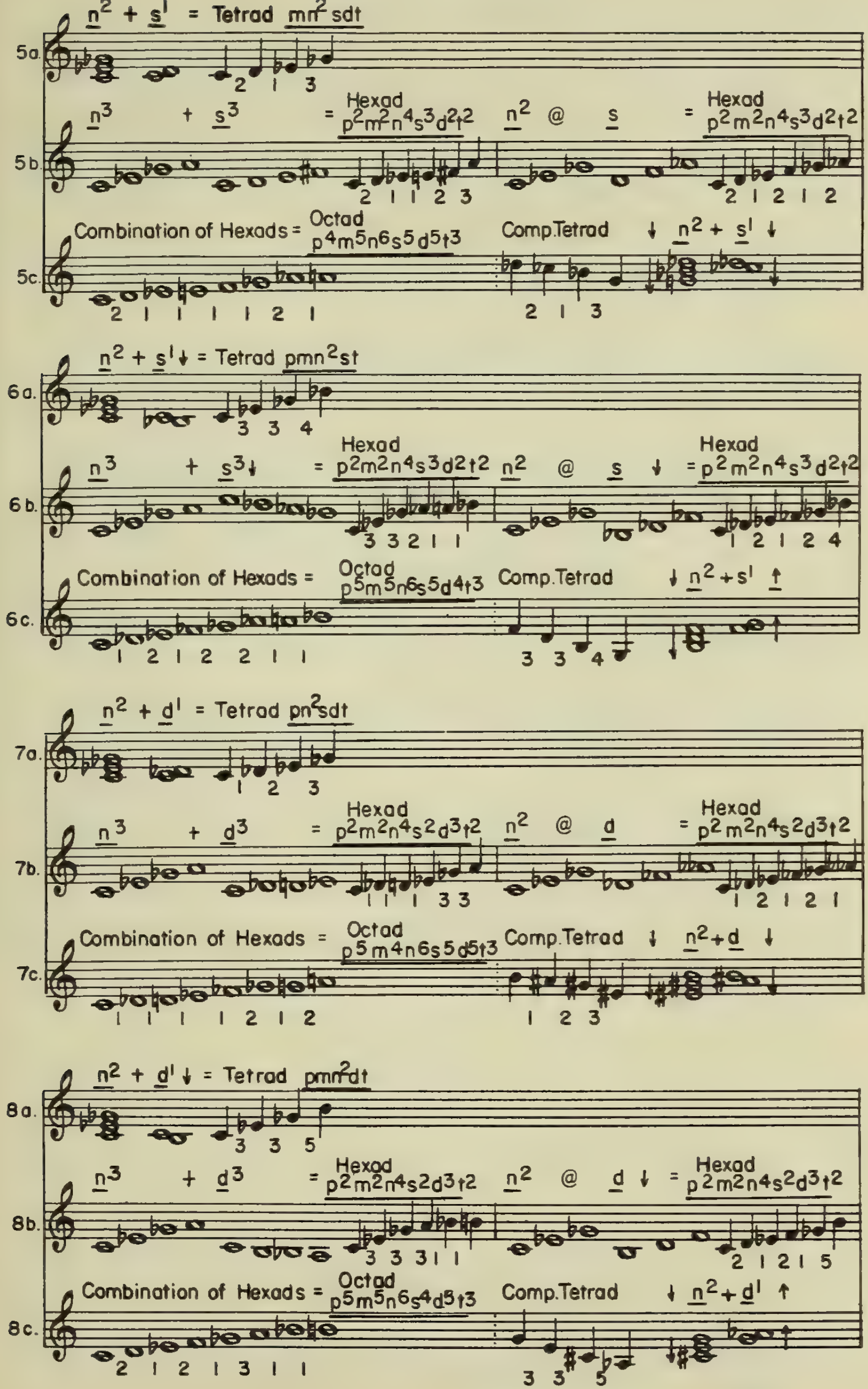
Lines 9*a*, 9*b*, and 9*c* and lines 10*a*, 10*b*, and 10*c* concern the projection of two perfect fifths and a major third.

Lines 11*a*, 11*b*, and 11*c* and lines 12*a*, 12*b*, and 12*c* show the projection of two minor seconds and a major third.

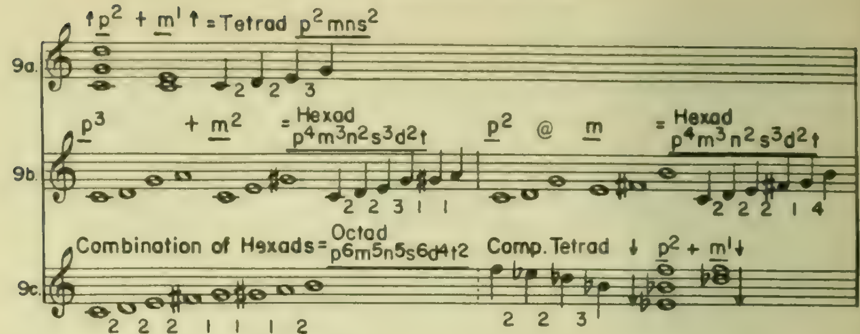
The relation of two perfect fifths and a minor second, or of two minor seconds and a perfect fifth, does not follow the same pattern. It is interesting, however, to observe in lines 13*a* and 13*b* that the combination of the hexads p^2 @ d and d^2 @ p form a seven-tone scale which is the involution of the basic perfect-fifth–tritone heptad.

EXAMPLE 48-4

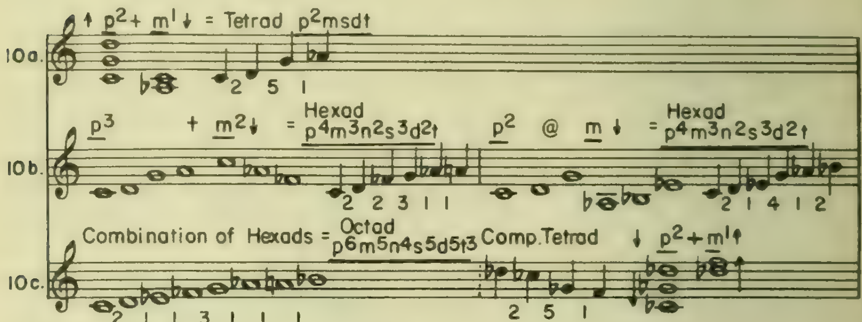

n2 + p1 = Tetrad pmn2dt
n3 + p3 = Hexad p3m2n4s2d2t2
n2 @ p = Hexad p3m2n4s2d2t2
Combination of Hexads = Octad
Comp. Tetrad
n2 + p = Tetrad pn2sdt
Combination of Hexads = Octad p5m4n6s5d5t3
n2 + m1 = Tetrad mn2sdt
n3 + m2 = Hexad p2m3n4s2d2t2
n2 @ m = Hexad p2m3n4s2d2t2
Combination of Hexads = Octad p4m5n6s5d5t3
n2 + m1 = Tetrad pmn2st
Combination of Hexads = Octad



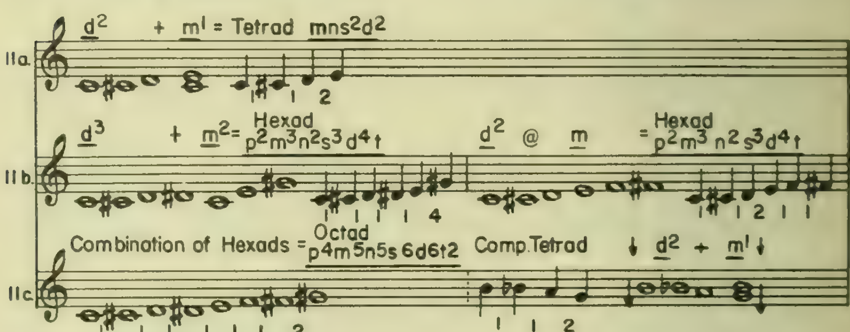
n2 + s1 = Tetrad mn2sdt
5a.
2 1 3
n3 + s3 = Hexad p2m2n4s3d2t2
n2 @ s = Hexad p2m2n4s3d2t2
5b.
2 1 1 2 3
2 1 2 1 2
Combination of Hexads = Octad p4m5n6s5d5t3
Comp.Tetrad n2 + s1
5c.
2 1 1 1 1 2 1
2 1 3
n2 + s1 = Tetrad pmn2st
6a.
3 3 4
n3 + s3 = Hexad p2m2n4s3d2t2
n2 @ s = Hexad p2m2n4s3d2t2
6b.
3 3 2 1 1
1 2 1 2 4
Combination of Hexads = Octad p5m5n6s5d4t3
Comp.Tetrad n2 + s1
6c.
1 2 1 2 2 1 1
3 3 4
n2 + d1 = Tetrad pn2sdt
7a.
1 2 3
n3 + d3 = Hexad p2m2n4s2d3t2
n2 @ d = Hexad p2m2n4s2d3t2
7b.
1 1 1 3 3
1 2 1 2 1
Combination of Hexads = Octad p5m4n6s5d5t3
Comp.Tetrad n2 + d
7c.
1 1 1 1 2 1 2
1 2 3
n2 + d1 = Tetrad pmn2dt
8a.
3 3 5
n3 + d3 = Hexad p2m2n4s2d3t2
n2 @ d = Hexad p2m2n4s2d3t2
8b.
3 3 3 1 1
2 1 2 1 5
Combination of Hexads = Octad p5m5n6s4d5t3
Comp.Tetrad n2 + d1
8c.
2 1 2 1 3 1 1
3 3 5



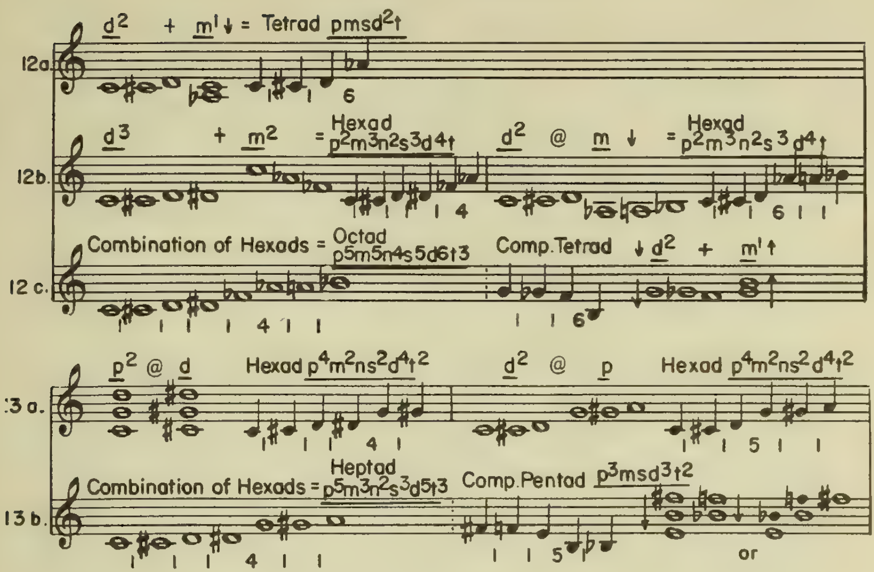
9a.
↑p2 + m1 ↑ = Tetrad p2mns2
2 2 3
9b.
p3 + m2 = Hexad p4m3n2s3d2t
2 2 3 1 1
p2 @ m = Hexad p4m3n2s3d2t
2 2 2 1 4
9c.
Combination of Hexads = Octad p6m5n5s6d4t2
2 2 2 1 1 1 2
Comp. Tetrad ↓ p2 + m1↓
2 2 3



10a.
↑ p2 + m1 ↓ = Tetrad p2msdt
2 5 1
10b.
p3 + m2↓ = Hexad p4m3n2s3d2t
2 2 3 1 1
p2 @ m ↓ = Hexad p4m3n2s3d2t
2 1 4 1 2
10c.
Combination of Hexads = Octad p6m5n4s5d5t3
2 1 1 3 1 1 1
Comp. Tetrad ↓ p2 + m1↑
2 5 1



11a.
d2 + m1 = Tetrad mns2d2
1 1 2
11b.
d3 + m2 = Hexad p2m3n2s3d4t
1 1 1 1 4
d2 @ m = Hexad p2m3n2s3d4t
1 1 2 1 1
11c.
Combination of Hexads = Octad p4m5n5s6d6t2
1 1 1 1 1 1 2
Comp. Tetrad ↓ d2 + m1 ↓
1 1 2



Note: The tetrads of Example 48-4 have all been discussed in Chapter 46 as projection by involution. For example, tetrad 1*a* of Example 48-4, ($n^2 + p^1$), is the same chord as the tetrad of Example 46-6*b*, line 2, ($\updownarrow n^2m^1\uparrow$), and is itself the involution of tetrad 8*a* of Example 48-4, ($n^2 + d\downarrow$), which appears in Chapter 46-6*b*, line 3, as $\updownarrow n^2m^1\downarrow$.

49

Relationship of Tones in Equal Temperament

We come finally to the formidable but fascinating task of attempting to show the relationship of these galaxies of tones within the system of equal temperament. The most complete presentation, and in many ways the most satisfactory, would seem to be that involving the abstract symbolism which I have employed in the large diagram accompanying this text.

Although this symbolism may at first glance seem foreign to the musician's habit of thinking tones only through the symbolism of written notes, and may, therefore, seem "mathematical" rather than musical, it has the great advantage of presenting a graphic, all-embracing picture of tone relationship divorced from the artificial and awkward complexity of musical notation.

For example, the symbol p^2s^2 indicates the simultaneous projection of two perfect fifths and two major seconds on *any* tone, *up or down*, and in any position. This one symbol therefore represents the sonority C-D-E-G in any of its four positions: C-D-E-G, D-E-G-C, E-G-C-D, and G-C-D-E, together with its involution ↓C-B♭-A♭-F, in its four positions: C-B♭-A♭-F, B♭-A♭-F-C, A♭-F-C-B♭, and F-C-B♭-A♭, plus the transposition of these sonorities to the other eleven tones of the chromatic scale. The one symbol therefore represents ninety-six sonorities. The presentation of such a chart *using musical notation* would assume a size beyond the realm of the practical. It should be noted that the order of half-steps of this sonority, represented in the chart as 223(5)-C-D-E-G-(C)—may also appear in the ver-

sions 235(2), 352(2) or 522(3); and in involution as 322(5), 225(3), 253(2), or 532(2).

I cannot overemphasize the statement which has reappeared in different forms throughout this text that my own concern is not with symbolism but with *sound*. The symbols are a means to an end, a device to aid in clarity of thinking. They have value to the composer only if they are associated with sound. To me the symbol p^2s^2 represents a very beautiful sound having many different connotations according to its position, doubling, and relationship with other sounds which precede and follow it.

One other word of caution should be added before we take off into the vast realm of tonal space which the chart explores. The student who has worked his way slowly and perhaps painfully through the preceding chapters cannot fail to be impressed, not only with the vast number of possibilities within the chromatic scale, but also with the subtleties involved in the change or the addition of one tone. He may feel overwhelmed both by the amount and the complexity of the material available to him in the apparently simple chromatic scale, and wonder how any one person can possibly arrive at a complete assimilation of this material in one lifetime.

The answer, of course, is that he cannot. For if a composer were to have a complete aural comprehension of all of the tonal relationships here presented, he would know more than all of the composers of occidental music from Bach to Bartok combined. This would be a formidable assignment for any young composer and should not be attempted in a one-year course!

The young composer should use this study rather as a means of broadening his tonal understanding and gradually and slowly increasing his tonal vocabulary. He may find one series of relationships which appeals to his esthetic tastes and set about absorbing this material until it becomes a part of himself. He will then speak in this "new" language as confidently, as naturally, and as communicatively, as Palestrina wrote in his idiom, providing, of course, that he has Palestrina's talent.

One of the greatest curses of much contemporary music is that it uses a wide and complicated mass of undigested and unassimilated tonal material. The end result becomes tonal chaos not only to the listener but, I fear, often to the composer himself. The complete assimilation of a small tonal vocabulary used with mastery is infinitely to be preferred to a large vocabulary incompletely understood by the composer himself.

Let us now turn to an examination of the large chart in the pocket of this text. Beginning at the extreme right-hand lower corner we find the letters *p*, *d*, *s*, *n*, *m*, and *t*, symbolizing the six basic intervals: the perfect fifth or perfect fourth, the minor second or major seventh, the major second or minor seventh, the minor third or major sixth, the major third or minor sixth, and the augmented fourth or diminished fifth.

Below each of the letters you will find a number of crosses, 5 under *p*, 5 under *d*, 6 under *s*, 5 under *n*, 6 under *m*, and 3 under *t*. These crosses serve as abbreviations of the interval symbol, that is, every cross under the letter *p* represents that interval. A cross indicates that the interval, of which the symbol appears at the top of the vertical column, is included in the triad, of which the symbol appears to the left of the horizontal line in which the cross is located.

Proceeding *laterally to the left* we find the section of the chart devoted to triad formations, III. Here, again, the crosses represent abbreviations of the triad symbols. In other words, each cross laterally on the line with the triad symbol p^2s represents the triad p^2s. The same thing is true of the crosses marking the positions of triads *pns*, *pmn*, *pmd*, and so forth. These triads are divided by dotted lines into groups—the first four all contain the perfect fifth; the next three all contain the minor second; ms^2 is the basic major-second triad; n^2t is the basic minor-third triad; m^3 is the basic major-third triad; and the last two triads are those in which the interval of the tritone predominates. The numbers to the right of the triad symbols indicate the order of half-steps which form this triad in its basic

position—p^2s above the tone C becomes $C_2D_5G_5$(C), having the order of half-steps 25(5). Each cross in this section of the chart indicates that the triad, whose symbol appears at the left of the horizontal line, is included in the tetrad identified by the symbol at the top of the vertical column in which the cross occurs.

Proceeding *upward* from the triads, we find immediately above them the section of the chart devoted to tetrads, IV. Here again the crosses represent the tetrad symbol proceeding *vertically downward.* The tetrad P^3, for example, will be found below the symbol on the first, second, fourth, and fifth spaces of the chart.

For the sake of space the interval analysis of the tetrad is given as six numbers, without the interval letters *p, m, n, s, d,* and *t.* The numbers to the right of the interval analysis represent again the order of the sonority in half-steps. The tetrad P^3 should therefore be read: three perfect fifths, having the analysis 301,200, three perfect fifths, no major thirds, one minor third, two major seconds, no minor seconds, and no tritones; the order of half-steps being 252(3), that is, above C; $C_2D_5G_2A_{(3)}$ (C). Each cross in this section of the chart indicates that the tetrad, whose symbol appears at the top of the vertical column, is included in the pentad identified by the symbol at the extreme left of the horizontal column in which the cross occurs.

Proceeding *laterally and to the left* we come to the section of pentads, V, which occupies the large lower left-hand section of the chart. Here, again, the crosses indicate the pentad on the same *lateral* line. The pentad P^4, for example, is found on the first, second, fourth, and sixth spaces of the *lateral* line following the symbol P^4 This pentad has the analysis 412,300, and the order of half-steps 2232(3), which might be represented by the tones C-D-E-G-A-(C). Each cross in this section of the chart indicates that the pentad, whose symbol appears at the left of the horizontal line, is included in the hexad identified by the symbol at the top of the vertical column in which the cross occurs.

The six-tone scales, or hexads, VI, will be found *above* the

pentads and forming a connection between the pentads below and the heptads above. The crosses, again, indicate of which hexads the individual pentads below are a part. The pentad P^4 will be seen to be a part of the hexads P^5, *pns*, $\updownarrow p^2s^2d^1$, and p^3m^3. P^5 has the analysis 523,410, indicating the presence of five perfect fifths, two major thirds, three minor thirds, four major seconds, one minor second, and no tritones. It has the indicated order of half-steps 22322(1), which would give the scale, above C, of the tones $C_2D_2E_3G_2A_2B_{(1)}(C)$.

The portion of the chart *above* the hexads gives the heptads, VII. These scales are the involutions of the complementary scales of the pentads below and are so indicated by the letter "C." The heptad VII p^6 is, therefore, the corresponding scale of the pentad V P^4. The scale C, *pns/s*, corresponds to the pentad *pns/s*, the heptad C, *pmn/p*, corresponds to the pentad *pmn/p*, and so forth. (*Pns/s* is used as an abbreviated form of the symbol, *pns* @ *s*.) Here each cross in this section of the chart indicates that the heptad, whose symbol appears at the left of the horizontal column, *contains* the hexad identified by the symbol below the vertical column in which the cross occurs.

Proceeding now *laterally to the right* we find the octads, VIII, above the tetrads. These scales are all the corresponding scales of the tetrads below, so that it is not necessary to repeat the symbol, but only to give the intervallic analysis and the order of half-steps. For example, the corresponding scale to the tetrad, P^3, is the octad opposite, with the analysis 745,642 and the order 1122122(1), giving the scale, above C, of $C_1C\sharp_1D_2E_2F\sharp_1G_2A_2B_{(1)}(C)$.

Proceeding *vertically upward* to the top of the chart are the nonads, IX, which are the counterparts of the triads at the bottom of the chart.

Proceeding *horizontally to the right*, we find the relationship between the nine-and ten-tone scales. It will be observed that the six ten-tone scales which are on the upper right hand of the chart are the counterparts of the six intervals which are repre-

sented at the lower right hand portion of the chart.

At first glance, this chart may seem to be merely an interesting curiosity, but careful study will indicate that it contains a tremendous amount of factual information regarding tone relationship. For example, the relation of two-tone, three-tone, four-tone, and five-tone sonorities to their corresponding ten-, nine-, eight-, and seven-tone scales will be discovered to be exact. If we begin with the pentads on the left of the chart and, reading down, we add 2 to the number of intervals present in each sonority—except in the case of the last figure, the tritone, where we add one-half of two, or one—we automatically produce the intervallic composition of the sonority's corresponding heptad. For example, the first pentad has the intervallic analysis 412,300. If we add to this the number 222,221, we produce 634,521, which will be found to be the analysis of the corresponding heptad. The second pentad has the analysis 312,310. Adding to this the intervals 222,221, we produce the analysis 534,531, which is the analysis of the heptad C. *pns/s*. In like manner, the analyses of all of the heptads may be produced directly from that of their corresponding pentads.

Proceeding further, we have already pointed out that the tetrads and octads have a corresponding relationship. This may be expressed arithmetically by adding to the intervallic analysis of the tetrad four of each interval, except the tritone, where we again add half of four, or two. The analysis of the four-tone perfect-fifth chord we observe to be 301,200. Adding to this 444,442, we produce 745,642, which proves to be the analysis of the corresponding octad. The second tetrad, p^2s^2, has the analysis 211,200. Adding the intervals 444,442, we produce 655,642, which proves to be the analysis of the corresponding octad. This is true, again, of all tetrad-octad relationships.

The triad-nonad relationship is expressed by the addition to the triad analysis of six of each interval except the tritone, where the addition is one-half of six, or three. The first triad at the bottom of the chart is p^2s or, expressed arithmetically, 200,100.

Adding to this 666,663, we produce 866,763, which will be found to be the analysis of the corresponding nine-tone scale at the top of the chart. The triad *pns*, 101,100, becomes in its nine-tone relationship 101,100 plus 666,663, or 767,763, and so forth.

The single interval may be projected to its ten-tone counterpart by the addition of eight of each interval, *p, m, n, s,* and *d*, and four tritones. The decad projection of the perfect fifth therefore becomes 100,000 plus 888,884, or 988,884. The projection of the major third becomes 898,884; of the minor third, 889,884, and so forth.

Since this chart is of necessity biaxial, it may take some practice to read it accurately. If we begin with the interval of the fifth, *p*, at the lower right hand of the chart we find by proceeding laterally to the left that it is contained in five triads p^2s, *pns*, *pmn*, *pmd*, and *pdt*. Conversely, we find that the perfect-fifth triad, p^2s, contains the intervals *p* and *s*. Proceeding now upward from the triads to the tetrads we find that the triad p^2s is contained in the tetrads p^3, p^2s^2, $p^2m^1\updownarrow$, $p^2d^1\updownarrow$, and p^2d^2. Conversely the perfect-fifth tetrad p^3 will be seen to contain the triads p^2s and *pns*.

Proceeding *laterally to the left*, from the tetrads to the pentads, we observe that the tetrad P^3 is found in the pentads P^4, *pns/s*, $\uparrow p^2n^2\downarrow$, and p^3d^2. Conversely, the pentad P^4 contains the tetrads P^3, p^2s^2, and *p/n*.

Proceeding *upwards*, from the pentads to the hexads, we find that the pentad P^4 is contained in the hexads P^5, *pns*, $p^2s^2d^1\updownarrow$, and p^3m^3. Conversely, the hexad, P^5, contains the pentads P^4, *pns/s*, and *pmn/p*.

Proceeding again upwards, from the hexads to the heptads, we find that the hexad P^5 is a part of the three heptads P^6, C. *pns/s*, and C. *pmn/p*. Conversely, the heptad P^6 contains the hexads P^5, *pns*, $n^2s^2p^1\updownarrow$, and p^2/m.

Proceeding now laterally and to the right, from the heptads to the octads, we find that the heptad P^6 is a part of the octads

P^7, C. p^2s^2, and C. p/n. Conversely, the octad P^7 contains the heptads P^6, C. pns/s C. $\uparrow p^2n^2\downarrow$, and C. p^3d^2.

Proceeding upward, from the octads to the nonads, we find that the octad P^7 is found in the nonads P^8 (C. p^2s) and C. pns. Conversely, the nonad (P^8) contains the octads P^7, C. p^2s^2, C. $p^2m^1\updownarrow$, C. $p^2d^1\updownarrow$, and C. p^2d^2.

Finally, proceeding laterally, from the nonads to the decads, we find that the nonad P^8 is contained in the decads C. p, and C. s. Conversely, the decad C. p (or P^9) contains the nonads P^8 (C.p^2s), C. pns, C. pmn, C. pmd, and C. pdt.

The arrows on the chart which indicate the progression from the intervals to the triads, from the triads to the tetrads, the tetrads to the pentads, and so forth, may be helpful in tracing various "paths" of tonal relationship.

As the student examines the analyses of the various sonorities or scales, he will find that they differ in complexity. The analysis of the triads is simple. The analysis of the tetrads is comparatively simple, but there are several forms that have at least two possible analyses. The second tetrad, p^2 s^2, for example, may be analyzed as the simultaneous projection of two perfect fifths and two major seconds (p^2s^2); or as the projection of a perfect fifth above and below an axis tone, together with the projection of a minor third above *or* below the same axis ($p^2n^1\updownarrow$); or, again, as the projection of a major second above and below an axis tone, together with the projection of a perfect fifth above or below the same axis (s^2p^1). The tetrad p @ n may also be analyzed as n @ p, since the result is the same. The basic tetrad of the tritone–perfect-fifth projection may also be analyzed as p @ t, and so forth.

The pentads have several members which have a double analysis, as indicated on the chart. The hexads are more complicated, some of them having three or more valid analyses. There are still other possible analyses which have not been specifically indicated, since their inclusion would add nothing of vital importance.

One curiosity might be noted. In Chapter 48 the subject of a "diagonal" relationship was discussed in the case of the isomeric "twins" and "quartets" among the hexads. The chart makes this relationship visually clear. The twins and quartets are indicated by brackets. Now if we examine the position of the crosses indicating the doads, triads, tetrads, octads, nonads, and decads we find that the upper half of the chart is an exact mirror of the lower part of the chart. In the case of the pentads and heptads, the upper half of the chart is a mirror of the lower *except* where the connecting hexad is a member of the "twin" or "quartet" relationship, *where the order is exactly reversed.* In the vertical column at the extreme left of the chart, the three crosses indicating pentads one, two, and three are mirrored above by the heptads one, two, and three, in ascending order. In the second column from the left the crosses marking pentads, one, two, four, fourteen, fifteen, and twenty are mirrored by heptads in the same ascending order. The third and fourth columns, however, are connected with their corresponding heptads by the isomeric hexad "quartets." Here it will be seen that the *third* column of pentads is "mirrored" in the *fourth* column of heptads, and, conversely, the *fourth* column of pentads is mirrored in the *third* column of heptads. This same diagonal relationship will be observed wherever the twins and quartets occur, although there are four cases where there is a "double diagonal," that is, where one pentad and one heptad are related to *both* members of a quartet family.

As far as the *order* of presentation of the sonorities is concerned, I have tried to make the presentation as logical as possible. The hexads, for example, are arranged in seven groups. In the first of these, the perfect fifth predominates or, as in the case of the second hexad, has equal strength with its concomitant major second. In the second group the minor second predominates, except in the case of the second of the series where the minor second has equal strength with its concomitant major second. In group three the major second predominates, or has

equal strength with the major third and tritone. In group four the minor third predominates with its concomitant tritone. In group five the major third predominates throughout. In group six the tritone predominates, or has equal valency with the perfect fifth and/or the minor second. In the last group no interval dominates the sonority, since in all of them four of the six intervals have equal representation.

This grouping is indicated by the dotted vertical lines and the solid "stair-steps" which should make the chart more easily readable.

50

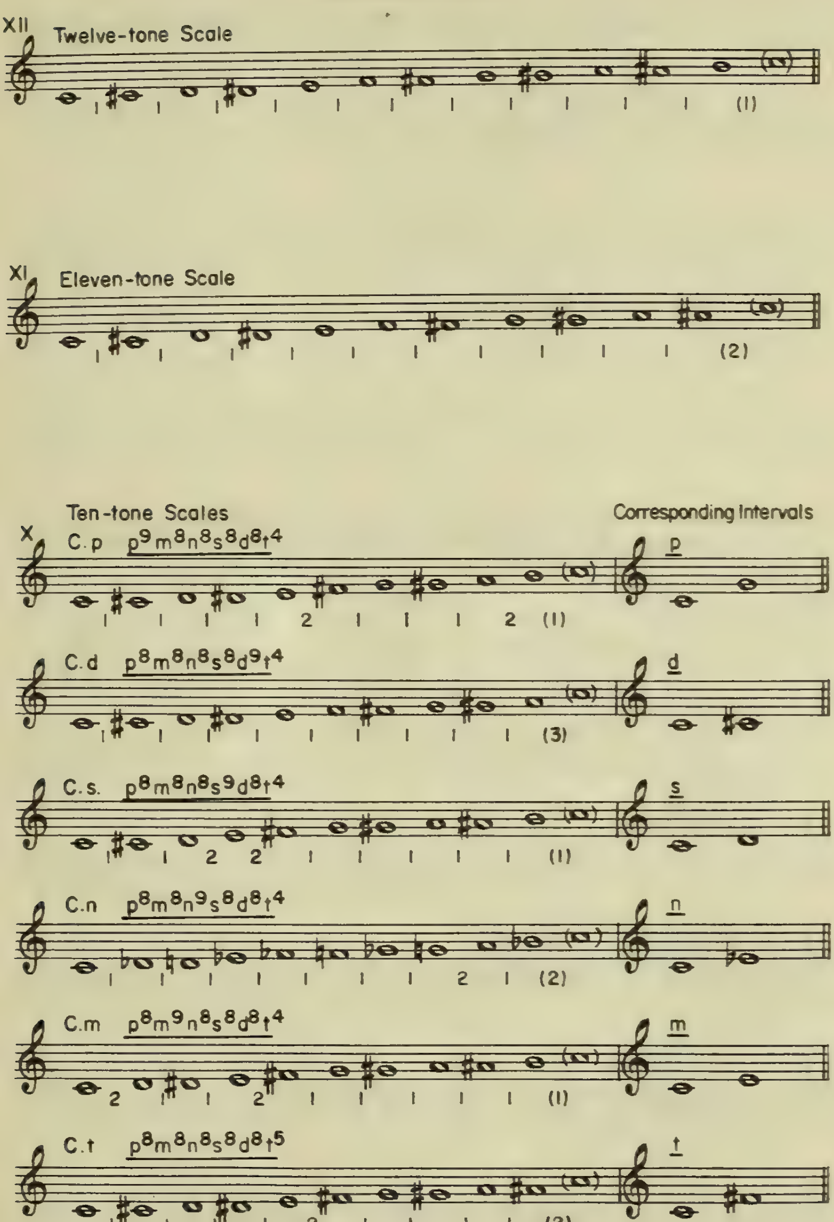
Translation of Symbolism into Sound

FOR THOSE COMPOSERS who have difficulty in grasping completely the symbolism of the preceding chapter, I am attempting here to translate the chart of the relationship of sonorities and scales in equal temperament back again to the symbolism of musical notation. It should be stated again that this translation cannot possibly be completely satisfactory. A nine-tone scale, for example, will have nine different versions. If the scale has an involution, that involution will also have nine positions. Each of these eighteen scales may be formed on any of the twelve tones of the chromatic scale. Therefore, in the cases of such nine-tone scales, one symbol represents 216 different scales in musical notation, although only one *scale form*.

The musical translation of the chart can therefore give only one translation of the many translations possible and must be so interpreted.

Example 50-1 begins with the twelve-tone and the eleven-tone scale, each of which is actually only one scale form, and then proceeds to the six ten-tone scales. Each of these scales, as we have seen, corresponds to a two-tone interval. The ten-tone scale C, *p* is presented with the interval *p* of which it is the projection. The ten-tone scale C, *d* is presented with its corresponding interval *d*, and so forth. The order of presentation will be seen to conform with the order of presentation in the chart. Since all of the scales are isometric, no involutions are given.

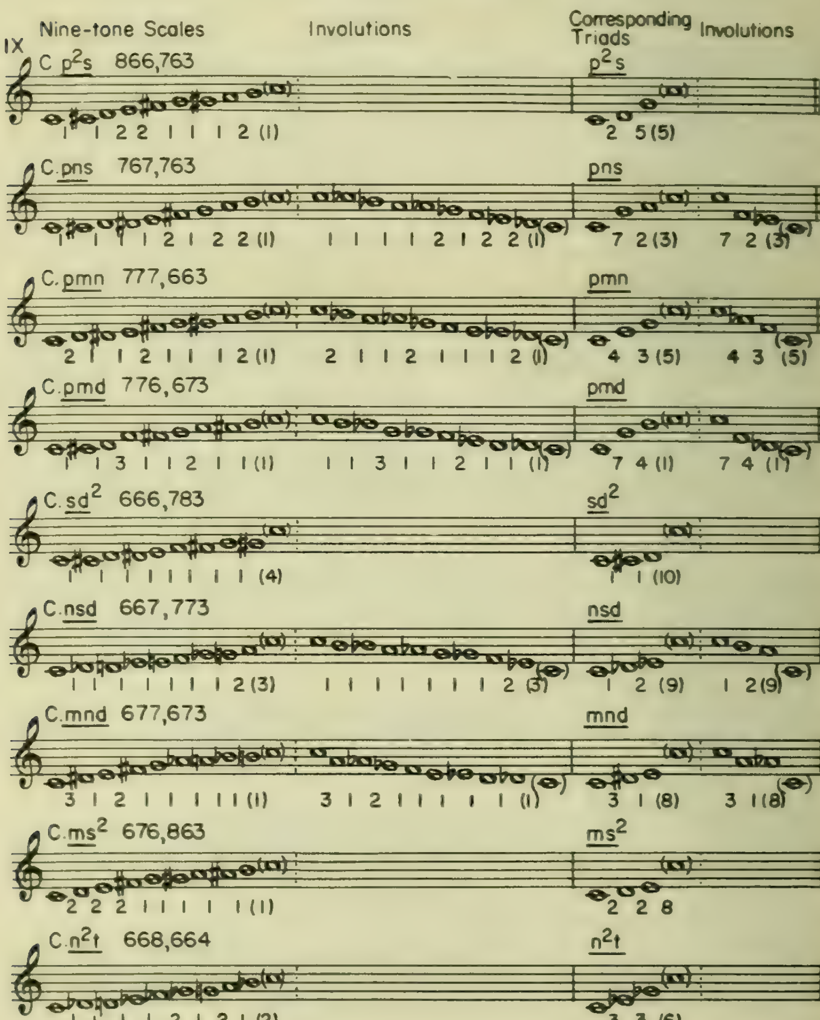
EXAMPLE 50-1

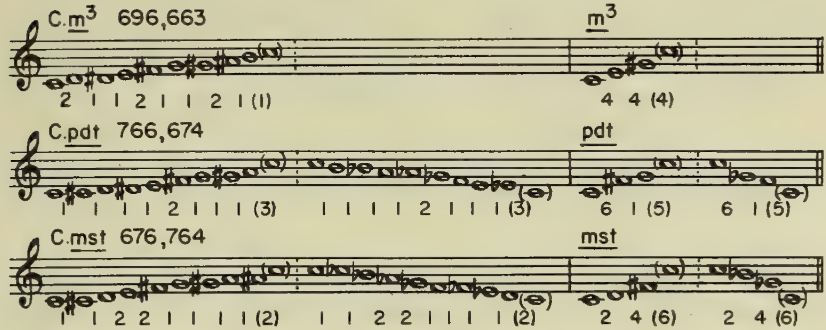

XII
Twelve-tone Scale
XI
Eleven-tone Scale
Ten-tone Scales
Corresponding Intervals
X
C.p $p^9m^8n^8s^8d^8t^4$
p
C.d $p^8m^8n^8s^8d^9t^4$
d
C.s. $p^8m^8n^8s^9d^8t^4$
s
C.n $p^8m^8n^9s^8d^8t^4$
n
C.m $p^8m^9n^8s^8d^8t^4$
m
C.t $p^8m^8n^8s^8d^8t^5$
t

Example 50-2 gives the nine-tone scales with their involutions, where they exist, and with the corresponding triads, of which they are projections, and the triad involution, if any.

The order of presentation is, again, the same as that of the chart for ready comparison,

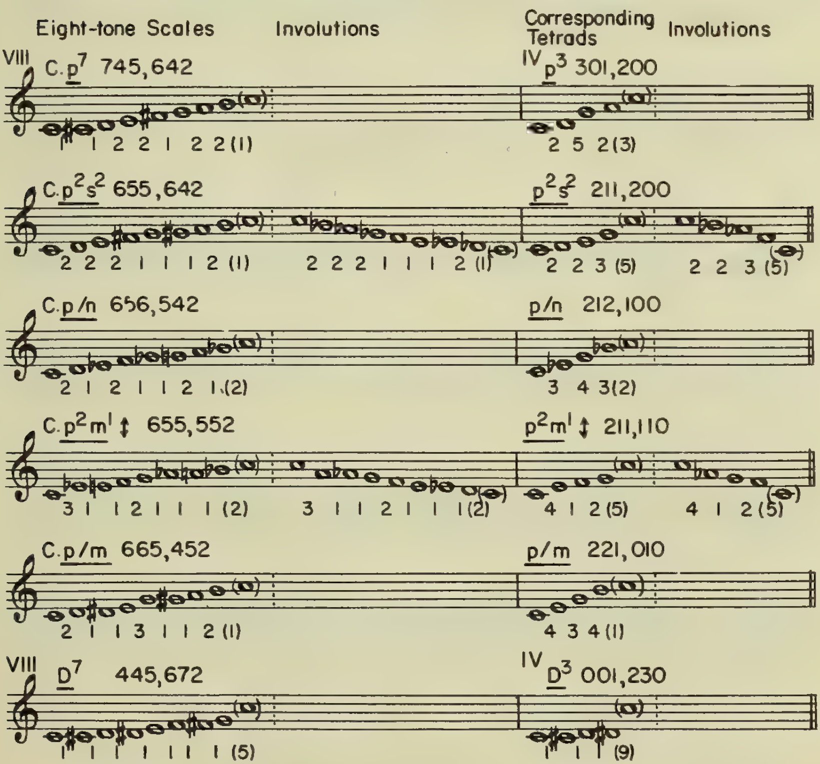
EXAMPLE 50-2

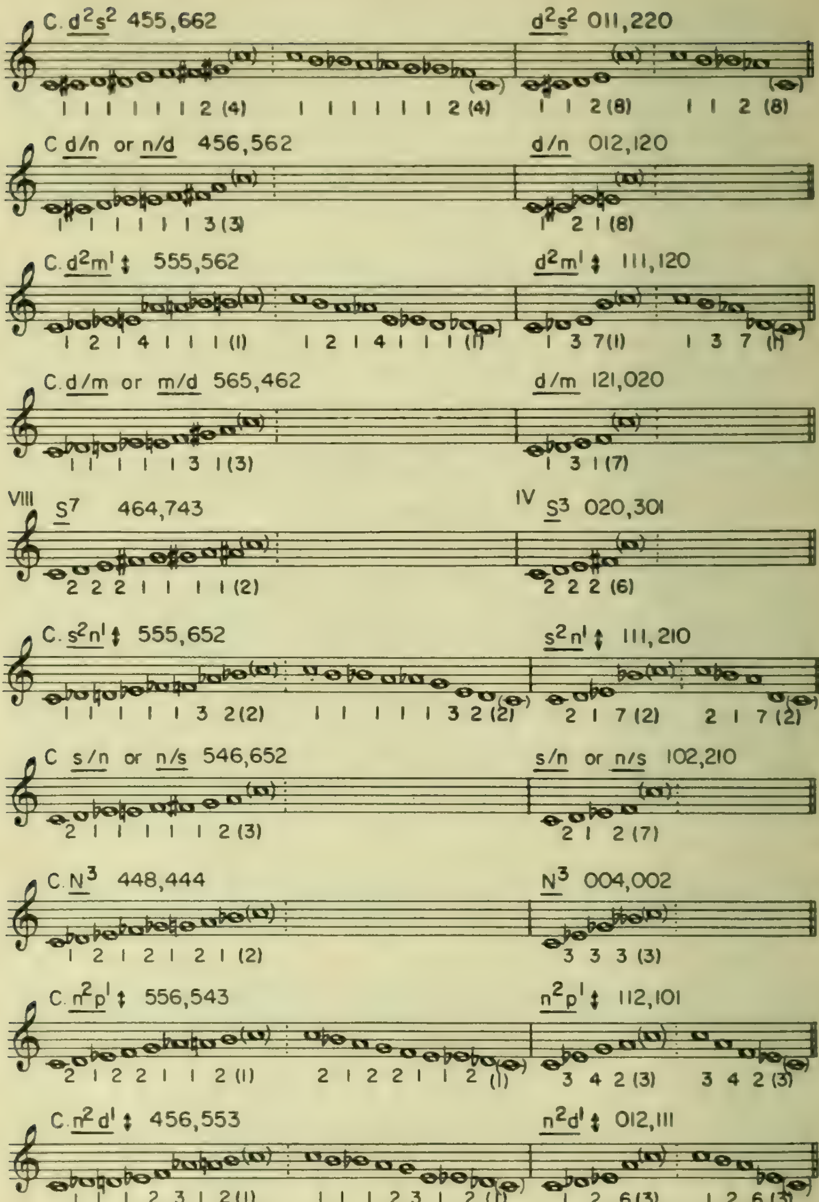


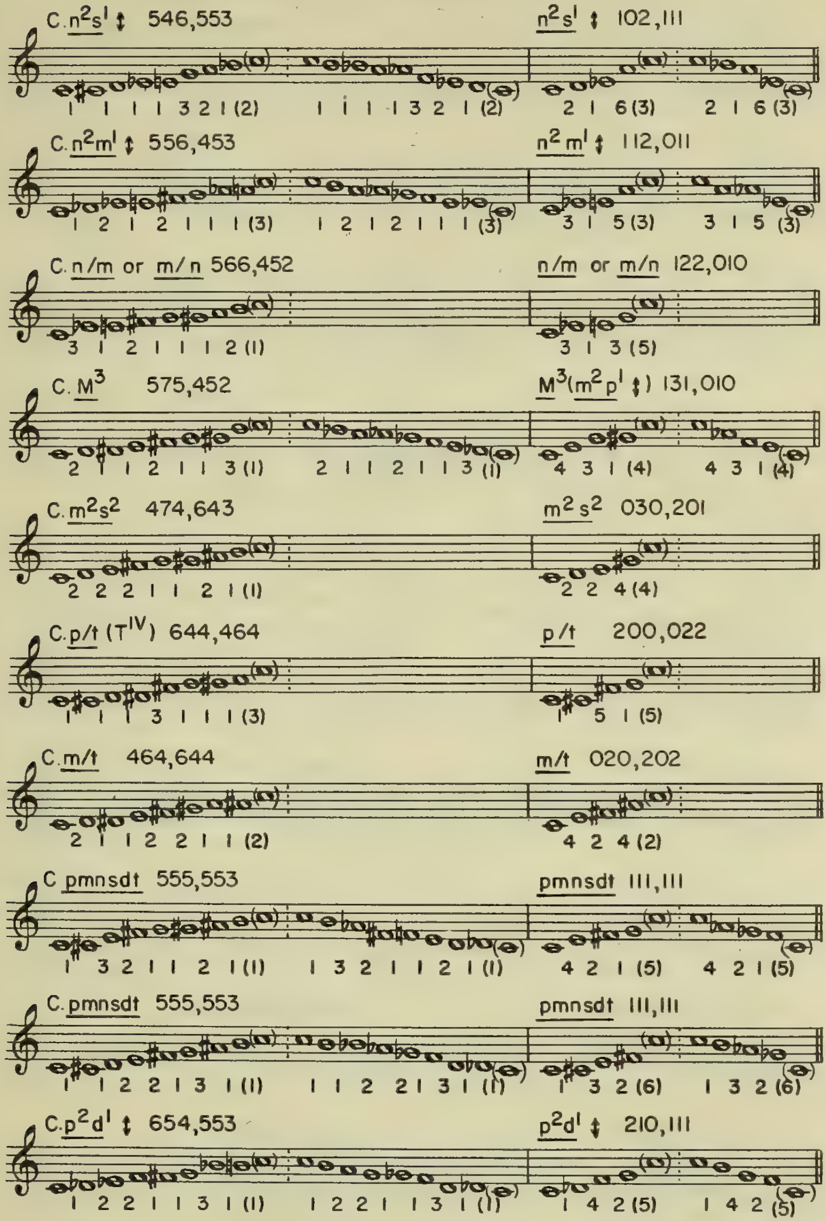


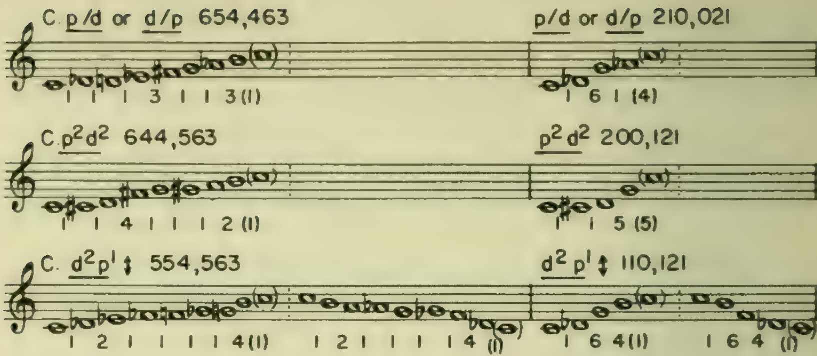
Example 50-3 gives the octads with their corresponding tetrads in the same order as that of the chart.

EXAMPLE 50-3



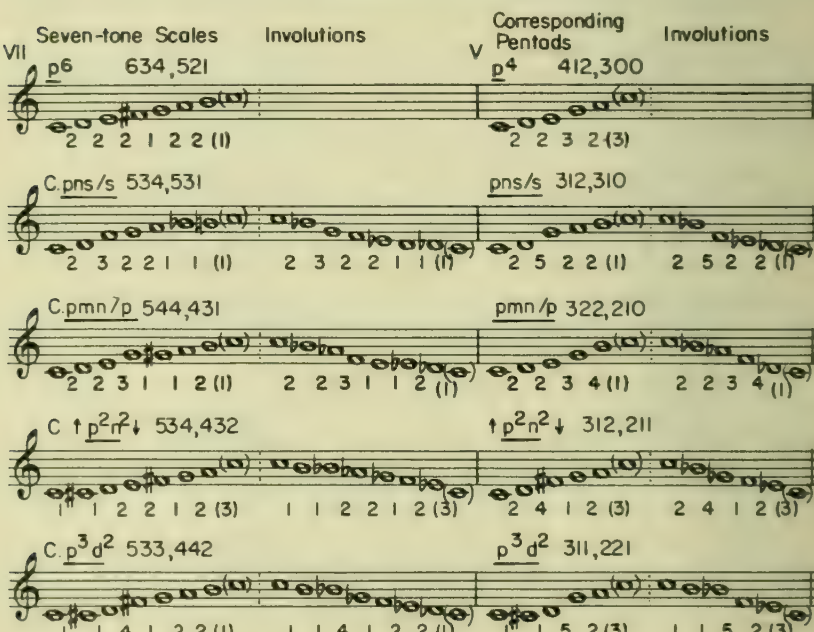

C. d2s2 455,662
1 1 1 1 1 1 2 (4)
1 1 1 1 1 1 2 (4)
d2s2 011,220
1 1 2 (8)
1 1 2 (8)
C d/n or n/d 456,562
1 1 1 1 1 1 3 (3)
d/n 012,120
1 2 1 (8)
C. d2m1 555,562
1 2 1 4 1 1 1 (1)
1 2 1 4 1 1 1 (1)
d2m1 111,120
1 3 7 (1)
1 3 7 (1)
C. d/m or m/d 565,462
1 1 1 1 1 3 1 (3)
d/m 121,020
1 3 1 (7)
VIII S7 464,743
2 2 2 1 1 1 1 (2)
IV S3 020,301
2 2 2 (6)
C. s2n1 555,652
1 1 1 1 1 3 2 (2)
1 1 1 1 1 3 2 (2)
s2n1 111,210
2 1 7 (2)
2 1 7 (2)
C s/n or n/s 546,652
2 1 1 1 1 1 2 (3)
s/n or n/s 102,210
2 1 2 (7)
C. N3 448,444
1 2 1 2 1 2 1 (2)
N3 004,002
3 3 3 (3)
C. n2p1 556,543
2 1 2 2 1 1 2 (1)
2 1 2 2 1 1 2 (1)
n2p1 112,101
3 4 2 (3)
3 4 2 (3)
C. n2d1 456,553
1 1 1 2 3 1 2 (1)
1 1 1 2 3 1 2 (1)
n2d1 012,111
1 2 6 (3)
1 2 6 (3)


C. n2s1 ↕ 546,553
1 1 1 1 3 2 1 (2) 1 1 1 1 3 2 1 (2)
n2s1 ↕ 102,111
2 1 6 (3) 2 1 6 (3)
C. n2m1 ↕ 556,453
1 2 1 2 1 1 1 (3) 1 2 1 2 1 1 1 (3)
n2m1 ↕ 112,011
3 1 5 (3) 3 1 5 (3)
C. n/m or m/n 566,452
3 1 2 1 1 1 2 (1)
n/m or m/n 122,010
3 1 3 (5)
C. M3 575,452
2 1 1 2 1 1 3 (1) 2 1 1 2 1 1 3 (1)
M3(m2p1 ↕) 131,010
4 3 1 (4) 4 3 1 (4)
C. m2s2 474,643
2 2 2 1 1 2 1 (1)
m2s2 030,201
2 2 4 (4)
C. p/t (TIV) 644,464
1 1 1 3 1 1 1 (3)
p/t 200,022
1 5 1 (5)
C. m/t 464,644
2 1 1 2 2 1 1 (2)
m/t 020,202
4 2 4 (2)
C pmnsdt 555,553
1 3 2 1 1 2 1 (1) 1 3 2 1 1 2 1 (1)
pmnsdt 111,111
4 2 1 (5) 4 2 1 (5)
C. pmnsdt 555,553
1 1 2 2 1 3 1 (1) 1 1 2 2 1 3 1 (1)
pmnsdt 111,111
1 3 2 (6) 1 3 2 (6)
C. p2d1 ↕ 654,553
1 2 2 1 1 3 1 (1) 1 2 2 1 1 3 1 (1)
p2d1 ↕ 210,111
1 4 2 (5) 1 4 2 (5)

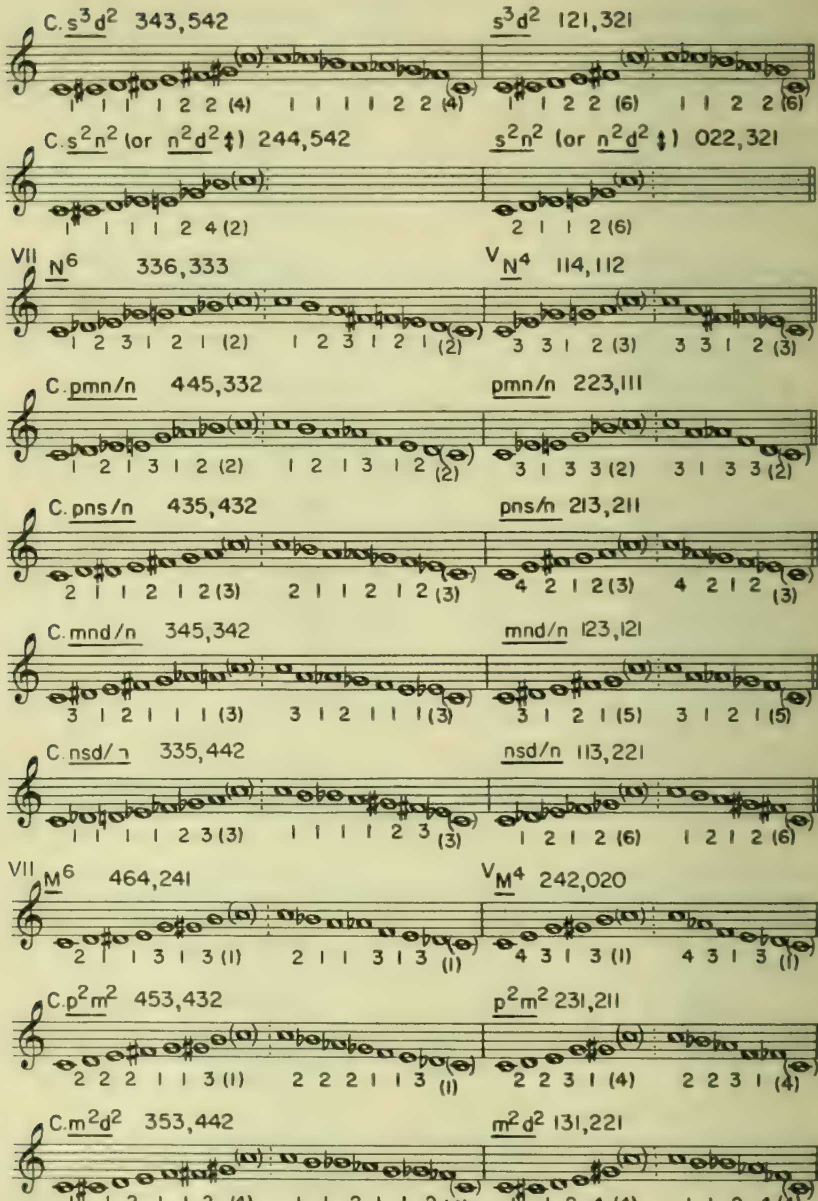


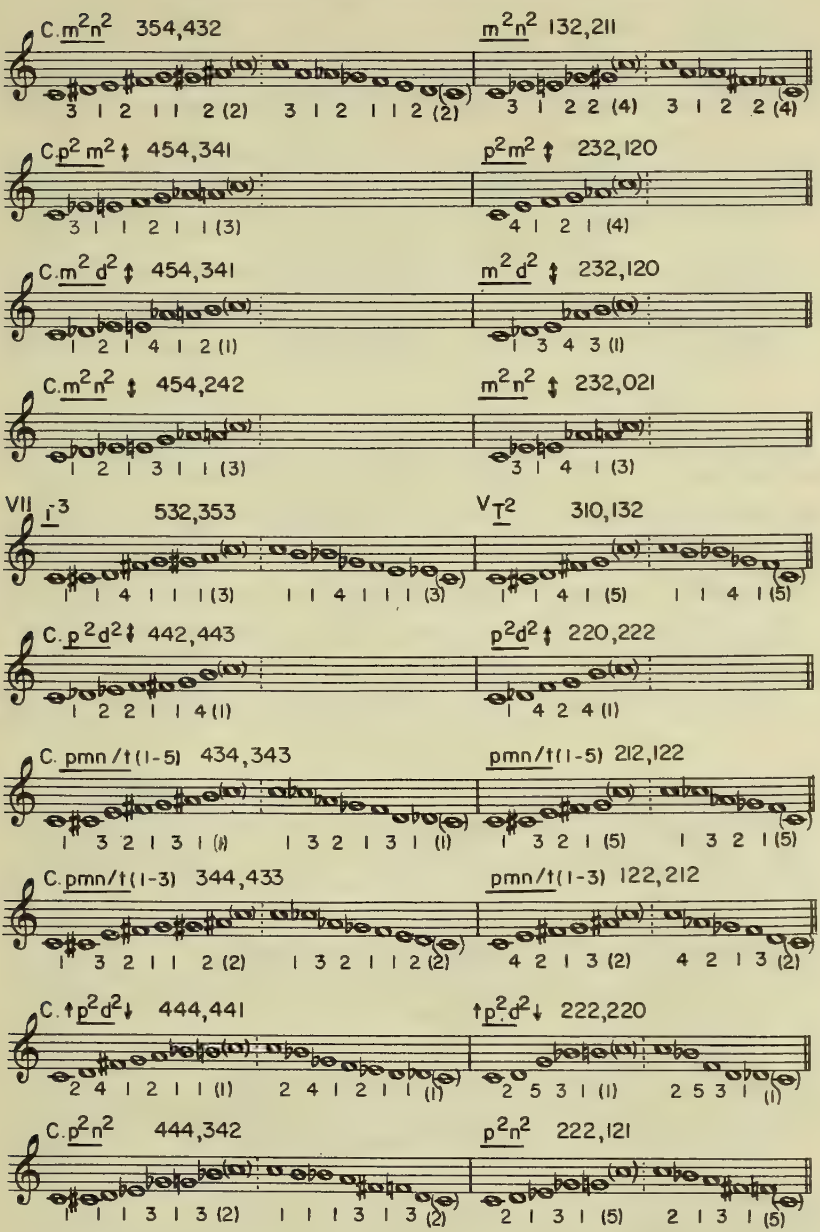
Example 50-4, in like manner, shows the relation of the heptads to their corresponding pentads and involutions.

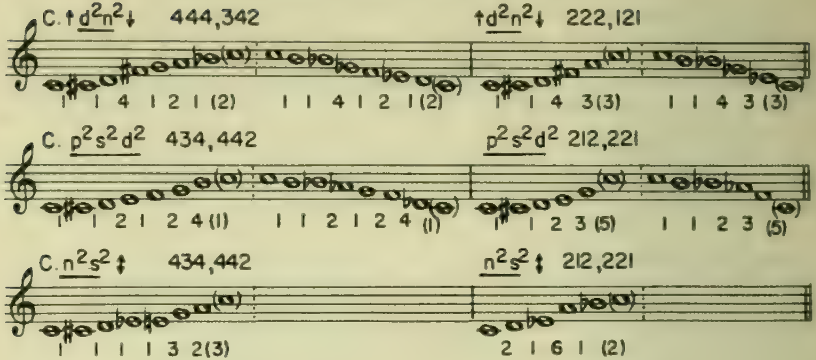
Example 50-4




C.pmd/p 543,342
2 3 1 1 3 1 (1) 2 3 1 1 3 1 (1)
pmd/p 321,121
2 4 1 4 (1) 2 4 1 4 (1)
VII
D^6 234,561
1 1 1 1 1 1 (6)
V
D^4 012,340
1 1 1 1 (8)
C.nsd/s 334,551
1 1 1 2 5 1 (1) 1 1 1 2 5 1 (1)
nsd/s 112,330
1 1 1 2 (7) 1 1 1 2 (7)
C.mnd/d 344,451
1 1 1 1 1 4 (3) 1 1 1 1 1 4 (3)
mnd/d 122,230
1 2 1 1 (7) 1 2 1 1 (7)
C.d^2n^2 334,452
1 1 1 1 2 1 (5) 1 1 1 1 2 1 (5)
d^2n^2 112,231
1 1 1 3 (6) 1 1 1 3 (6)
C.d^3p^2 433,452
1 1 1 2 1 1 (5) 1 1 1 2 1 1 (5)
d^3p^2 211,231
1 1 1 4 (5) 1 1 1 4 (5)
C.pmd/d 443,352
1 1 4 1 3 1 (1) 1 1 4 1 3 1 (1)
pmd/d 221,131
6 1 3 1 (1) 6 1 3 1 (1)
VII
S^6 262,623
2 2 2 1 1 2 (2)
V
S^4 040,402
2 2 2 2 (4)
C. ↑s^2n^2↓ (or p^2n^2↕) 444,522
2 2 2 1 2 1 (2)
↑s^2n^2↓ or p^2n^2↕) 222,301
2 2 2 3 (3)
C.s^3p^2 443,532
2 2 2 1 1 1 (3) 2 2 2 1 1 1 (3)
s^3p^2 221,311
2 2 2 1 (5) 2 2 2 1 (5)

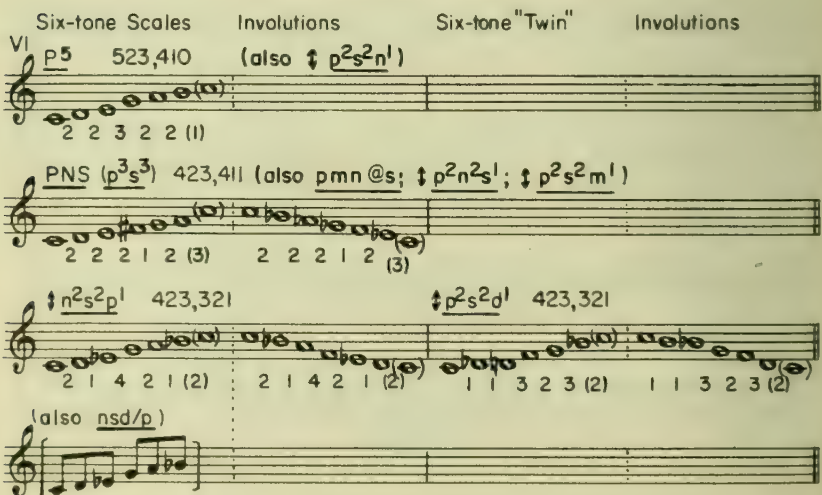

C. s³d² 343,542
s³d² 121,321
C. s²n² (or n²d²♮) 244,542
s²n² (or n²d²♮) 022,321
VII N⁶ 336,333
V N⁴ 114,112
C. pmn/n 445,332
pmn/n 223,111
C. pns/n 435,432
pns/n 213,211
C. mnd/n 345,342
mnd/n 123,121
C. nsd/n 335,442
nsd/n 113,221
VII M⁶ 464,241
V M⁴ 242,020
C. p²m² 453,432
p²m² 231,211
C. m²d² 353,442
m²d² 131,221

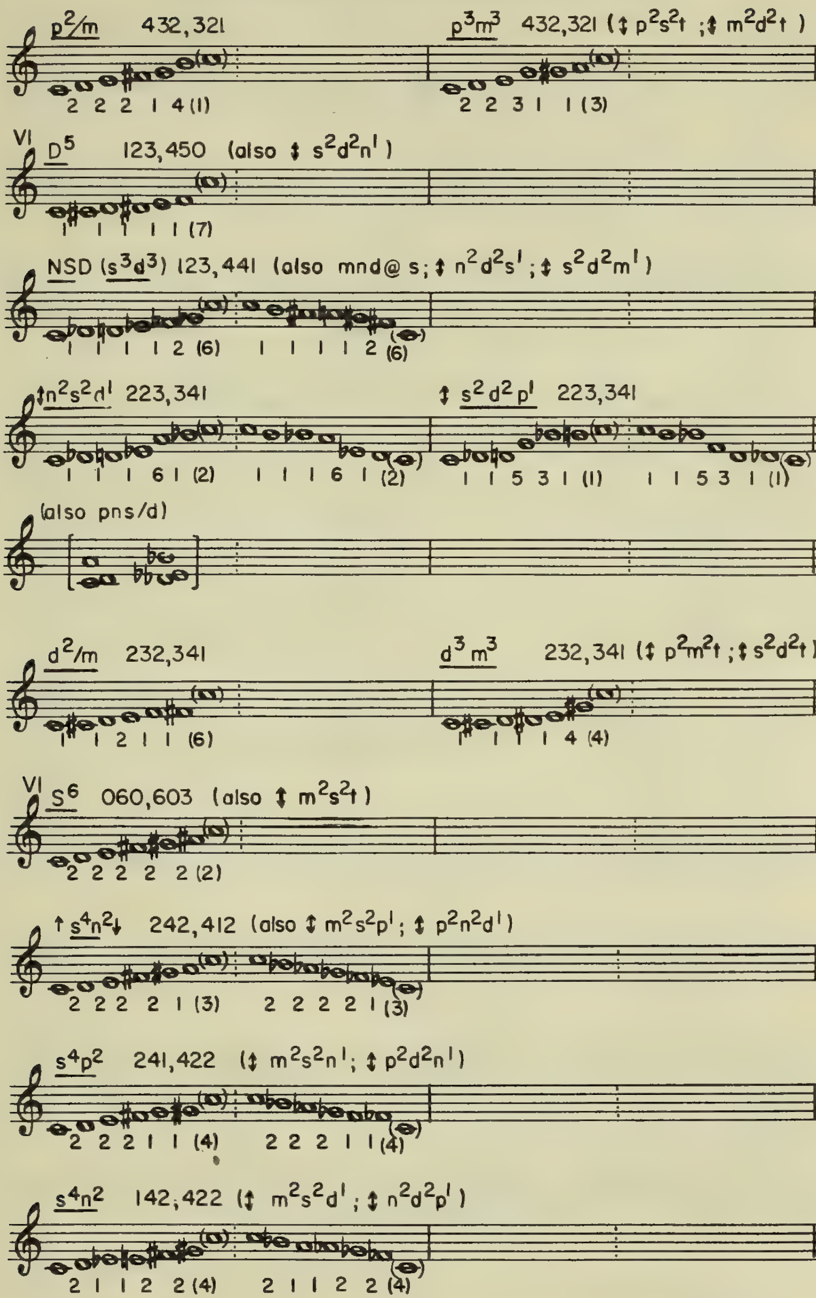

C.m2n2 354,432
m2n2 132,211
3 1 2 1 1 2 (2) 3 1 2 1 1 2 (2)
3 1 2 2 (4) 3 1 2 2 (4)
C.p2 m2 454,341
p2m2 232,120
3 1 1 2 1 1 (3)
4 1 2 1 (4)
C.m2 d2 454,341
m2 d2 232,120
1 2 1 4 1 2 (1)
1 3 4 3 (1)
C.m2n2 454,242
m2n2 232,021
1 2 1 3 1 1 (3)
3 1 4 1 (3)
VII I-3 532,353
VT2 310,132
1 1 4 1 1 1 (3) 1 1 4 1 1 1 (3)
1 1 4 1 (5) 1 1 4 1 (5)
C. p2d2 442,443
p2d2 220,222
1 2 2 1 1 4 (1)
1 4 2 4 (1)
C. pmn/t(1-5) 434,343
pmn/t(1-5) 212,122
1 3 2 1 3 1 (1) 1 3 2 1 3 1 (1)
1 3 2 1 (5) 1 3 2 1 (5)
C.pmn/t(1-3) 344,433
pmn/t(1-3) 122,212
1 3 2 1 1 2 (2) 1 3 2 1 1 2 (2)
4 2 1 3 (2) 4 2 1 3 (2)
C. p2d2 444,441
p2d2 222,220
2 4 1 2 1 1 (1) 2 4 1 2 1 1 (1)
2 5 3 1 (1) 2 5 3 1 (1)
C.p2n2 444,342
p2n2 222,121
1 1 1 3 1 3 (2) 1 1 1 3 1 3 (2)
2 1 3 1 (5) 2 1 3 1 (5)

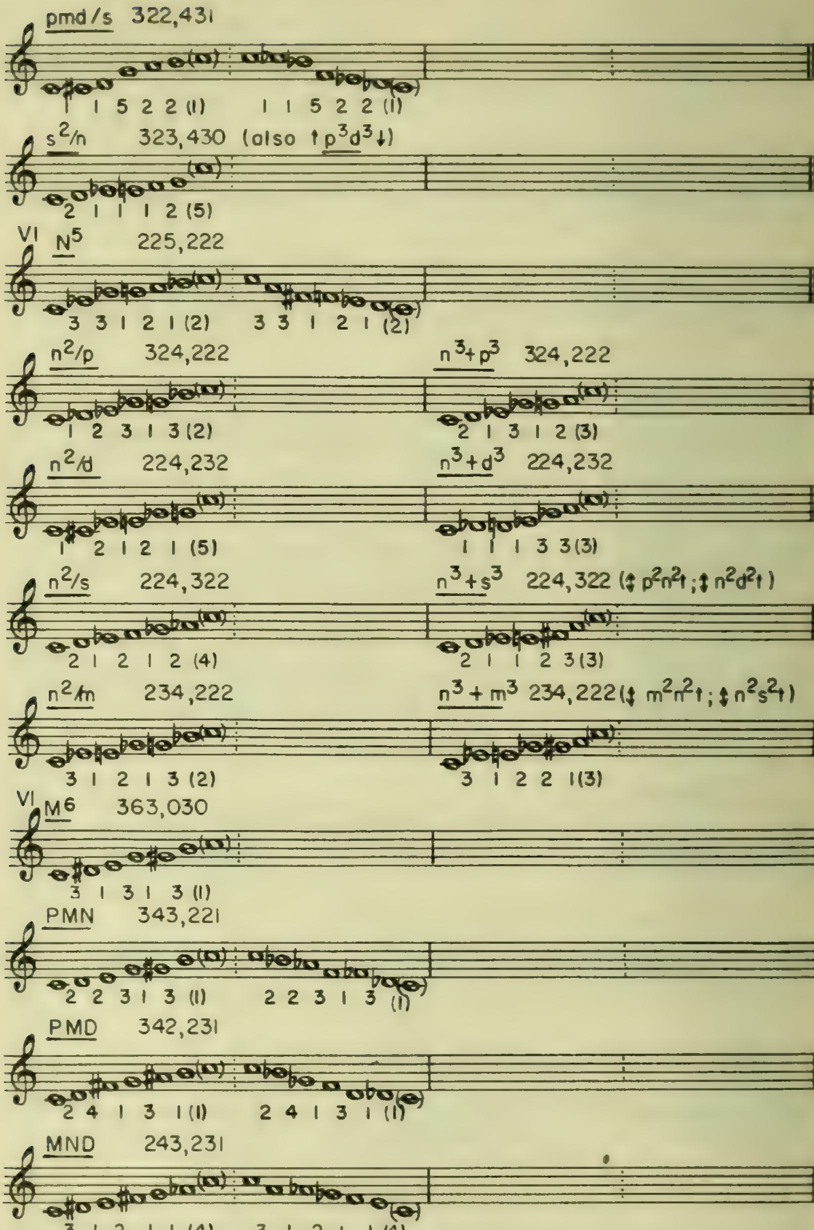


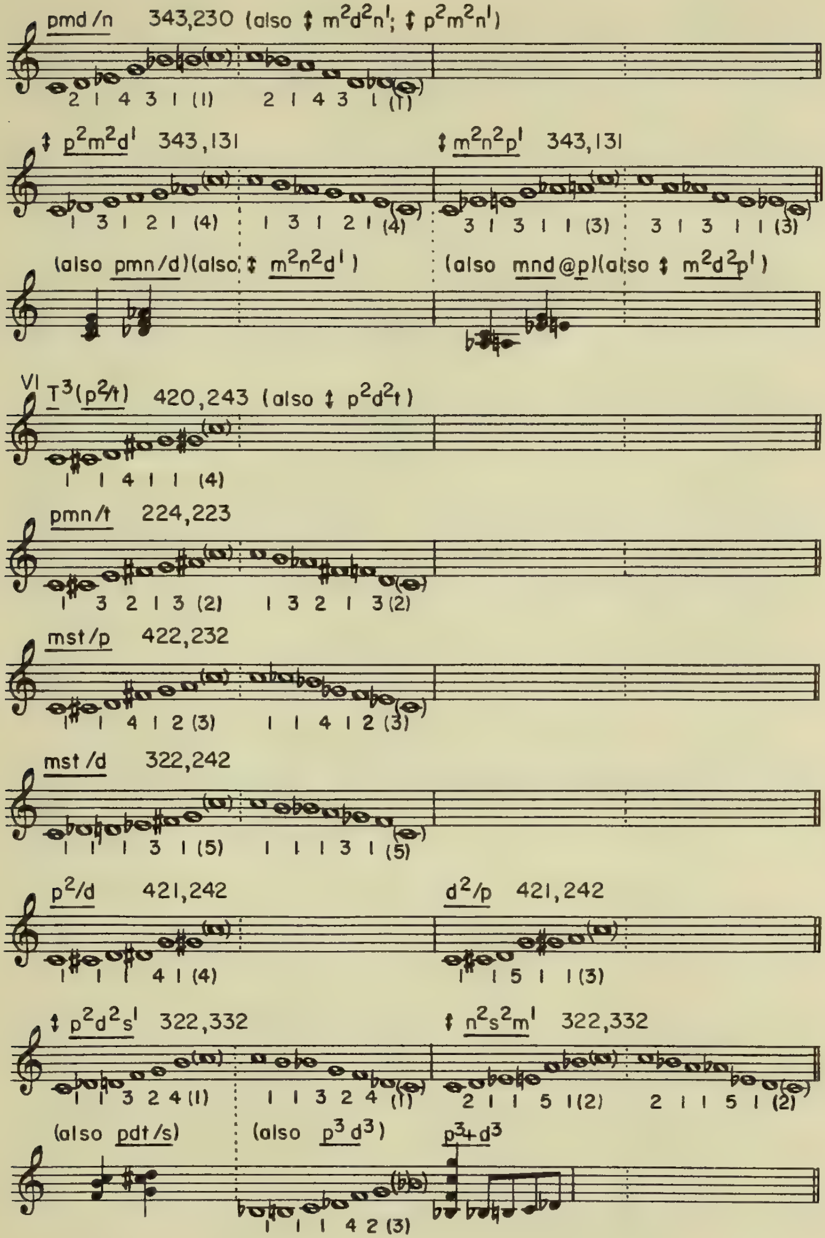
Finally Example 50-5 presents the six-tone scales with their involutions. In most cases, as we have already seen, the involution of the hexad is also its complementary scale. In the cases of the isomeric "twins," the complementary scale is given in the third part of the line. Where the original scale is a part of a "quartet," the scale is given with its involution, followed by the complementary scale, followed in turn by its involution.

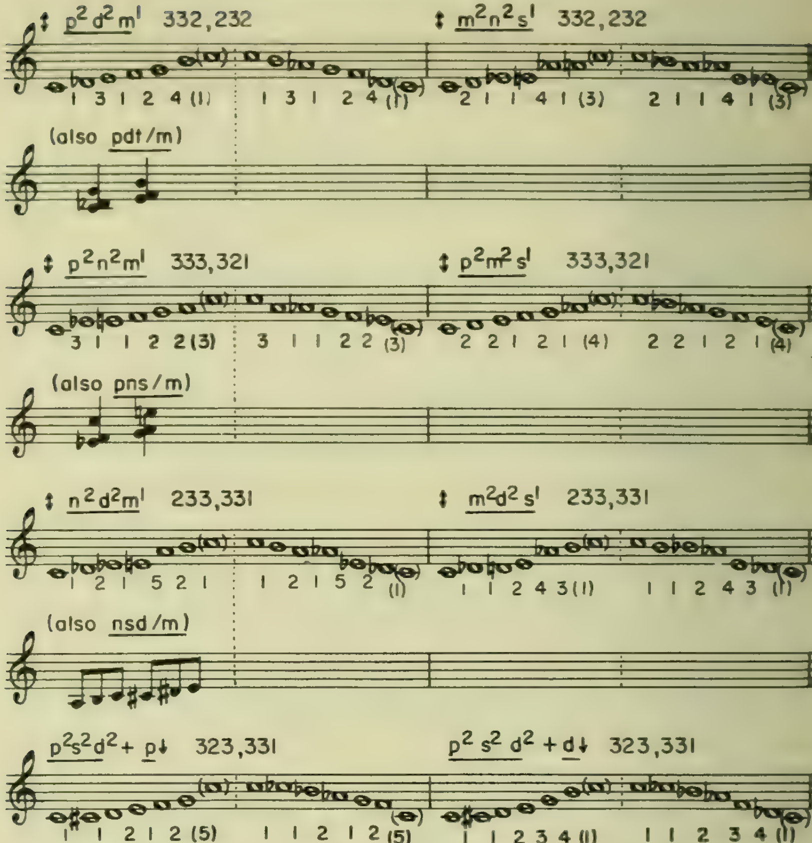
Example 50-5




p^2/m 432,321
2 2 2 1 4 (1)
p^3m^3 432,321 (↕ p^2s^2t ; ↕ m^2d^2t)
2 2 3 1 1 (3)
VI
D^5 123,450 (also ↕ $s^2d^2n^1$)
1 1 1 1 1 (7)
NSD (s^3d^3) 123,441 (also mnd@ s; ↕ $n^2d^2s^1$; ↕ $s^2d^2m^1$)
1 1 1 1 2 (6) 1 1 1 1 2 (6)
↕ $n^2s^2d^1$ 223,341
1 1 1 6 1 (2) 1 1 1 6 1 (2)
↕ $s^2d^2p^1$ 223,341
1 1 5 3 1 (1) 1 1 5 3 1 (1)
(also pns/d)
d^2/m 232,341
1 1 2 1 1 (6)
d^3m^3 232,341 (↕ p^2m^2t ; ↕ s^2d^2t)
1 1 1 1 4 (4)
VI
S^6 060,603 (also ↕ m^2s^2t)
2 2 2 2 2 (2)
↑ s^4n^2 ↓ 242,412 (also ↕ $m^2s^2p^1$; ↕ $p^2n^2d^1$)
2 2 2 2 1 (3) 2 2 2 2 1 (3)
s^4p^2 241,422 (↕ $m^2s^2n^1$; ↕ $p^2d^2n^1$)
2 2 2 1 1 (4) 2 2 2 1 1 (4)
s^4n^2 142,422 (↕ $m^2s^2d^1$; ↕ $n^2d^2p^1$)
2 1 1 2 2 (4) 2 1 1 2 2 (4)


pmd/s 322,431
1 1 5 2 2 (1) 1 1 5 2 2 (1)
s²/n 323,430 (also ↑p³d³↓)
2 1 1 1 2 (5)
VI N⁵ 225,222
3 3 1 2 1 (2) 3 3 1 2 1 (2)
n²/p 324,222
1 2 3 1 3 (2)
n³+p³ 324,222
2 1 3 1 2 (3)
n²/d 224,232
1 2 1 2 1 (5)
n³+d³ 224,232
1 1 1 3 3 (3)
n²/s 224,322
2 1 2 1 2 (4)
n³+s³ 224,322 (↕ p²n²↑; ↕ n²d²↑)
2 1 1 2 3 (3)
n²/m 234,222
3 1 2 1 3 (2)
n³ + m³ 234,222 (↕ m²n²↑; ↕ n²s²↑)
3 1 2 2 1 (3)
VI M⁶ 363,030
3 1 3 1 3 (1)
PMN 343,221
2 2 3 1 3 (1) 2 2 3 1 3 (1)
PMD 342,231
2 4 1 3 1 (1) 2 4 1 3 1 (1)
MND 243,231
3 1 2 1 1 (4) 3 1 2 1 1 (4)


pmd/n 343,230 (also ‡ m²d²n¹; ‡ p²m²n¹)
2 1 4 3 1 (1) 2 1 4 3 1 (1)
‡ p²m²d¹ 343,131
1 3 1 2 1 (4) 1 3 1 2 1 (4)
‡ m²n²p¹ 343,131
3 1 3 1 1 (3) 3 1 3 1 1 (3)
(also pmn/d)(also ‡ m²n²d¹)
(also mnd@p)(also ‡ m²d²p¹)
VI T³(p²/t) 420,243 (also ‡ p²d²t)
1 1 4 1 1 (4)
pmn/t 224,223
1 3 2 1 3 (2) 1 3 2 1 3 (2)
mst/p 422,232
1 1 4 1 2 (3) 1 1 4 1 2 (3)
mst/d 322,242
1 1 1 3 1 (5) 1 1 1 3 1 (5)
p²/d 421,242
1 1 1 4 1 (4)
d²/p 421,242
1 1 5 1 1 (3)
‡ p²d²s¹ 322,332
1 1 3 2 4 (1) 1 1 3 2 4 (1)
‡ n²s²m¹ 322,332
2 1 1 5 1 (2) 2 1 1 5 1 (2)
(also pdt/s)
(also p³d³)
1 1 1 4 2 (3)
p³+d³



These relationships of tone will repay endless study and absorption, for within them lies all of the tonal material of occidental music, classic and modern, serious and popular. Within them lie infinite and subtle variations, from the most sensuously luxuriant sounds to those which are grimly ascetic; from the mildest of gentle sounds to the most savagely dissonant.

Each scale or sonority encloses and enfolds its own character. In parting, let us look at one combination of sounds which we have used before as an example, the tetrad p^2s^2 and its octad projection. It is a sweet and gentle sound used thousands of times by thousands of composers. It has, for me, a strong per-

sonal association as the opening sonority of the "Interlochen theme" from my "Romantic" symphony. You will find it and its octad projection on the second line of Example 50-3. Note that the tetrad has the sound of C-D-E-G. Notice that its octad is saturated with this pleasant sound, for the octad contains not only the tetrad C-D-E-G but also similar tetrads on D, D-E-F♯-A; on E, E-F♯-G♯-B, and on G, G-A-B-D. In the hands of an insensitive composer, it could become completely sentimental. In the hands of a genius, it could be transformed into a scale of surpassing beauty and tenderness.

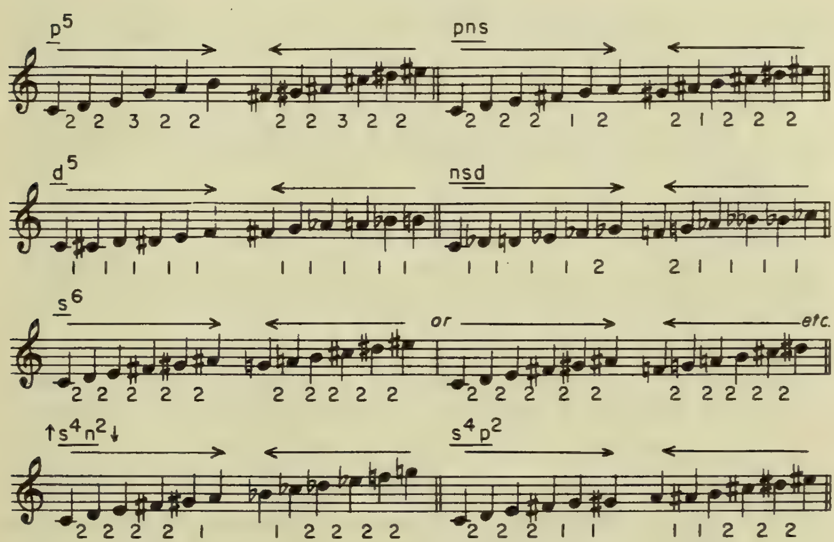
In conclusion, play for yourself gently and sensitively the opening four measures of Grieg's exquisitely beautiful song, "En Svane." Note the dissonance of the second chord as contrasted with the first. Then note again the return of the consonant triad followed by the increasingly dissonant sound, where the D♭ is substituted for the D. Listen to it carefully, for this is the mark of genius. It took only the change of one tone to transform the sound from its gentle pastoral quality to one of vague foreboding. But it had to be the *right* note! If this text is of any help in assisting the young composer to find the *right* note, the labor of writing it will not have been in vain.

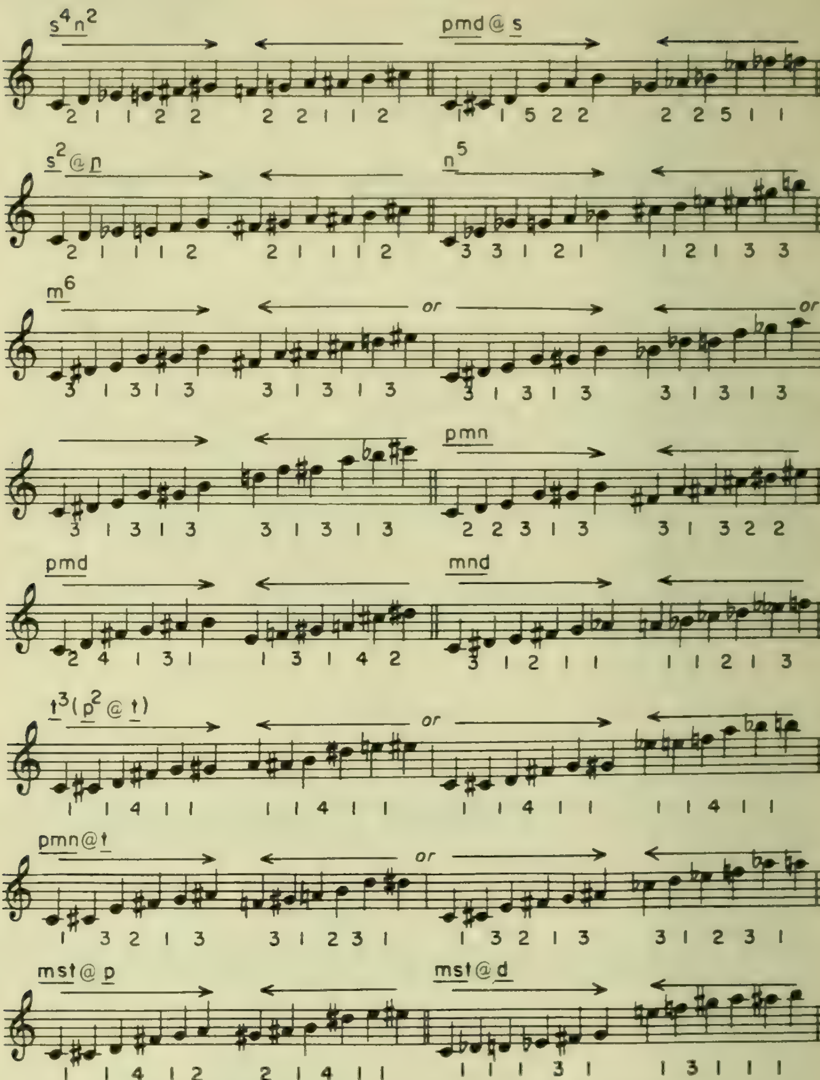
Appendix

Symmetrical Twelve-Tone Forms

For the composer who is interested in the type of "tone row" which uses all of the twelve tones of the chromatic scale without repetition, nineteen of the six-tone scales with their complementary involutions offer interesting possibilities for symmetrical arrangement. If we present these scales, as in Example 1, each followed by its complementary involution, we produce the following symmetrical twelve-tone scales:

Example 1





In any of the above scales, any series of consecutive tones from two to five will be found to be projected to its corresponding ten, nine, eight, or seven-tone scale. For example, in the first scale, p^5, not only are the twelve tones the logical projection of the original hexad but the first ten tones are the projection of the first two tones; the first nine tones will be seen to be the projection of the first three; the first eight tones are the

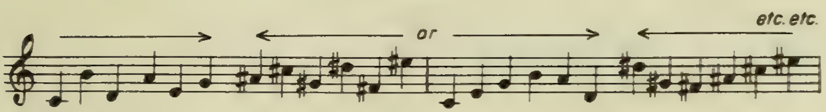
projection of the first four, and the first seven tones are the projection of the first five.

In other words, the seven-tone scale C-D-E-F♯G-A-B is the projection of C-D-E-G-A, the eight-tone scale C-D-E-F♯-G-G♯-A-B is the projection of C-D-E-G, and so forth, as illustrated in Example 2:

Example 2

It should be clear that the above relationship remains true *regardless of the order of tones* in the original hexad as long as the series is in the form of a six-tone scale—or sonority—with its complementary involution. For example, the scale of Example 2 might be rearranged as in Example 3:

Example 3



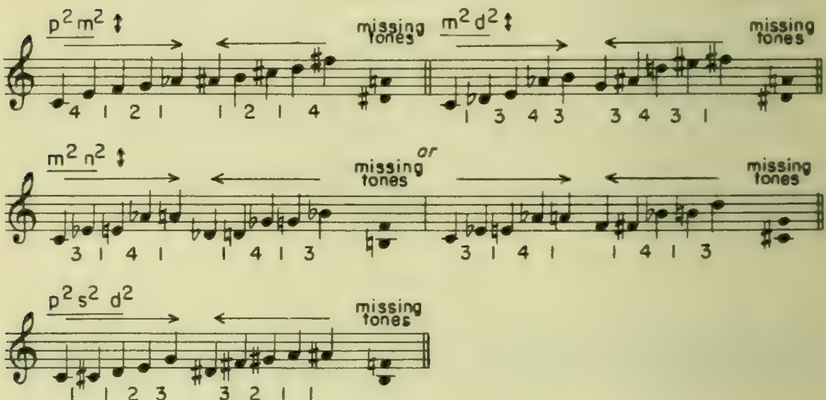
The method of determining the "converting tone"—that is, the tone on which we begin the descending complementary scale—was discussed in Chapter 40, pages 266 to 269. A quicker, although less systematic, method is by the "trial and error" process, that is, by testing all of the possibilities until the tone is found which, used as a starting point, will reproduce the same order of intervals downward without duplicating any of the original tones. Referring, again, to Example 1, p^5, it will be clear that E♯, or F, is the *only* tone from which we can project downward the intervals 22322 *without duplicating* any of the tones of the original hexad.

The hexad "twins" and "quartets" cannot be arranged in this manner for reasons previously explained. This is also true of the hexad *pmd* @ *n* which follows the general design of the

"quartets" although, unlike them, its complementary scale proves to be its own transposition at the interval of the tritone.

The nineteen hexads of Example 1 contain in their formation all of the triads, tetrads and pentads of the twelve-tone scale except the five pentads, $p^2m^2\updownarrow$, $m^2d^2\updownarrow$, $m^2n^2\updownarrow$, $p^2s^2d^2$, and $n^2s^2\updownarrow$, the last of which will be recognized as the "maverick" sonority of Chapter 47. The first four may be projected to a symmetrical ten-tone row as in Example 4:

Example 4



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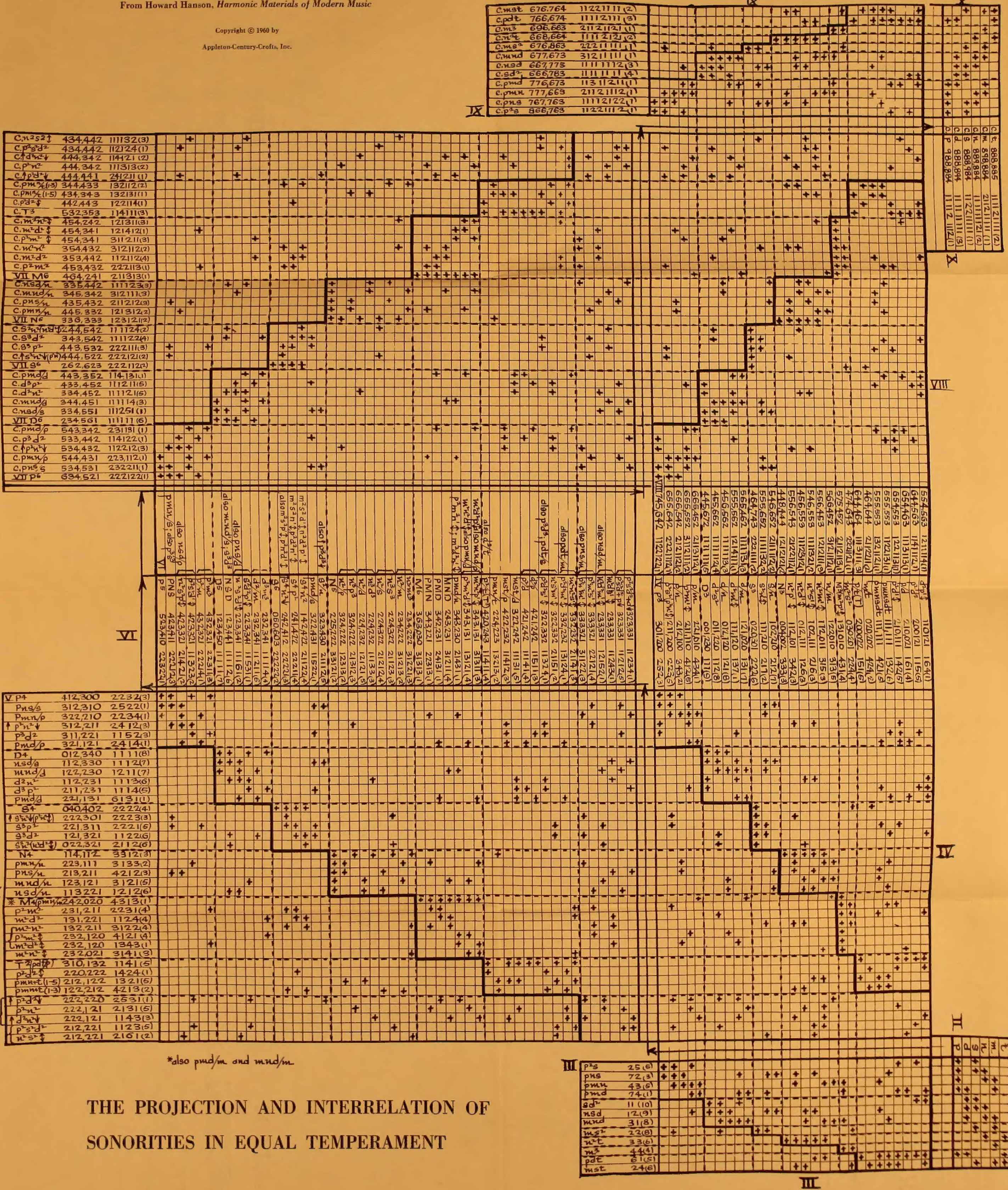

THE PROJECTION AND INTERRELATION OF SONORITIES IN EQUAL TEMPERAMENT